Theoretical Consideration on the Fundamental Behavior of Conducting Magnetic Fluids* (Couette-like Flow)

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Generally, a conducting magnetic fluid has a considerably complex behavior that depends upon the electromagnetic field. Therefore, even the fundamental conducting magnetic fluid flow has not been sufficiently examined. In the present paper, a Couette-like flow is discussed, which is so important and fundamental in engineering problems. At first, some nondimensional parameters caused by magnetization are newly defined, and some relations between various nondimensional parameters defined in usual magnetohydrodynamics and the parameters newly defined here are given. The influences of these newly defined parameters on an electromagnetic field and a flow field are clarified. Furthermore, a pressure distribution caused by electromagnetic forces is analytically obtained which does not appear in the nonconducting magnetic fluid.

Key Words: Conducting Magnetic Fluid, Non-Newtonian Fluid, Magnetohydrodynamics, Couette Flow, Theoretical Consideration, Fundamental Behavior

1. Introduction

Conducting fluids have been sufficiently examined in magnetohydrodynamics. However, the behavior of magnetic fluids whose areas of application have been rapidly expanding have yet to be adequately explained. A magnetic fluid is produced by scattering ferromagnetic substances (e.g. magnetite), whose particle diameter is about 10 nm, into a nonconducting fluid such as water or kerosine. On the one side, conducting diamagnetic fluids are treated in plasma physics. On the other side, however, usual magnetohydrodynamics treat only nonconducting magnetic fluids (strictly speaking, paramagnetic fluids). The trial productions of magnetic fluids with conductivity have recently been made, thus expanding the fields of application. In keeping with this, a system of basic equa-

tions of conducting magnetic fluids was derived by Shizawa and Tanahashi⁽¹⁾ using the thermodynamical method. The behavior of conducting magnetic fluids, however, is considerably complex; the phenomena are generally concerned with the electromagnetic and flow fields. Hence, even for fundamental flows, solutions of electromagnetic and flow fields are derived only in a few special cases under a few assumptions. Therefore, the fundamental conducting magnetic fluid flow has not yet been adequately examined. In the present paper, using the equations proposed in the previous report, the analytical solutions of the Couette-like flow of conducting magnetic fluids (e.g., flow field, electromagnetic field, temperature field) are obtained in the case where velocity vector $\mathbf{v}$, induced electric field vector $\mathbf{E}_0$ and induced magnetic field $\mathbf{H}_0$ are perpendicularly intersected with each other (see Fig. 1). Then, a few new nondimensional parameters which originate in the magnetization of the fluid are defined. Furthermore, some relations between nondimensional parameters defined in usual magnetohydrodynamics and those newly defined here are given. The influences of these newly defined parameters on the electromagnetic field and flow field are clarified.

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Furthermore, pressure distributions caused by electromagnetic forces, which do not appear in nonconducting magnetic fluids, are analytically obtained. The mother liquors of conducting magnetic fluids are liquid metals (e.g. mercury or mixture of sodium and potassium), so that their thermal conductivities are remarkably high compared with the usual magnetic fluids. It is considered that these properties can be applied technologically to energy conversion, heat exchange and oil pressure technology, etc. Few papers, however, obtain the temperature field by using the exact energy equation. In the present paper, the influences of these flow fields and of electromagnetic fields on temperature fields are particularly discussed, in regard to applications in heat exchangers and the like. Formerly, the effects of an applied electric field on a flow field were not analyzed theoretically. In this study, it is shown analytically that the influence of an induced electric field on a flow field changes with respect to the induced magnetic field. As to the Couette flow of a conducting magnetic fluid, some physical quantities, i.e., velocity, temperature, flow flux, electromagnetic field and pressure, etc., which are of technical interest are shown analytically. Moreover it is shown that these are controlled artificially by changing the induced electromagnetic field. The various solutions obtained in this analysis are all strict analytical solutions, so would be available the estimation of various technological models.

Nomenclature

- $\mathbf{B}$: magnetic flux density vector
 $\mathbf{D}$: electric flux density vector
 $\mathbf{D}^{(v)}$: rate of deformation tensor
 d : distance of two parallel plates
 $\mathbf{E}$: electric field vector
 $\mathbf{H}$: magnetic field vector
 H_{a1} : Hartmann's number
 $= \mu H_0 d \sqrt{\sigma/\eta}$
 H_{a2} : Hartmann's number of magnetization =
 $H_0 d \sqrt{\sigma \mu \chi_m / \eta}$
 $\mathbf{j}$: electric current density vector
 $\mathbf{M}$: magnetization vector

Fig. 1 Flow in two parallel plates (Analytical model)

