Turbulent Structure of a Flat Plate Boundary Layer Diffusion Flame*

(Production and Transfer Mechanism of a Turbulent Kinetic Energy)

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Turbulent structures of a diffusion flame formed in a flat plate boundary layer have been investigated experimentally. The flame is formed in the boundary layer with a 10 m/s free stream velocity, when a fuel (hydrogen/nitrogen mixture) is injected uniformly through a porous wall. Simultaneous measurements of two velocity components parallel and normal to a flat plate have been made, using a polarized four beam laser-Doppler velocimetry, and statistical properties up to third order cross correlations have been calculated. A mean density profile has also been obtained from temperature data measured by a thermocouple, and stable species concentration data measured by gas chromatography. From these data, statistical properties have been obtained, taking into account the effect of the mean density variation. Results obtained suggest that the turbulent production and transfer mechanism can be fundamentally identified with that of an isothermal shear flow even around the flame zone.

Key Words: Boundary Layer, Diffusion Flame, Turbulence, Turbulent Kinetic Energy.

1. Introduction

Boundary layer combustions over a solid wall have been widely investigated in connection with various applications, such as ablative cooling, erosive burning, surface combustion of liquid fuels, and fires^{(1)~(15)}. Turbulence-flame interactions in boundary layer combustions over a solid wall are especially important not only because of its practical importance, mentioned above, but also because a wall boundary layer is one of the typical turbulent flows^{(8)~(15)}.

The authors have measured velocity parallel to the flat plate^{(12)~(15)} and simultaneously, that normal to the flat plate⁽¹⁵⁾, using a laser-Doppler velocimetry. As a result, mean velocities and turbulent intensities of velocity fluctuation parallel^{(12)~(15)} and normal⁽¹⁵⁾ to the flat plate, a covariance⁽¹⁴⁾⁽¹⁵⁾, and third order correlations

of two velocity components⁽¹⁵⁾, have been obtained. These results show that the mean velocity profile has a local acceleration near the flame zone, and the profile of turbulent intensity of velocity fluctuation has two maxima and one minimum⁽¹²⁾. Furthermore, results show that profiles of the covariance and the mean velocity gradient are qualitatively similar except near the flame zone⁽¹⁴⁾. This shows that the turbulence can be treated as an "ordinary shear generated turbulence"⁽¹⁶⁾, except near the flame zone.

In recent years, discussion concerning the budget of turbulent kinetic energy in combustion fields has been conducted on the basis of experimental results⁽¹⁷⁾⁽¹⁸⁾. As is well known, it is impossible in the present stage to elucidate a full budget of turbulent kinetic energy in combustion fields. However, discussion on the budget has clarified some important turbulent structures, even though some terms in the budget were unknown.

In the present study, the budget of turbulent kinetic energy is estimated, based on the experimental results measured by the authors, and the turbulent structure is discussed. The discussion is mainly focused on the production and transfer mechanism of

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a turbulent kinetic energy around the flame zone, since the turbulent structure around the flame zone has not been elucidated at all.

Nomenclature

E	: predominant term
P	: pressure
U	: velocity in x -direction
u	: fluctuating velocity in x -direction
V	: velocity in y -direction
v	: fluctuating velocity in y -direction
W	: velocity in z -direction
w	: fluctuating velocity in z -direction
x	: coordinate defined in Figure 1
y	: coordinate defined in Figure 1
z	: coordinate normal to the x - y plane

Greek Symbols

δ	: small value
ε	: viscous dissipation
ρ	: density

Superscript

—	: mean value
'	: turbulent intensity

Subscript

∞	: free stream
A_d	: advection
D_i	: diffusion
P_r	: production

2. Experimental Apparatus

2.1 Flow field

A schematic illustration of the flow field in the present series of investigations is shown in Fig. 1. Since the experimental setup has been shown in detail in Refs. (12)~(15), only an outline is given here. The coordinate system is defined as shown in Fig. 1. The experiment was done in a wind tunnel (cross section 100 mm $\times$ 200 mm). The free stream velocity was 10 m/s. A hydrogen/ nitrogen mixture, the mass concentration of H_2 of which was 4%, was injected through a porous plate (98 mm $\times$ 200 mm in surface area) and a diffusion flame was formed in a boundary layer. Measurements were made at $x=180$ mm, where the variation in the x -direction was much smaller than that in the y -direction, as shown in Refs. 12 and 13.

