Digital Optimal Control of Elastic Structure Systems

(Positioning Control of an Elastic Rotating Arm)*

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By treating the positioning control problem of a cantilever-type arm rotating horizontally, the digital optimal control problem of an elastic structure system is theoretically and experimentally considered in this study. Especially, a method for examining the influence of spillover due to a finite dimensional approximation of the elastic structure and a calculation delay in the digital control is presented. By the practical development of control equipment and the various control experiments, the usefulness of the proposed design method is verified, and important information about this kind of control is obtained. Furthermore, the influence of the friction in the DC servo-motor is investigated, mainly by digital simulations.

Key Words: Application of Control, Optimal Control, Digital Control, Elastic System, Positioning Control, Rotating Arm, Spillover, Calculation Delay

1. Introduction

There have been many studies made of the optimal control problems of elastic structure systems since those of Komkov⁽¹⁾; and practical studies dealing with the control techniques of positioning and vibration applied for building structures, mechanical structures, space structures and so on have recently increased. The necessity of the optimal control of structure systems has increased with the demand for high efficiency, high speed, high accuracy, high reliability, energy conservation, cost efficiency, etc.. Since the control problem of structure systems leads essentially to the control problem of distributed parameter systems, it contains many problems which don't occur in the case of lumped systems; and its complexity increases in the case of digital control. The modeling relating to various non-linearities often becomes an important problem. These problems are related to each other and the main features of the control prob-

lem of elastic structures are as follows:

- (1) the problem of modeling;
- (2) the problem of finite approximation (spillover problem);
- (3) the digital control sampling problem;

Although problem (2) is connected with problem (1), it is a central problem of distributed parameter systems; that is, a problem due to the necessity to practically truncate the modes for the infinite-dimensional system. Although the truncation order is often chosen practically, Balas⁽²⁾⁽³⁾ analyzed the influences of omitted higher modes, that is, the spillover; but the digital control problem was not considered. Problem (3), connected closely with problem (2) is a problem relating sample delay due to calculation time as well as to the effect of higher modes. Besides the above problems, there are ordinary problems of the criterion for optimality and the non-linearity of system as well as in lumped systems.

The control problem of a positioning rotating arm in robots is one of the typical vibration control problems of elastic structures, and many studies^{(4)~(7)} of this problem have recently been carried out. However, there are few studies of the digital or optimal

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control problems of such a system. This study deals with the position control problem of an elastic rotating arm and considers its digital optimal control experimentally.

2. An Elastic Rotating Arm and Its Modeling

Figure 1 shows the elastic rotating arm model dealt with in this study. This model is constructed such that a cantilever-type arm fixed with an axis rotates around the point O . The axis whose radius equals R_s is connected directly with the output axis of a DC servo-motor. The rotation angle of the motor and the deflection of the beam at the point which is a distance r away from the fixed edge are expressed respectively by $\theta(t)$ and $z(r, t)$. Then, the trajectory $s(r, t)$ at the arbitrary point is expressed as

$$S(r, t) = (r + R_s)\theta(t) + z(r, t) \quad (1)$$

From the equilibrium of moment about the point O ,

$$\begin{aligned} & \rho \int_0^l (r + R_s)^2 dr \cdot \ddot{\theta}(t) + \rho \int_0^l (r + R_s) \\ & \times \frac{\partial^2 z(r, t)}{\partial t^2} dr + c \int_0^l (r + R_s)^2 dr \cdot \dot{\theta}(t) \\ & + c \int_0^l (r + R_s) \frac{\partial z(r, t)}{\partial t} dr = T_m(t) \end{aligned} \quad (2)$$

From the equilibrium of force for an infinitesimal element,

$$\begin{aligned} & \rho \frac{\partial^2 z(r, t)}{\partial t^2} + c \frac{\partial z(r, t)}{\partial t} + EI \frac{\partial^4 z(r, t)}{\partial r^4} \\ & + c_s I \frac{\partial^5 z(r, t)}{\partial t \partial r^4} = -\rho(r + R_s)\ddot{\theta}(t) \\ & - c(r + R_s)\dot{\theta}(t) \end{aligned} \quad (3)$$