$\mathbf{P}$: polarization vector

p : pressure

Q : volume flow rate

Re_m : magnetic Reynolds number $= \sigma \mu V d$

r : intensity of heat source per unit mass

s : entropy per unit mass

T : absolute temperature

t : time

$\mathbf{v}$: velocity vector $= (v(z), 0, 0)$

V : speed of upper wall

α : thermal effect parameter of viscosity

$$= V \sqrt{\frac{\eta}{\kappa(T_2 - T_1)}}$$

β : thermal effect parameter of Joule heating =

$$\mu V H_0 d \sqrt{\frac{\sigma}{\kappa(T_2 - T_1)}}$$

γ : electric field effect parameter $= E_0 / (\mu V H_0)$

η : shear viscosity

Θ : magnetic field gradient parameter

$$= H_0 \sqrt{\chi_m d / \eta V}$$

κ : coefficient of heat conductivity

μ : magnetic permeability

μ_0 : magnetic permeability in a vacuum

ρ : mass density

ρ_e : electric charge density

σ : electric conductivity

χ_m : magnetic susceptibility

Suffixes

0 : induced amounts

e : amounts concerned with electric field

$*$: nondimensional amounts

m : amounts concerned with magnetic field

2. Analytical Model and Basic Equations

2.1 Analytical model

An incompressible ($\nabla \cdot \mathbf{v} = 0$) and nonpolarized conducting magnetic fluid fills the space between quasi-infinite parallel plates separated by distance d . One plate continues linear and constant motion with velocity V parallel to the other plate. The temperatures of the plates are assumed to be T_1 and T_2 , respectively, and the difference of temperature in the fluid small (see Fig. 1).

2.2 Basic equations

The equations shown in the previous report for conducting magnetic fluids are as follows.

(Equation of motion)

$$\begin{aligned} \rho \frac{d\mathbf{v}}{dt} = & -\nabla p + \eta \nabla^2 \mathbf{v} + \rho_e \mathbf{E} + \mathbf{j} \times \mathbf{B} \\ & + (\mathbf{M} \cdot \nabla) \mathbf{H} + \mathbf{M} \times (\nabla \times \mathbf{H}) + (\mathbf{P} \cdot \nabla) \mathbf{E} \\ & + \mathbf{P} \times (\nabla \times \mathbf{E}) + \rho \mathbf{b} \end{aligned} \quad (1)$$

(Equation of energy)

$$\rho T \frac{ds}{dt} = \kappa \nabla^2 T + 2\eta \mathbf{D}^{(v)} : \mathbf{D}^{(v)}$$

$$+(\mathbf{j} - \rho_e \mathbf{v})^2 / \sigma + \rho r \quad (2)$$

(Equation of electromagnetic field)

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (3)$$

$$\nabla \times \mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t} \quad (4)$$

$$\nabla \cdot \mathbf{D} = \rho_e \quad (5)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (6)$$

$$\mathbf{j} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}) + \rho_e \mathbf{v} \quad (7)$$

2.3 Boundary conditions

Boundary conditions of each unknown are as follows:

$$v(0) = 0; \quad v(d) = V$$

$$T(0) = T_1; \quad T(d) = T_2$$

As to the induced magnetic field, which will be shown later, only x -component H_x is induced, and the following boundary condition is obtained:

$$-H_x(0) = H_x(d) = \frac{1}{2} \int_0^d j_y dz$$

At the point $z = z_0$, the boundary condition is $H_x(z_0) = 0$. Here, z must obey the following equation.

$$\int_{z_0}^d j_y dz = \int_0^{z_0} j_y dz$$

As to the pressure p , it is standardized by the pressure of the point $z = z_0$ where the magnetic pressure is zero:

$$p(z_0) = 0$$

2.4 Assumptions

The assumptions used in the present paper are as follows.

- (1) The fluid is nonpolarized and incompressible.
- (2) Dependency of the viscosity and thermal conductivity on the temperature is neglected.
- (3) Flow and electromagnetic fields are steady, so the electric charge density in the fluid is kept zero⁽²⁾.
- (4) There is no pressure gradient.
- (5) Magnetic susceptibility does not depend on a magnetic field.
- (6) Gravity is neglected.
- (7) Upper and lower walls are made of insulators.
- (8) Electric conductivity and thermal conductivity do not depend on a magnetic field⁽³⁾⁽⁴⁾.
- (9) Polarization of the fluid is neglected.
- (10) Hall effect is neglected.