2.2 Velocity measurement

Simultaneous measurements of two velocity components (U and V) were made, using a polarized four beam laser-Doppler velocimetry with counter-type signal processors. 4 000 pairs of U and V were measured for one measurement point, and various

turbulent parameters (mean value, root mean square, cross correlation, and higher order correlation) were calculated by a microcomputer. Scattering particles (TiO_2) were seeded in either the free air stream or injectant gas. Details of the measurement method, data analysis method and particle seeding method are mentioned in Refs. (12)~(15).

2.3 Temperature measurement

The measurement of mean temperature was made with an uncoated 50 μ m Pt/Pt13 % Rh thermocouple. The measured values were not corrected for thermometric errors due to radiation, conduction, and catalytic activity.

2.4 Concentration measurement of stable chemical species

The mean value of the molar concentration of stable chemical species was measured by gas-chromatography. Concentrations of three species, hydrogen, oxygen, and nitrogen were measured, and the concentration of steam was calculated, using measured data.

3. Results and Discussion

3.1 Aerodynamic structure at $x=180$ mm

Figure 2 shows the aerodynamic structure at $x=180$ mm. The boundary layer thickness (L) is 52 mm. Figures 2(a)~2(g) show statistical values of two velocity components U and V . $\bar{U}$, $\bar{V}$ and u , v are the mean and fluctuating values of U and V , respectively. White datum marks, measured by particles seeded in the free stream, show the values originating in the main flow. Black datum marks, measured by particles seeded in the injected gas flow, show the value originating in the injected gas flow. The black and white marks overlap near the flame zone. In this area, the main flow and the injected gas flow are mixed, though the mixing is stratified. Figure 2(h) shows profiles of the mean temperature and the mean value of molar concentration of stable chemical species. Figure 2(i) shows the mean density profile, which is calculated with the assumption of being a perfect gas,

Fig. 1 Schematic of the flow field

Table 1 Comparison between the order of magnitude of higher order correlations estimated and that obtained from experimental results.

	the order of magnitude estimated*	the order of magnitude from experimental results
$\overline{uv}$	0.01-0.0025	0.002
$\overline{uuu}$		0.0006
$\overline{vvv}$	0.001-0.000125	0.0001
$\overline{uvv}$		0.0001
$\overline{uuv}$		0.0001

* the order of magnitude of $\bar{V}$, u' , and v' is assumed to δ , when that of $\bar{U}$ is unity.

Fig. 2 Aerodynamic structure at $x=180$ mm

using the data of temperature and molar concentration.

Typically, the mean velocity ($\bar{U}$) is locally accelerated, and the sign of the velocity gradient is changed, as shown in Fig. 2(a). The sign of $-\overline{uv}$ is also changed, corresponding to the change in the sign of the velocity gradient, as shown in Fig. 2(c). This shows the applicability of widely used gradient diffusion type turbulent models⁽¹⁴⁾. Figures 2(a) ~ 2(c) also show that the location of the maximum of u' does not correspond to that of $-\overline{uv}$, and the location of the maximum of $-\overline{uv}$ does not correspond to that of maximum $d\bar{U}/dy$. This suggests that the profile of $-\overline{uv}$ can not be predicted directly from the profile of $d\bar{U}/dy$ near the flame zone⁽¹⁴⁾.

The profiles of temperature and species concentration show the typical profiles of a diffusion flame, as shown in Fig. 2(h). Figure 2(g) shows that the mean density profile reaches the minimum at the flame zone.