The axial torque T_m of the DC-servo motor satisfies the following equation:

$$T_m(t) = K_t i(t) - J_0 \ddot{\theta}(t) - c_0 \dot{\theta}(t) \quad (4)$$

$$L \frac{di(t)}{dt} + Ri(t) + K_E \dot{\theta}(t) = e(t) \quad (5)$$

where,

- ρ : line density of beam (kg/m)
- E : Young's modulus (N/m²)
- I : moment of inertia of area (m²)

Fig. 1 Model of elastic rotating arm

R_s : radius of motor shaft (m)

c : external viscous damping coefficient of beam (N · s/m²)

c_s : internal viscous damping coefficient of beam (N · s/m²)

K_T : torque constant (N · m/A)

J_0 : moment of inertia of motor shaft (N · m · s²)

c_0 : viscous damping coefficient of motor shaft (N · m · s)

L : inductance of DC servo-motor armature coil (H)

R : resistance of DC servo-motor armature coil (Ω)

K_E : induced voltage constant (V · s)

l : length of beam (m)

$i(t)$: current in armature coil of DC servo-motor (A)

$e(t)$: input voltage in armature coil of DC servo-motor (V)

t : time (s)

The boundary condition of the cantilever beam is as follows:

$$z(0, t) = \frac{\partial z}{\partial r} \Big|_{r=0} = \frac{\partial^2 z}{\partial r^2} \Big|_{r=l} = \frac{\partial^3 z}{\partial r^3} \Big|_{r=l} = 0 \quad (6)$$

The eigenfunction $\phi_i(r)$ satisfying the above boundary condition is

$$\begin{aligned} \phi_i(r) = & (\sin h \nu_i l + \sin \nu_i l)(\cos h \nu_i r - \cos \nu_i r) \\ & - (\cos h \nu_i l + \cos \nu_i l)(\sin h \nu_i r - \sin \nu_i r) \\ & i = 1, 2, \end{aligned} \quad (7)$$

where ν_i is the root of the following frequency equation.

$$1 + \cos \nu_i l \cosh \nu_i l = 0, \quad i = 1, 2, \dots \quad (8)$$

$z(r, t)$ is made into modal expansion by using the eigenfunctions; that is, by replacing $z(r, t)$ as

$$z(r, t) = \sum_{i=1}^{\infty} \phi_i(r) \cdot x_i(t) \quad (9)$$

and approximating it with N modes and further choosing the following state vector

$$x(t) = [x_1, x_2, \dots, x_N, \dot{x}_1, \dot{x}_2, \dots, \dot{x}_N, \theta, \dot{\theta}, i]^T \quad (10)$$

the following state equation is obtained.

$$\dot{M}x(t) = A_K x(t) + B_K u(t) \quad (11)$$

$$M = \begin{bmatrix} & & & & 0_{N \times 3} \\ & & & & 0 \quad \phi_1 \quad 0 \\ & & & & 0 \quad \phi_2 \quad 0 \\ & & & & \vdots \quad \vdots \quad \vdots \\ & & & & 0 \quad \phi_N \quad 0 \\ \hline & 0 & 0 & \dots & 0 \\ 0_{3 \times N} & \rho \eta_1 / J & \rho \eta_2 / J & \dots & \rho \eta_N / J \\ & 0 & 0 & \dots & 0 \end{bmatrix} \quad \begin{matrix} I_{2N \times 2N} \\ I_{3 \times 3} \end{matrix}$$

$$J = J_0 + \rho \int_0^l (r + R_s)^2 dr,$$

$$\eta_i = \int_0^l (r + R_s) \phi_i(r) dr,$$

$$\phi_i = \eta_i / \int_0^l \phi_i^2(r) dr$$

$$A_k = \begin{bmatrix} 0_{N \times N} & I_{N \times N} & 0_{N \times 3} \\ -\omega_1^2 & 0 & -2(\xi_{e1} + \xi_1)\omega_1 & 0 & 0 & -c\psi_1/\rho & 0 \\ -\omega_2^2 & & -2(\xi_{e2} + \xi_2)\omega_2 & & 0 & -c\psi_2/\rho & 0 \\ \vdots & & \ddots & & \vdots & \vdots & \vdots \\ 0 & -\omega_N^2 & 0 & -2(\xi_{eN} + \xi_N)\omega_N & 0 & -c\psi_N/\rho & 0 \\ 0_{3 \times N} & 0 & 0 & \cdots & 0 & 0 & 1 & 0 \\ & -c\eta_1/J & -c\eta_2/J & \cdots & -c\eta_N/J & 0 & -\Delta & K_t/J \\ & 0 & 0 & \cdots & 0 & 0 & -K_E/L & -R/L \end{bmatrix}$$

