Heat Transfer During Melting Around a Vertical Cylinder*

(Effects of Aspect Ratio of the Cylinder and Subcooling)

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Experiments and numerical analyses of the melting around a vertical cylinder with or without subcooling were carried out for different cylinder diameters. In the experiments, *n*-octanecane was used as a phase change material (PCM). The measured shapes of the melting front agreed well with the analytical results. In the analysis, the laminar natural convection in the liquid phase and the heat conduction in the solid phase were calculated simultaneously by means of a finite difference method. The subcooling degree significantly depresses the melting rate. It is not until the subcooling degree is large that it decreases the rate of the total thermal energy storage. The relation between the average Nusselt number on the cylinder and the average thickness of the molten region does not depend on the subcooling degree. The molten volumes with different cylinder diameters without subcooling are arranged by non-dimensional time, including the aspect ratio of the cylinder.

Key Words: Phase Change, Melting, Natural Convection, Thermal Conduction, Thermal Energy Storage, Subcooling, Vertical Cylinder, Aspect Ratio

1. Introduction

The heat transfer associated with melting or solidification is actively studied^{(1)~(6)} for the purpose of obtaining fundamental information concerning the efficient design of the latent heat-of-fusion thermal energy storage system. It is well known that the natural convection in the molten region is of particular importance in such phase change problems, especially in the melting process. A pioneering analysis of melting with natural convection in the molten region was performed by Sparrow et al.⁽⁷⁾, though the thickness of the molten region was relatively thin compared with the height. In the analysis, the melting process around a vertical cylinder for the non-subcooled case was analyzed. The non-subcooled case is meant to be that the initial temperature of the phase change material (PCM) is its fusion temperature. The same problem was studied analytically and experimentally

over a large range of ratios of the thickness to the height of the molten region by the author, and the first report⁽⁸⁾ showed variations of the Nusselt number on the heated cylinder and the molten volume with a Rayleigh number and a dimensionless time. The effects of the subcooling on the melting around a vertical cylinder were studied experimentally by Kemink and Sparrow⁽⁹⁾ and Kalhori and Ramadhyani⁽¹⁰⁾. It was found that the subcooling delays the onset of natural convection dominance and decreases the steady state heat transfer coefficient on the heated cylinder, and decreases not only the melting rate but also the rate of the total thermal energy storage. As these results were obtained from experiments with only one subcooling degree, the effects of different subcooling degrees on the melting and energy storage rates were not clarified. Also, the relation between the thickness of the molten region and the Nusselt number on the heated cylinder was not shown. The effects of the subcooling on the melting process from a vertical plate were studied analytically and experimentally by the author⁽¹¹⁾ and the following results were obtained. The Nusselt number on the heated plate is determined by the Rayleigh number and the

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thickness of the molten region, but it is independent on the subcooling degree. The subcooling does not significantly affect the total thermal energy stored. In the present paper, the melting around a vertical cylinder with laminar natural convection in the molten region is studied, and the effects of the aspect ratio of the cylinder and the subcooling on the Nusselt number on the cylinder, the melting rate, and the energy storage rate are clarified by an analysis and by experiments.

2. Analysis

Figure 1 shows the analytical model where the cylindrical co-ordinates system is used. A disk-like solid (height H and $r_0 < r < r_1$) is initially at a uniform temperature, T_i ($T_i < T_F$ where T_F is the fusion temperature). The melting is initiated by a step-wise temperature increase of the vertical cylinder with radius r_0 at the center of the disk. The surface temperature of the cylinder is maintained at T_0 ($T_0 > T_F$). The other boundaries of the disk are adiabatic. The assumptions in the analysis are as follows.

(1) The natural convection in the molten region is quasi-steady during the melting process and is laminar and two-dimensional. Boussinesq approximation is assumed and the viscous heat dissipation is neglected.

(2) The heat conduction in the solid region is two-dimensional and unsteady.