From these data, the order of magnitude of the variables can be determined. Figures 2(a) and 2(b) show that when $\bar{U}_\infty$ is assumed to be in the order of unity, the order of magnitude of $\bar{V}$, u' , and v' is about $1/10$ – $1/20$, while $\bar{U}$ is unity. This is almost the same as the ratio in the case of an isothermal boundary layer⁽¹⁹⁾. When the order of magnitude of δ is assumed to be $1/10$ – $1/20$, the order of magnitude of the higher order correlations can be determined, as shown in Table 1. The orders of magnitude obtained from Figures 2(d) ~ 2(g) are also listed. The estimated order of magnitude agrees fairly well with that obtained experimentally. This result shows that if the order of magnitude of $\bar{V}$, u' , and v' is assumed to be δ , while $\bar{U}$ is unity, the order of magnitude of the higher order correlations can be estimated reasonably well.

3.2 Estimation of advection, production by shear, and turbulent diffusion terms in a turbulent kinetic energy equation

Since the order of magnitude of the variables can be determined, as mentioned in the previous section, equations of turbulent kinetic energy can be simplified.

In order to estimate the order of magnitude of each term, the following characteristics should be taken into account :

1. As mentioned in the previous section, the order of magnitude of variables can be estimated as follows :

$$\bar{U}/\bar{U}_\infty=1, \bar{V}/\bar{U}_\infty=\delta, u'/\bar{U}_\infty=\delta, v'/\bar{U}_\infty=\delta.$$

2. The order of magnitude of mean density and density fluctuation are assumed as follows, similar to the estimation of the velocity component :

$$\bar{\rho}=1, \rho'=\delta.$$

3. At $x=180\text{mm}$, the derivative in the x -direction is much smaller than that in the y -direction, as shown in Ref. 12.

Equations for turbulent kinetic energy of steady two-dimensional flow, and an equation of $-\bar{\rho} \overline{uv}$, which plays an important role for the production of $\bar{\rho} \overline{u^2}$, are generally described as follows :

$$\begin{aligned} & \frac{1}{2} \left(\frac{\partial}{\partial x} \overline{\rho uu} \bar{U} + \frac{\partial}{\partial y} \overline{\rho uu} \bar{V} \right) \\ & \quad \text{a} \\ & + \overline{\rho uu} \frac{\partial \bar{U}}{\partial x} + \overline{\rho uv} \frac{\partial \bar{U}}{\partial y} + \overline{\rho u} \bar{U} \frac{\partial \bar{U}}{\partial x} + \overline{\rho u} \bar{V} \frac{\partial \bar{U}}{\partial y} \\ & \quad \text{b} \\ & + \frac{1}{2} \frac{\partial}{\partial x} \overline{\rho uuu} + \frac{1}{2} \frac{\partial}{\partial y} \overline{\rho uuv} + u \frac{\partial \bar{P}}{\partial x} + \epsilon_u = 0 \\ & \quad \text{c} \quad \text{d} \quad \text{e} \end{aligned} \quad (1)$$

$$\begin{aligned} & \frac{1}{2} \left(\frac{\partial}{\partial x} \overline{\rho vv} \bar{U} + \frac{\partial}{\partial y} \overline{\rho vv} \bar{V} \right) \\ & \quad \text{a} \\ & + \overline{\rho uv} \frac{\partial \bar{V}}{\partial x} + \overline{\rho vv} \frac{\partial \bar{V}}{\partial y} + \overline{\rho v} \bar{U} \frac{\partial \bar{V}}{\partial x} + \overline{\rho v} \bar{V} \frac{\partial \bar{V}}{\partial y} \\ & \quad \text{b} \\ & + \frac{1}{2} \frac{\partial}{\partial x} \overline{\rho uvv} + \frac{1}{2} \frac{\partial}{\partial y} \overline{\rho vvv} + v \frac{\partial \bar{P}}{\partial y} + \epsilon_v = 0 \\ & \quad \text{c} \quad \text{d} \quad \text{e} \end{aligned} \quad (2)$$