$$\Delta = \left\{ c_0 + c \int_0^l (r + R_s)^2 dr \right\} / J, B_K = [0, 0, \dots, 0, 0, 0, 1/L]^T,$$

$$u(t) = e(t), \omega_i^2 = \frac{EI}{\rho} \nu_i^4, 2\xi_i \omega_i = \frac{c_s I}{\rho} \nu_i^4, 2\xi_{ei} \omega_i = \frac{c}{\rho}$$

Further, multiplying Eq.(11) by the inverse matrix of M from the left-hand side, we have

$$\dot{x}(t) = A_c x + B_c u(t), A_c = M^{-1} A_K,$$

$$B_c = M^{-1} B_K$$

(12)

where T denotes the transpose of the matrix.

Assuming that it is possible to measure the rotation angle $\theta(t)$, the rotation angular velocity $\dot{\theta}(t)$, the current $i(t)$ and the strain ε at the position r_e far from the fixed point, we have the following output equation:

$$y(t) = Cx(t), y(t) = [\varepsilon(t), \theta(t), \dot{\theta}(t), i(t)]$$

(13)

$$C = \begin{bmatrix} \frac{d}{2} \cdot \frac{\partial^2 \phi_1}{\partial r^2} \Big|_{r=r_s}, \dots, \frac{d}{2} \cdot \frac{\partial^2 \phi_N}{\partial r^2} \Big|_{r=r_s}, 0, \dots, 0 & 0, 0, 0 \\ 0_{3 \times 2N} & I_{3 \times 3} \end{bmatrix}$$

where d denotes the thickness of the beam.

3. Formulation of Criterion Function

The choice of criterion function, upon which the control method depends very much, is not unique. It is reported⁽⁸⁾ that the quadratic criterion function is suitable for the position control. In the case of elastic systems, it is necessary to examine the stability of the controlled system since the control law obtained from the optimal control theory does not guarantee stability, because of the spillover. Since it is not easy to examine the stability for the nonstationary control law, the following criterion is chosen in this study:

$$V(u) = \lim_{t_b \rightarrow \infty} \int_0^{t_b} \left[\alpha \int_0^l \{ (r + R_s) \theta(t) + z(r, t) \}^2 \right. \\ \times dr + \beta \dot{z}^2(t) + \xi \left\{ \frac{d}{2} \left(E \frac{\partial^2 z(r, t)}{\partial r^2} \Big|_{r=0} \right. \right. \\ \left. \left. + c_s \frac{\partial^3 z(r, t)}{\partial t \partial r^2} \Big|_{r=0} \right) \right\}^2 + \gamma u^2(t) \Big] dt \quad (14)$$

where α, β, ξ and γ are weighting coefficients.

In this criterion function, the quantity obtained by integrating the square error from the objective position over the span of the beam is taken as an objective function, and the stress at the fixed point, the current $i(t)$ in the DC-servo motor and the control force $u(t)$ are taken as constraints.

Rewriting Eq.(14) using the state vector expressed in Eq. (10), we have the following general quadratic criterion function.

$$V(u) = \lim_{t_b \rightarrow \infty} \int_0^{t_b} \{ x^T(t) Q x(t) + \gamma u^2(t) \} dt \quad (15)$$

The elements of Q in the above equation are omitted.