(3) The density difference between the solid and liquid phases of the PCM is neglected.

The following dimensionless governing equations are similar to those in the first report⁽⁸⁾, where the location of the melting front was represented by $r = f(z, t)$ and the governing equations were rewritten in dimensionless forms by using H as the characteristic length and $T_0 - T_F$ as the temperature difference. For the liquid region,

$$\begin{aligned} \frac{\partial \Omega}{\partial F_0} + Pr \left(\frac{U}{R} \frac{\partial \Omega}{\partial R} + \frac{V}{R} \frac{\partial \Omega}{\partial Z} \right) &= \frac{Ra}{R} \frac{\partial \theta}{\partial R} \\ + Pr \left(\frac{\partial^2 \Omega}{\partial R^2} + \frac{3}{R} \frac{\partial \Omega}{\partial R} + \frac{\partial^2 \Omega}{\partial Z^2} \right) & \end{aligned} \quad (1)$$

Fig. 1 Cylindrical co-ordinates system for the melting around a vertical cylinder

$$\begin{aligned} \frac{\partial \theta}{\partial F_0} + Pr \left(\frac{U}{R} \frac{\partial \theta}{\partial R} + \frac{V}{R} \frac{\partial \theta}{\partial Z} \right) \\ = \frac{\partial^2 \theta}{\partial R^2} + \frac{1}{R} \frac{\partial \theta}{\partial R} + \frac{\partial^2 \theta}{\partial Z^2} \end{aligned} \quad (2)$$

$$\frac{\partial^2 \Psi}{\partial R^2} - \frac{1}{R} \frac{\partial \Psi}{\partial R} + \frac{\partial^2 \Psi}{\partial Z^2} = -R^2 \Omega \quad (3)$$

$$U = \frac{\partial \Psi}{\partial z}, \quad V = -\frac{\partial \Psi}{\partial R}$$

For the solid region,

$$\frac{\alpha}{a_s} \frac{\partial \theta_s}{\partial F_0} = \frac{\partial^2 \theta_s}{\partial R^2} + \frac{1}{R} \frac{\partial \theta_s}{\partial R} + \frac{\partial^2 \theta_s}{\partial Z^2} \quad (4)$$

The energy balance equation at the melting front (at $R = F$),

$$\begin{aligned} \frac{\partial F}{\partial \tau} &= -\frac{\partial \theta}{\partial R} + \frac{\partial F}{\partial Z} \frac{\partial \theta}{\partial Z} \\ &- \frac{k_s}{k} \left(-\frac{\partial \theta_s}{\partial R} + \frac{\partial F}{\partial Z} \frac{\partial \theta_s}{\partial Z} \right) \end{aligned} \quad (5)$$

The initial condition: $F = R_0$ and $\theta_s = \gamma$ at $F_0 = 0$.

The boundary conditions: $\theta = 1$ at $R = R_0$, $\theta = \theta_s = 0$ at $R = F$, and the other boundaries are adiabatic. $U = V = 0$ and $\Psi = 0$ at $R = R_0$, $R = F$, $Z = 0$, and $Z = 1$.

The following dimensionless numbers and variables were used.

$$\begin{aligned} R &= r/H, \quad Z = z/H, \quad F_0 = at/H^2, \\ \theta &= (T - T_F)/(T_0 - T_F), \quad \theta_s = (T_s - T_F)/(T_0 - T_F), \\ \Omega &= H^2 \omega / (\nu R), \quad \Psi = \psi / (\nu H), \\ U &= ru/\nu, \quad V = rv/\nu, \quad Pr = \nu/\alpha, \\ Ra &= g\beta(T_0 - T_F)H^3 Pr/\nu^2, \quad Ste = c(T_0 - T_F)/\Delta h, \\ \tau &= F_0 \cdot Ste, \quad \gamma = (T_i - T_F)/(T_0 - T_F), \\ F &= f/H, \quad R_0 = r_0/H, \quad R_1 = r_1/H. \end{aligned}$$