$$\begin{aligned} & \frac{1}{2} \left(\frac{\partial}{\partial x} \overline{\rho ww} \bar{U} + \frac{\partial}{\partial y} \overline{\rho ww} \bar{V} \right) \\ & \quad \text{a} \\ & + \frac{1}{2} \frac{\partial}{\partial x} \overline{\rho uww} + \frac{1}{2} \frac{\partial}{\partial y} \overline{\rho vww} + w \frac{\partial \bar{P}}{\partial z} + \epsilon_w = 0 \\ & \quad \text{c} \quad \text{d} \quad \text{e} \end{aligned} \quad (3)$$

$$\begin{aligned} & \frac{\partial}{\partial x} \overline{\rho uv} \bar{U} + \frac{\partial}{\partial y} \overline{\rho uv} \bar{V} \\ & \quad \text{a} \\ & + \overline{\rho uu} \frac{\partial \bar{V}}{\partial x} + \overline{\rho uv} \frac{\partial \bar{V}}{\partial y} + \overline{\rho u} \bar{U} \frac{\partial \bar{V}}{\partial x} + \overline{\rho u} \bar{V} \frac{\partial \bar{V}}{\partial y} \\ & \quad \text{b} \end{aligned}$$

$$\begin{aligned} & + \overline{\rho uv} \frac{\partial \bar{U}}{\partial x} + \overline{\rho vv} \frac{\partial \bar{U}}{\partial y} + \overline{\rho v} \bar{U} \frac{\partial \bar{U}}{\partial x} + \overline{\rho v} \bar{V} \frac{\partial \bar{U}}{\partial y} \\ & \quad \text{b} \\ & + \frac{\partial}{\partial x} \overline{\rho uuv} + \frac{\partial}{\partial y} \overline{\rho uvv} + v \frac{\partial \bar{P}}{\partial x} + u \frac{\partial \bar{P}}{\partial y} \\ & \quad \text{c} \quad \text{d} \\ & + \epsilon_{uv} = 0 \\ & \quad \text{e} \end{aligned} \quad (4)$$

where, (a) stands for convection part, (b) production part, (c) (turbulent) diffusion part, (d) pressure-velocity interaction part, and (e) viscous dissipation.

When characteristics (1)-(3) are applied, the order of magnitude of each term is estimated, and the term of the largest order of magnitude of each part can be selected as follows :

$$\begin{aligned} \bar{\rho} \overline{u^2} \quad E_{Ad,u^2} &= -\frac{1}{2} \frac{\partial}{\partial y} \overline{\rho uu} \bar{V}, \\ E_{Pr,u^2} &= -\overline{\rho uv} \frac{\partial \bar{U}}{\partial y}, \\ E_{Di,u^2} &= -\frac{1}{2} \frac{\partial}{\partial y} \overline{\rho uuv}. \end{aligned} \quad (5)$$

$$\begin{aligned} \bar{\rho} \overline{v^2} \quad E_{Ad,v^2} &= -\frac{1}{2} \frac{\partial}{\partial y} \overline{\rho vv} \bar{V}, \\ E_{Pr,v^2} &= -\overline{\rho vv} \frac{\partial \bar{V}}{\partial y}, \\ E_{Di,v^2} &= -\frac{1}{2} \frac{\partial}{\partial y} \overline{\rho vvv}. \end{aligned} \quad (6)$$

$$\begin{aligned} \bar{\rho} \overline{w^2} \quad E_{Ad,w^2} &= -\frac{1}{2} \frac{\partial}{\partial y} \overline{\rho ww} \bar{V}, \\ E_{Pr,w^2} &= \text{---}, \\ E_{Di,w^2} &= -\frac{1}{2} \frac{\partial}{\partial y} \overline{\rho vww}. \end{aligned} \quad (7)$$

$$\begin{aligned} \bar{\rho} \overline{uv} \quad E_{Ad,uv} &= \frac{\partial}{\partial y} \overline{\rho uv} \bar{V}, \\ E_{Pr,uv} &= \overline{\rho vv} \frac{\partial \bar{U}}{\partial y}, \\ E_{Di,uv} &= \frac{\partial}{\partial y} \overline{\rho uvv}. \end{aligned} \quad (8)$$

where subscripts *Ad*, *Pr*, and *Di* mean advection, production and diffusion, respectively.