4. Application of Digital Optimal Control Theory

4.1 Controller

Transforming the state Eq. (12) and the output Eq. (13) into the discrete time system with a sampling interval T , we have

$$x(k+1) = Ax(k) + Bu(k) \\ k=1, 2 \quad (16)$$

$$A = \exp[A_c T]$$

$$B = \int_0^T \exp[A_c \tau] d\tau \cdot B_c$$

$$y(k) = Cx(k) \quad (17)$$

The approximation of the zero-order hold of the criterion function (15) is

$$V(u) = \lim_{m \rightarrow \infty} \sum_{k=0}^m \{ x^T(k) Q x(k) + \gamma u^2(k) \} \\ m = t_b/T \quad (18)$$

The stationary optimal control law obtained from the digital optimal control theory is given as follows⁽⁹⁾:

$$u(k) = -Fx(k) \quad (19)$$

$$F = \lim_{k \rightarrow \infty} (\gamma + B^T P(k) B)^{-1} B^T P(k) A \quad (20)$$

$$P(k+1) = A^T P(k) A + Q \\ - A^T P(k) B (\gamma + B^T P(k) B)^{-1} B^T P(k) A \quad (21) \\ P(0) = 0$$

4.2 Observer

Since the control law of Eq.(19) has a structure for the feedback of all state variables, it is necessary to design an observer to estimate the state vector from the output equation. In this study, the state vector is estimated by the minimum-order observer

according to the following procedure.

First, by putting

$$G = \begin{bmatrix} C \\ W \end{bmatrix}$$

and after choosing an appropriate matrix W such that $|G| \neq 0$, the following transform is carried out.

$$GAG^{-1} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, GB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

Then, the minimum-order observer is given by the following equation:

$$\hat{x}(k) = D\lambda(k) + Hy(k) \quad (22)$$

$$\lambda(k+1) = \hat{A}\lambda(k) + \hat{K}y(k) + \hat{B}u(k) \quad (23)$$

where $\lambda(k)$ is the $(N-4)$ -dimensional vector when $x(k)$ is the N -dimensional vector, and

$$\hat{A} = A_{22} - LA_{12}$$

$$\hat{K} = \hat{A}L + A_{21} - LA_{11}$$

$$\hat{B} = B_2 - LB_1$$

$$D = G^{-1} \begin{bmatrix} 0 \\ \vdots \\ I_{(N-4) \times (N-4)} \end{bmatrix}$$

$$H = G^{-1} \begin{bmatrix} I_{4 \times 4} \\ L \end{bmatrix}$$

Here, if the observer is designed as the optimal observer⁽⁹⁾, the design parameter L becomes as follows:

$$L^T = (\bar{R} + A_{12}\Lambda(k)A_{12}^T)^{-1}A_{12}\Lambda(k)A_{22}^T \quad (24)$$

$$\Lambda(k+1) = \bar{Q} + A_{22}\Lambda(k)A_{22}^T - A_{22}\Lambda(k)A_{12}^T \times \{\bar{R} + A_{12}\Lambda(k)A_{12}^T\}^{-1}A_{12}\Lambda(k)A_{22}^T \quad (25)$$

where $\bar{Q}$ and $\bar{R}$ should be chosen appropriately. In this study unit matrices are chosen.

If the control law of Eq. (19) is expressed by using the state variables of Eq. (22),

$$u(k) = -F\hat{x}(k) \quad (26)$$

However, because of time lag due to calculation, it is impossible to obtain k -th $u(k)$ according to Eq. (26) after calculating $x(k)$ in the observer from the detection quantity $y(k)$ of k -th sampling. In order to solve this problem, using the following predicting observer, k -th $u(k)$ is calculated in the sampling interval from $(k-1)$ -th $y(k-1)$.

$$u(k) = -FAD\lambda(k-1) - FAHy(k-1) - FBu(k-1) \quad (27)$$

$$\lambda(k) = \hat{A}\lambda(k-1) + \hat{K}y(k-1) + Bu(k-1) \quad (28)$$

In this case, the sampling period depends on the calculation time of the computer.

4.3 Stability of the controlled system

In the above observer and controller, stability is not guaranteed owing to the influences of the spillover caused by the finite dimensional approximation, and the property of the predicting observer. Therefore, it is necessary to examine the stability of the M -dimensional controller for the $N(N > M)$ -dimensional system which is modeled so that the modeling error is negligibly small.