The vorticity and the stream functions are defined respectively as

$$\omega = \frac{\partial v}{\partial r} - \frac{\partial u}{\partial z}, \quad ru = \frac{\partial \psi}{\partial z}, \quad rv = -\frac{\partial \psi}{\partial r}$$

In the above equations, u and v are the velocities in the r - and z -directions respectively, g is the gravitational acceleration, Δh is the latent heat-of-fusion; and as for the physical properties of the liquid phase, α is the thermal diffusivity, c is the specific heat, k is the thermal conductivity, β is the thermal expansion coefficient, ν is the kinematic viscosity, and ρ is the density. Subscript S denotes the solid phase.

The procedure of the analysis is similar to that for the non-subcooled case⁽⁸⁾; especially, the analysis of the liquid region is completely the same as that for the non-subcooled case. The boundary fixed method was used and the above equations were solved by a finite difference method. The steady state solution at every time step was obtained in the liquid region, and the unsteady solution in the solid region was obtained by a backward time difference method.

3. Experiments

The experimental enclosure is shown in Fig. 2. The experimental apparatus and procedure are simi-

lar to those in the first report. The central cylinder is made of a 18 mm O.D. copper tube. The height of PCM, H , is 25 mm and the outer diameter is 140 mm; therefore, $R_0=0.36$ and $R_1=2.8$. The enclosure is made of 25 mm thick styrene foam. The temperatures along the one-half height of the PCM were measured by the thermo-couples. The PCM used was 98 percent pure n -octadecane. Five enclosures filled with the PCM were placed horizontally in a temperature controlled box and the initial temperature distributions in the PCM were made uniform. The melting was initiated by circulating hot water through the cylinders in all the five enclosures from a constant temperature bath. After the melting was initiated, the temperature in the box was raised and kept about the fusion temperature. The melts were drained from the enclosures in turn at predetermined time intervals, and the shapes of the melting front were obtained by measuring the cross-sections of the left solid phases. The initial temperatures of the PCM were a little bit lower than the fusion temperature for the non-subcooled case, and 10 or 20°C lower for the subcooled case. The temperatures of the heated cylinder were 10 or 20°C higher than the fusion temperature.

Fig. 2 Experimental enclosure

Fig. 3 Melting front obtained from the analysis and the experiments. ($T_0 - T_F = 10^\circ\text{C}$, $\gamma=0$)

Fig. 4 Melting front obtained from the analysis and the experiments ($T_0 - T_F = 10^\circ\text{C}$, $\gamma=-2$)

Fig. 5 Melting front obtained from the analysis and the experiments ($T_0 - T_F = 20^\circ\text{C}$, $\gamma=-1$)

4. Results and Discussion

Both the experimental and analytical results of the locations of the melting front are shown in Figs. 3~5. The temperature difference in Figs. 3 and 4 is $T_0 - T_F = 10^\circ\text{C}$ (or $Ra=3.17 \times 10^6$, $Ste=0.094$). Figure 3 is for the non-subcooled case ($T_i - T_F=0$) and Fig. 4 is for the case with a large subcooling degree ($\gamma=-2$, or $T_i - T_F=-20^\circ\text{C}$). In Fig. 5, $T_0 - T_F=20^\circ\text{C}$ ($Ra=6.24 \times 10^6$, $Ste=0.187$) and $\gamma=-1$. The analytical results agree well with the experimental ones. The subcooling markedly depresses the melting rate. Though the subcooling delays the growth of the melting front at the lower part much more than at the upper part, it has a small effect on the shapes of the melting front. Both the experimental and analytical results of the temperature distributions along $Z=1/2$ corresponding to Fig. 5 are shown in Fig. 6. In the early stage of melting, the analytical results agree well with the experimental ones. As time passes, the measured temperatures in the solid region reach the fusion temperature sooner than the analyzed ones. In