The term selected, is regarded as a representative term of each part in the present study. The terms of velocity- pressure correlation and dissipation are not estimated since the present data are insufficient.

3.3 Production and transfer of turbulent kinetic energy

Profiles of $\bar{\rho} \overline{u^2}$, $\bar{\rho} \overline{v^2}$, and $\bar{\rho} \overline{uv}$ are obtained using experimental data as shown in Figs. 3, 4, and 5. A derivative was determined by the third spline interpolation method.

Figures 3(a) and 3(b) show profiles of $\bar{\rho} \overline{u^2}$ and its advection, production and diffusion. The profile of $\bar{\rho} \overline{u^2}$ has two maxima and one minimum. The flat plate side maximum is located in the fuel side region of the flame zone, and the value of $\bar{\rho} \overline{u^2}$ remains high

between the maximum and the flame zone. Figure 3(b) shows that significant production occurs at the region around the flat plate side maximum of the $\bar{\rho}\bar{u}^2$ profile. In this region, on the other hand, the advection and diffusion are negative. Around the flame zone, all terms, that is, advection, production, and diffusion become positive. In the region between the flame zone and the boundary layer edge, the values of advection, production and diffusion are small. Close to the flame zone, $\bar{\rho}\bar{u}^2$ reaches a minimum, where high viscous dissipation is expected. Near the boundary layer edge, the profile of $\bar{\rho}\bar{u}^2$ keeps the upstream profile, since there is no effect of the flame.

The production value of $\bar{\rho}\bar{u}^2$ is large on the flat plate side of the flame zone, where the value of $\bar{\rho}\bar{u}^2$ is large. Advection and diffusion, on the other hand, become negative there. This indicates that part of $\bar{\rho}\bar{u}^2$ is lost by the advection and diffusion. Around the flame zone, the advection and diffusion become positive. Thus, the $\bar{\rho}\bar{u}^2$ appears to be transferred by the advection and diffusion from the region around the maximum production to the region around the flame

Fig. 3 Profiles of $\bar{\rho}\bar{u}^2$ and its advection, production and diffusion

zone. This transfer keeps $\bar{\rho}\bar{u}^2$ high around the flame zone, although a high level of viscous dissipation due to an increase in temperature is expected there.

Figures 4(a) and 4(b) show the profile of $\bar{\rho}\bar{v}^2$ and profiles of its production, advection and diffusion. The profile of $\bar{\rho}\bar{v}^2$ is similar to that of $\bar{\rho}\bar{u}^2$, though the value of $\bar{\rho}\bar{v}^2$ is about 1/2 - 1/3 of $\bar{\rho}\bar{u}^2$. This ratio is almost the same as that of an isothermal flat plate boundary layer⁽¹⁹⁾. The profile of E_{Pr,v^2} is quite different from that of E_{Pr,u^2} ; that is, no efficient gain by E_{Pr,v^2} can be found in the whole boundary layer. This suggests that $\bar{\rho}\bar{v}^2$ is scarcely produced by shear. Other terms (E_{Ad,v^2} and E_{Di,v^2}) also have no effective gain or loss in the boundary layer.

Figures 5(a) and 5(b) show the profile of $\bar{\rho}\bar{uv}$ and profiles of its production, advection and diffusion. The production has a large positive value in the flat plate side region. In the free stream side region, the production becomes negative close to the flame zone and it becomes a small positive value with an increase in y . The values of the advection and diffusion are small. This shows that the value of $-\bar{\rho}\bar{uv}$ near the flame zone is attributed to the production by shear and in the free stream side region, the profile keeps the upstream one, similar to $\bar{\rho}\bar{u}^2$.