The stability of the control system can be inves-

tigated by the following procedure: The augmented system consisting of Eqs. (16), (27) and (28) is defined as

$$\begin{bmatrix} x(k+1) \\ u(k+1) \\ \lambda(k+1) \end{bmatrix} = \begin{bmatrix} A & B & 0 \\ -\underline{F}\underline{A}\underline{H}\underline{C} & -\underline{F}\underline{B} & -\underline{F}\underline{A}\underline{D} \\ \underline{\hat{K}}\underline{C} & \underline{\hat{B}} & \underline{\hat{A}} \end{bmatrix} \begin{bmatrix} x(k) \\ u(k) \\ \lambda(k) \end{bmatrix}$$

where A, B and C are system matrices taking account of elastic modes up to the N -th mode, and $\underline{F}, \underline{A}, \underline{B}, \underline{D}, \underline{H}, \underline{\hat{K}}, \underline{\hat{B}}$ and $\underline{\hat{A}}$ are matrices of the system designed up to the M -th mode. They are underlined to distinguish the cases of up to the N -th mode. Representing the above equation as follows,

$$x_z(k+1) = A_z x_z(k) \quad (29)$$

if the matrix A_z is a stable matrix, the stability of the augmented system is guaranteed.

Figure 2 shows the results of eigenvalue loci of A_z for the change of the weighting coefficient γ in the Laplace plane for the case that the specification values of the experimental equipment are used; and $N=5$, $M=3$, and $T=0.025$. In this figure, ω_s denotes the sampling angular frequency. This figure is drawn within the range of stable eigenvalues, and the system becomes unstable as the weighting coefficient decreases further; that is, the control increases.

4.4 Influence of Coulomb friction

There is a case in which the friction at the mobile part causes a problem. Especially, it is serious when the shaft is driven directly, without a gear. By modeling the friction as Coulomb friction, the observer taking account of the friction is synthesized as follows:

First, the Coulomb Friction torque is modeled by the following equation.

$$T_f = \begin{cases} -T_{f0}; & \dot{\theta} > s_p \text{ or } |\dot{\theta}| \leq s_p, K_{Ti} > T_{f0} \\ -K_{Ti}; & |\dot{\theta}| \leq s_p, |K_{Ti}| \leq T_{f0} \\ T_{f0}; & \dot{\theta} < -s_p \text{ or } |\dot{\theta}| \leq -s_p, K_{Ti} < -T_{f0}, \\ s_p \geq 0 \end{cases} \quad (30)$$

For this kind of problem, Yamada⁽¹⁰⁾ has already proposed a control method and the present method is essentially the same, except that in the present method there is a dead space S_p . Adding the term of the Coulomb friction torque to property equation (4) of the DC-servo motor, we have

$$T_m(t) = f(\dot{\theta})K_{Ti}(t) - J_0\ddot{\theta}(t) - c_0\dot{\theta}(t) - g(\dot{\theta})T_{f0} \quad (31)$$

where

$$f(\dot{\theta}) = \begin{cases} 1; & \dot{\theta} > s_p \\ 0; & |\dot{\theta}| \leq s_p \\ 1; & \dot{\theta} < -s_p, \end{cases} \quad g(\dot{\theta}) = \begin{cases} 1; & \dot{\theta} > s_p \\ 0; & |\dot{\theta}| \leq s_p \\ -1; & \dot{\theta} < -s_p \end{cases}$$

Adding T_{f0} to the state variables as follows,

$$x_c(t) = [x_1, x_2, \dots, x_N, \dot{x}_1, \dot{x}_2, \dots, \dot{x}_N, \theta, \dot{\theta}, i, T_{f0}]^T$$

(32)

we have the following state equation.

$$M_c \dot{x}_c(t) = A_{c1} x_c(t) + B_{c1} u(t) \quad (33)$$

$$M_c = \begin{bmatrix} M & 0_{N \times 1} \\ 0_{1 \times N} & 1 \end{bmatrix}$$

$$A_{c1}(i,j) = A_k(i,j); \quad i, j = 1, 2, \dots, N$$

where

$$A_{c1}(N-2, N) = f(\dot{\theta}) \cdot K_T / J$$

$$A_{c1}(i, j) = 0;$$

$$i = N+1, j = 1, 2, \dots, N$$

$$i = 1, 2, \dots, N+1, j = N+1$$

$$A_{c1}(N-2, N+1) = -g(\dot{\theta}) / J$$

$$B_{c1} = [B_c, 0]$$

By multiplying Eq.(33) by the inverse matrix of M_c the same state equation as Eq.(12) can be obtained. Discretizing the obtained equation in the sampling period T , we have the same discrete time state equation as Eqs.(16) and (17). Furthermore, a discrete time minimum-order observer is synthesized as an adaptive observer, such that the values of each matrix change according to the value of $\dot{\theta}$.