the experiments, the ambient temperature of the enclosure was raised to about the fusion temperature after the start of melting. The heat which leaked into the PCM through the enclosure walls caused the deviation between the measured temperatures and the temperatures analyzed on the adiabatic boundary condition. In order to confirm the explanation mentioned above, an analysis and an experiment of heat conduction without melting were carried out. In the experiment, the same enclosure as shown in Fig. 2 was used, and the initial temperature of the PCM was $T_i - T_F = -20^\circ\text{C}$. The temperatures of the cylinder and the ambience were raised to $T_0 - T_F = -5^\circ\text{C}$. Figure 7 shows the temperature changes at $Z=1/2$. In Fig. 7 the measured temperatures are shown by the notes. The dashed line represents the results of the one-dimensional heat conduction analysis in which the

Fig. 6 Temperature distributions along $Z=1/2$, corresponding to Fig. 5

Fig. 7 Time-wise changes of the temperatures at $Z=1/2$ in the heat conduction without melting ($T_i - T_F = -20^\circ\text{C}$, $T_0 - T_F = 15^\circ\text{C}$)

enclosure walls are regarded as being adiabatic. The solid line represents the solution of the two-dimensional heat conduction analysis in which the heat conduction in the enclosure walls was taken into account and the heat transfer coefficient between the walls and the ambience was assumed to be $1.163 \text{ W}/(\text{m}^2 \cdot \text{K})$. As the experimental results agree with the results of the two-dimensional analysis, it is found that the deviation between the analyzed temperatures and the measured ones in Fig. 6 was caused by the fact that the enclosure walls were not completely adiabatic.

Figure 8 shows the variation of the dimensionless molten volume V_{oi} with dimensionless time, where the molten volume was divided by the volume of the heated cylinder.

$$V_{oi} = \frac{1}{R_0^2} \int_0^1 F^2 dZ - 1 \quad (6)$$

All analytical results agree well with the experimental ones. As $|\gamma|$ increases, the melting rate decreases markedly. Figure 9 shows the analytical results of the variation of the average Nusselt number $\overline{Nu}_0 = \overline{h}_0 H/k$ on the heated cylinder with dimensionless time, where the conditions are same as for Fig. 8, and the characteristic length is H , and the temperature

Fig. 8 Variation of the dimensionless molten volume with dimensionless time

difference is $T_0 - T_F$. In the early stage of melting, where the heat conduction dominates the heat transfer in the molten region, the average Nusselt number decreases rapidly. As the molten region expands, the natural convection becomes dominant. Therefore, the average Nusselt number increases after it reaches the minimum. As the subcooling decreases the melting rate, the increase of the average Nusselt number in the natural convection dominance period is depressed. This effect of subcooling is clearly shown by the result in the case of a large degree of subcooling, namely the result of $\gamma = -2$ in Fig. 2. The $\bar{Nu}_0$ of $\gamma = -2$ in Fig. 9 has a large increasing rate after nearly $\tau = 0.15$. This corresponds to the increase of the melting rate in Fig. 8. The subcooling delays the onset of natural convection dominance in the present melting from a cylinder with a constant temperature, which is similar to the melting from a cylinder with a constant heat flux⁽⁹⁾; but this effect of the subcooling is minimal in the present melting. The relation between the average Nusselt number and the average melting front, $\bar{F}$, obtained by the volumetric mean is shown by solid lines in Fig. 10, where

$$\bar{F} = \left(\int_0^1 F^2 dZ \right)^{1/2} \quad (7)$$

The dashed line in Fig. 10 is the result of the melting from a vertical plate⁽¹¹⁾, in which the heated surfaces are put together. The average Nusselt numbers with different subcooling degrees almost coincide with one another when each Rayleigh number is the same. This means that the difference of the shapes of the melting front based on the different subcooling degrees is small; and also the difference of the average Nusselt numbers based on the difference of the shapes of

Fig. 9 Variation of the average Nusselt number on the heated cylinder with dimensionless time

melting front is small. The $\bar{Nu}_0$ for the melting from a cylinder is the same as for the melting from a plate when the heat conduction dominates. As the area of the melting front enlarges with time more rapidly in the melting from a cylinder than from a plate, the increasing rate of the curve of $\bar{Nu}_0 - \bar{F}$ in the period of the natural convection dominance is larger in the melting from a cylinder than from a plate.