A turbulent structure of a simple isothermal shear flow is summarized, as shown in Fig. 6⁽²⁰⁾⁽²¹⁾. In a simple shear flow, all turbulent kinetic energies are produced as $\bar{u}^2$ by the production term of $\bar{u}^2$. The turbulent kinetic energy of $\bar{v}^2$ and $\bar{w}^2$ are transferred from $\bar{u}^2$ due to pressure-velocity interaction terms. $\bar{uv}$ which is necessary to produce $\bar{u}^2$ is produced by the production term of $\bar{uv}$ using $\bar{v}^2$. As a result, the $\bar{u}^2$

Fig. 4 Profiles of $\bar{\rho}\bar{v}^2$ and its advection, production and diffusion

component has more energy than the other components ($\overline{u^2}$ is roughly twice as large as $\overline{v^2}$ and $\overline{w^2}$).

Figures 3~5 show that similar characteristics exist even when the flame is formed; and they are summarized as follows:

- 1) The production term of $\overline{\rho u^2}$ is large; on the other hand, the production term of $\overline{\rho v^2}$ is almost zero.
- 2) The value of $\overline{\rho v^2}$ is about 1/2-1/3 of that of $\overline{\rho u^2}$.
- 3) The advection and diffusion of $\overline{\rho v^2}$ have no effective gain.
- 4) $\overline{\rho uv}$ is produced by the product of $\overline{\rho v^2}$ and $d\overline{U}/dy$.

These characteristics fit with those of isothermal simple shear flow. The comparison shows that a similar mechanism exists even when the flame is formed in the boundary layer. Characteristic (1) suggests that $\overline{\rho u^2}$ receives most of the production of turbulent kinetic energy, and the energy of $\overline{\rho v^2}$ is transferred from that of $\overline{\rho u^2}$. Characteristics (2) and (3) suggest that the pressure-velocity interaction may play a vital role in the transfer of turbulent kinetic energy to the other components. Characteristic (4) shows that $\overline{\rho uv}$ which plays an important role for the production of $\overline{\rho u^2}$, is produced by shear. This indicates that the production and transfer mechanism can be fundamen-

Fig. 5 Profiles of $\overline{\rho uv}$ and its advection, production and diffusion

tally identified with that of the isothermal shear flow, even around the flame zone.

4. Concluding Remarks

Measurements of statistical properties, up to third order cross correlations, in a flat plate turbulent boundary layer diffusion flame have been done, using a two-component laser-Doppler velocimetry. The order of magnitude of terms of production, advection and diffusion of turbulent kinetic energy has been estimated, on the basis of experimental data, and representative terms of the production, advection and diffusion have been selected. Using these representative terms, the production and transfer mechanism of the turbulent kinetic energy in a flat plate turbulent boundary layer diffusion flame has been discussed. Conclusions are summarized as follows:

- (1) The turbulent kinetic energy is mainly produced as a component parallel to the flat plate ($\overline{\rho u^2}$) in a flat plate side region of the flame zone. The part of the produced $\overline{\rho u^2}$ is transferred to around the flame zone by the advection and diffusion. This keeps the $\overline{\rho u^2}$ component high around the flame zone although the high level of viscous dissipation due to high temperature is expected there. The production, advection and diffusion of $\overline{\rho v^2}$ are almost zero. Thus, the energy of $\overline{\rho v^2}$ is transferred from $\overline{\rho u^2}$ by a mechanism other than production, advection and diffusion. The $\overline{\rho v^2}$ component is supposed to be transferred from $\overline{\rho u^2}$ by pressure-velocity interaction. A correlation of velocities parallel and normal to the flat plate ($\overline{\rho uv}$), which plays an important role in the production of $\overline{\rho u^2}$, is produced by shear.
- (2) This mechanism is fundamentally the same

Fig. 6 Turbulent structure in a simple isothermal shear flow

as that of isothermal shear flow. Consequently, it can be concluded that the turbulence production and transfer mechanism is fundamentally identified with that of the isothermal shear flow, even around the flame zone.

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