5. Control Experiment and Simulation

5.1 Experimental apparatus

Figure 3 shows the schematic of the experiment. The 1st, 2nd and 3rd natural frequencies of the elastic rotating arm are 1.13 Hz, 7.25 Hz and 20.3 Hz under the boundary condition of the fixed end. The other specifications of the experimental apparatus are as follows:

$$l = 9.12 \times 10^{-1} (\text{m})$$

$$\zeta_1 = 1.0 \times 10^{-3}, \quad \zeta_{01} = 1.0 \times 10^{-2}$$

$$J_0 = 1.05 \times 10^{-3} (\text{N} \cdot \text{m} \cdot \text{s}^2)$$

Fig. 2 Loci of eigenvalues of matrix for change of weighting coefficient

$$c_0 = 3.50 \times 10^{-2} (\text{N} \cdot \text{m} \cdot \text{s})$$

$$K_T = 6.46 \times 10^{-1} (\text{N} \cdot \text{m} / \text{A}),$$

$$K_E = 6.49 \times 10^{-1} (\text{V} \cdot \text{s})$$

$$R = 8.71 (\Omega), \quad L = 6.03 \times 10^{-2} (\text{H})$$

$$r_e = 2.25 \times 10^{-2} (\text{m})$$

The strain ϵ of the elastic rotating arm, the rotation angle θ of the DC-servo motor, its angular velocity $\dot{\theta}$ and the current i are measured, respectively, by a strain gauge, a pulse encoder, a tachogenerator and a resistance connected in cascade with the armature circuit. The total number of sensing quantities is four. These quantities are converted from the analog to the digital mode in the sampling interval $T = 0.025$ sec, and they are put into a computer.

The calculations of the observer and controller of Eqs.(27) and (28) are given to the A/D converted quantities and then they are put into a power amplifier through a D/A converter. That is, they are feedback to the DC servo-motor. The synthesis of the observer and controller were carried out for three cases taking account of (1) only a rigid mode of the elastic rotating arm, (2) up to the 1st elastic mode and (3) up to the 3rd elastic mode.

5.2 Results of the experiment and the simulation

The control experiment to position the rotating arm from the initial value $\theta_0 = 1.57$ rad (90°) was carried out for the above three cases. The results of the experiment and the simulation are described only for the case in which the Coulomb friction is not taken into consideration in the observer. The observer taking account of the Coulomb friction will be mentioned in the next chapter. However, the results of computer simulation are obtained by taking account of the friction torque in the dynamic characteristics of the control object.

(1) The case of taking account of only the rigid

Fig. 3 Scheme of experimental apparatus

mode. Figure 4 shows the control results for the case in which the feedback gain is determined such that the most effective control can be expected after examining the stability of Eq.(29) for the various weighting coefficients. The angle does not converge to the origin since the rotation torque of the DC servo-motor and the friction torque reach equilibrium on the way to the origin. A better control than the result shown in Fig. 4 could not be attained for the case in which only the rigid mode was taken into account.

(2) The case of taking account of modes up to the 1st elastic mode. Figure 5 shows the result of the control experiment carried out in the same manner as for the case of the rigid mode. In contrast with (1), the angle converges to the origin; however the settling time is about 2 seconds and the response is still slow. Better control than this could not be attained also in this case either.

(3) The case of taking account of modes up to the 3rd elastic mode. Figure 6 shows the result of a control experiment carried out in the same manner. The rotation angle of the DC servomotor converges in almost the same time interval as the natural period 1 second (fixed end) of the 1st elastic mode, and it converges the fastest in comparison with the other cases. However a little residual vibration occurs because of friction.