The thermal energies stored are represented as follows, where the dimensionless molten volume is V_{ol} and the dimensionless solid volume is V_{os} and the average temperature in the solid region is $\bar{T}_s$. Dividing the thermal energies by the latent heat-of-fusion thermal energy in the heated cylinder volume yields the following dimensionless expressions, where Q_s^* is the thermal energy stored as sensible heat in the solid region, and Q_L^* is the thermal energy stored as both the latent heat and sensible heat in the molten region.

$$Q_s^* = Ste \{ V_{os} \bar{\theta}_s - (V_{ol} + V_{os}) \gamma \} (c\rho)_s / (c\rho) \quad (8)$$

$$Q_L^* = V_{ol} (1 + Ste/2) \quad (9)$$

$$V_{os} = (R_1/R_0)^2 - (\bar{F}/R_0)^2 \quad (10)$$

$$\bar{\theta}_s = (\bar{T}_s - T_F) / (T_0 - T_F) \quad (11)$$

In Eq. (9) the average temperature in the molten region is assumed to be $(T_0 + T_F)/2$. The dimensionless total thermal energy storage Q_T^* is

$$Q_T^* = Q_s^* + Q_L^* \quad (12)$$

Figure 11 shows the variations of the dimensionless thermal energy storage with dimensionless time. When $|\gamma| \leq 1$, the subcooling does not significantly affect the total thermal energy storage, but the total thermal energy storage for the case of $\gamma = -2$ is 20–25% less than for the non-subcooled case. The experimental result obtained by Kalhori and Ramadhyani for the case of $\gamma = -2$ under different conditions from the present study was about 25–30% less than for the non-subcooled case. The decrease of the total thermal energy storage for the case of $\gamma = -2$ is due to the

Fig. 10 Relation between the average Nusselt number and the average location of melting front

delay of increase of the Nusselt number in the natural convection dominance period in Fig. 9. On the other hand, it has been found that the subcooling does not significantly affect the total thermal energy storage in the melting from a plate⁽¹¹⁾. This is because the change of Nusselt number in the natural convection dominance period is small, as shown by the dashed line in Fig. 10. In Fig. 11, the sensible heat storage in the solid region does not depend on the Rayleigh number, but depends mainly on the temperature difference $T_i - T_F$.

For the non-subcooled case, the effects of the aspect ratio of the cylinder are discussed below. Figure 12 shows the relation between the average Nusselt number on the heated cylinder and the ratio of the average location of the melting front to the radius of the cylinder, $\bar{F}/R_0$, for the different aspect ratios of the cylinder, or for $R_0=0.36$ and 0.6 . It is reported⁽¹²⁾ that the average Nusselt number of the natural convection in a vertical annulus (f =the outer radius, r_0 =the inner radius) is able to be determined by the Rayleigh number but it does not depend on the height-to-gap width ratio, if the height of the annulus, H , is chosen as the characteristic length and the radius ratio, f/r_0 , is fixed. Roughly speaking, even in the melting process, the $\bar{Nu}_0$ in the natural convection dominance period does not depend on R_0 . Therefore, it is determined by $\bar{F}/R_0$ and the Rayleigh number. A one-dimensional approximate analysis of the melting is carried out in order to find the dimensionless parameters. In the analysis, the period of heat conduction dominance is neglected because it is very short and the average Nusselt number is assumed to be

expressed by

$$\bar{Nu}_0 = CR_a^m (\bar{F}/R_0)^n \quad (13)$$

The following fundamental equation is obtained from the energy balance at the melting front.

$$\bar{F} \frac{d\bar{F}}{d\tau} = \bar{Nu}_0 R_0 \quad (14)$$

By putting $\bar{F}/R_0=1$ at $\tau=0$, the following solution is obtained.

$$\frac{1}{2-n} \left\{ \left(\frac{\bar{F}}{R_0} \right)^{2-n} - 1 \right\} = CR_a^m \frac{\tau}{R_0} \quad (15)$$

Substituting Eqs. (6) and (7) into Eq. (15) yields

$$\frac{1}{2-n} \left\{ (V_{oi} + 1)^{(2-n)/2} - 1 \right\} = CR_a^m \frac{\tau}{R_0} \quad (16)$$

Equation (16) shows that the molten volume V_{oi} is a function of τ/R_0 which is a dimensionless time. The variations of the dimensionless molten volume with this dimensionless time in the cases of the different R_0 are shown in Fig. 13. The results with the different diameters of the cylinder and with same Rayleigh number agreed approximately with each other when dimensionless time, τ/R_0 , was used.

5. Conclusions

The effects of the initial subcooling and the aspect ratio of the heated cylinder on the heat transfer during the melting around a vertical cylinder with the laminar natural convection in the molten region were studied analytically and experimentally. The following results were obtained.

(1) The melting rate is decreased markedly by the relatively small subcooling degree. The total thermal energy storage is not affected significantly by the subcooling when $\gamma=-1$ but it is decreased more than 20% when the subcooling degree is as large as $\gamma=-2$.

(2) The overage Nusselt number on the heated

Fig. 11 Variation of the dimensionless thermal energy storages with dimensionless time

Fig. 12 Relation between the average Nusselt number and the ratio of the average location of the melting front to the cylinder radius for the non-subcooled case

Fig. 13 Variation of the dimensionless molten volume with the dimensionless time, τ/R_0 , corresponding to Fig. 12

cylinder does not depend on the subcooling degree if the average locations of the melting front are the same.

(3) In the non-subcooled case, the effects of the difference of cylinder diameter on the molten volume are able to be arranged by the dimensionless time, τ/R_0 .

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References

- (1) Saitoh, T. and Hirose, K., Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 51, No. 462, B (1985), p. 705.
- (2) Sasaguchi, K., Imura, H. and Furusho, H., Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 52, No. 473, B (1986), p. 167.
- (3) Saito, A., Utaka, Y., Okawa, S. and Katayama, K., Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 52, No. 475, B (1986), p. 1070.
- (4) Imura, H. and Yoshida, M., Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 52, No. 476, B (1986), p. 1850.
- (5) Hayashi, Y., Takimoto, A. and Kawahara, S., Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 52, No. 477, B (1986), p. 2187.
- (6) Saito, A., Saito, A., Utaka, Y., Okuda, K. and Katayama, K., Trans. Jpn. Assoc. Refrig., (in Japanese), Vol. 3, No. 1, (1986), p. 51.
- (7) Sparrow, E. M., Patankar, S. V. and Ramadhyani, S., Trans. ASME, Vol. 99, C (1977), p. 520.
- (8) Okada, M., Bull. JSME., Vol. 28, No. 243 (1985), p. 2007.
- (9) Kemink, R. G. and Sparrow, E. M., Int. J. Heat Mass Transf., Vol. 24, No. 10 (1981), p. 1699.
- (10) Kalhori, B. and Ramadhyani, S., Trans. ASME, J. Heat Transf., Vol. 107, (1985), p. 44.
- (11) Okada, M. and Koyanagi, Y., Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 52, No. 481, B (1986), p. 3287.
- (12) Prasad, V. and Kulacki, F. A., Trans. ASME, J. Heat Transf., Vol. 107, (1985), p. 596.