6. Considerations

6.1 Effect of the finite-dimensional approximation and sampling period

The method of the stability test shown in Eq.(29) is indispensable for realizing the digital optimal control of the structure system. That is, because of influence of truncated higher modes and one sample delay in the digital control, the above method becomes necessary. The results obtained in this study indicate that the higher the truncation mode becomes, the better the control performance which can be achieved. This is also seen from Fig.7 showing the trade-off curve between the object function and the control.

For the case of up to the 1st elastic mode, there occurs the question of whether or not faster convergence can be attained by shortening the calculation time. Certainly, it becomes possible, even in the same sampling period to use the observer, taking account of the Coulomb friction, because a calculation time is smaller than that of up to the 3rd mode case. However, the improvement of convergence velocity can not be expected, from the viewpoint of stability as described in the previous chapter.

It is considered that the information about the elastic 3rd mode includes considerable error, since the natural frequency (fixed end) of 20.3 Hz of the elastic

Fig. 4 Experimental result
(case of only rigid mode)

Fig. 5 Experimental result
(case of up to the 3rd
elastic mode)

Fig. 6 Experimental result
(case of up to the 5th
elastic mode)

Fig. 7 Trade-off curve between the objective function and the control (analysis taking account of up to the 5th elastic mode)

3rd mode is close to the sampling frequency of 40 Hz. In spite of this fact, however, the results of experiment and simulation indicate that the modes up to the 3rd mode should be taken into consideration. Therefore, it was seen that it is important to decrease the influence of the higher modes by taking account of as many modes as possible. In other words, in this experiment, the effect on the control performance attained by shortening the sampling interval is smaller than the one obtained by making the truncation order higher.

6.2 The problem of criterion function

The criterion function of Eq.(14) includes the term of the stress of the elastic rotating arm. The comparison between the case with this term and the case without this term was carried out. The case in which the stress is taken account of is more liable to become unstable than the case without this term in the eigenvalue loci for the change of the weighting coefficient of the control in the criterion function. This is considered to be due to the fact that the influences of spillover become larger, since suppressing the stress affects the eigenvalues of the higher modes whose curvatures are large.

With respect to the criterion itself, better criteria can be considered as mentioned in chapter 3. The great defect of a case in which the stationary control law is applied to a transient control, is that the control quantity decreases, the nearer the response approaches to the origin, because of the constant feedback gain. Therefore, the considerable power is necessary at the initial stage, and more power than necessary is consumed. On the other hand, if the nonstationary control law is applied, it is not easy to examine analytically the effect of the spillover.

Fig. 8 Simulation result of the control compensating Coulomb friction (control taking account of up to the 3rd elastic mode)

6.3 Influence of friction

As seen from Fig. 4, the rotation angle does not converge to the origin owing to the friction on the shaft of the DC servo-motor. On the other hand, as shown in Fig. 6, the friction has conversely good effect on the positioning of the rotation angle, since the convergence velocity is faster; but residual vibration occurs. Although the experiment for the case of using the observer and the controller taking account of the Coulomb friction could not be carried out because of the increase of calculation time, a computer simulation for the case of using the observer described in section 4.4 was performed. An example of the results is shown in Fig. 8. It is seen from this figure that the effect of compensating for the friction appears in the neighborhood of the origin. Although the present method is essentially the same as that of Yamada, it has made clear that inserting dead space, as indicated in Eq. (30), leads to a further improvement of control performance in the digital control.

7. Conclusion

This study dealt with the control problem of positioning the elastic rotating arm, and considered experimentally the digital optimal control problem of the elastic structure system. It is a feature of the digital control of the elastic structure system that the problem of spillover due to finite-dimensional approximation, and the problem of one sample delay due to the sampling of digital control are complicatedly related. For such a problem, this study presented a method for synthesizing a control system by examining the stability of the whole system, including the controller and the observer. Further, by designing a practical digital control system for an elastic rotating arm by means of this method, and by performing experiments and computer simulations, its usefulness was verified.

Furthermore, by considering the results obtained, the useful information about this kind of control could

be obtained. In addition, by considering the influence of friction which often becomes a problem in practice, it was confirmed that the influence of friction can be removed by using an adaptive observer. Finally, the authors are grateful to Mr. J. Inose for assistance with the experiments.

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