3. Normalization of Equations

3.1 Nondimensional expressions of physical variables

Each physical variable is normalized in the following way:

$$\text{coordinate : } z = dz^*$$

$$\text{velocity : } v = Vv^*$$

$$\text{electric current : } \mathbf{j} = \sigma \mu V H_0 \mathbf{j}^*$$

$$\text{temperature : } T = (T_2 - T_1) T^* + T_1$$

$$\text{induced magnetic field : } H_x = H_0 H_x^*$$

$$\text{magnetization : } \mathbf{M}_x = \chi_m H_0 \mathbf{M}_x^*$$

$$\text{pressure : } p = (\eta V/d) p^*$$

$$\text{volume flow rate : } Q = VdQ^*$$

3.2 Electromagnetic field equations

By the previous assumptions, Maxwell's equations concerned with the electromagnetic field are expressed as follows:

$$\nabla^* \times \mathbf{E}^* = 0 \quad (8)$$

$$\nabla^* \times \mathbf{H}^* = R_{em} \mathbf{j}^* \quad (9)$$

$$\nabla^* \cdot \mathbf{D}^* = 0 \quad (10)$$

$$\nabla^* \cdot \mathbf{B}^* = 0 \quad (11)$$

where

$$R_{em} = \sigma \mu Vd \quad (12)$$

The following equation is obtained by the Ohm's law.

That is,

$$\mathbf{j}^* = \gamma \mathbf{E}^* + \mathbf{v}^* \times \mathbf{B}^* \quad (13)$$

where

$$\gamma = E_0 / (\mu V H_0) \quad (14)$$

This nondimensional parameter γ expresses the ratio of the applied electric field and the induced electric field, which is defined as electric field effect parameter.

3.3 Equation of motion

By applying the previous assumptions and normalizations to Eq. (1), Eq. (15) is obtained:

$$0 = -\nabla^* p^* + \nabla^{*2} \mathbf{v}^* + (H_{a1}^2 - H_{a2}^2) \mathbf{j}^* \times \mathbf{H}^* + \Theta^2 (\mathbf{M}^* \cdot \nabla^*) \mathbf{H}^* \quad (15)$$

where

$$H_{a1} = \mu H_0 d \sqrt{\sigma / \eta} \quad (16)$$

$$H_{a2} = H_0 d \sqrt{\sigma \mu \chi_m / \eta} = H_{a1} \sqrt{\chi_m / \mu} \quad (17)$$

$$\Theta = H_0 \sqrt{\frac{\chi_m d}{\eta V}} = H_{a2} / \sqrt{R_{em}} \quad (18)$$

Here, H_{a1} is Hartmann's number, and the other two parameters, H_{a2} and Θ , are newly defined, because the fluids have magnetization. H_{a2} means the ratio of magnetic viscosity force by magnetization to viscosity force. H_{a2} is defined as Hartmann's number of magnetization. Θ means the ratio of magnetic force caused by magnetic field gradients to viscosity force. Θ is defined as the magnetic field gradient parameter. The definitions of H_{a1} , H_{a2} and Θ , and the relations between the usual nondimensional parameters are expressed in Eqs. (16)–(18).

3.4 Equation of energy

By applying the previous assumptions and normalizations to Eq. (2), Eq. (19) is obtained:

$$0 = \nabla^{*2} T^* + 2\alpha^2 \mathbf{D}^{(v)*} : \mathbf{D}^{(v)*} + \beta j^{*2} \quad (19)$$

where

$$\alpha = V \sqrt{\frac{\eta}{\kappa(T_2 - T_1)}} \quad (20)$$

$$\beta = \mu V H_0 d \sqrt{\frac{\sigma}{\kappa(T_2 - T_1)}} = \alpha H_{a1} \quad (21)$$

Here, α means the ratio of the thermal conduction by

the gradient of temperature to the thermal dissipation by viscosity. α is defined as the thermal effect parameter of viscosity. α does not become zero, by the physical requirement. (If $\alpha=0$, then the flow cannot be Couette flow.) β means the ratio of the thermal conduction by the gradient of temperature to the thermal dissipation by Joule heating. β is defined as the thermal effect parameter of Joule heating.

4. Component Expressions of Equations

4.1 Equations of electromagnetic field

By using Eq.(22) derived from Maxwell's equations and Ohm's law, let us consider each component of an electromagnetic field.

$$\nabla \times \mathbf{H} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad (22)$$

The component expressions are as follows:

$$-\frac{dH_y}{dz} = \sigma E_x \quad (23)$$

$$\frac{dH_x}{dz} = \sigma(E_y - v\mu H_z) \quad (24)$$

$$0 = \sigma(E_z + v\mu H_y) \quad (25)$$

As the electric current does not flow in the upper and lower walls, the z -component of the magnetic field is uniform. By using Stokes' law in regard to the magnetic field, we obtain $|H_x(d) - H_x(0)| = I$. Here, I is the quantity of electric current passing through a cross section of the channel in the y -direction, per unit width. As to the y -component of the magnetic field, $H_y(0) = H_y(d) = 0$, because there is no electric current in the direction of fluid flow, both inside and outside of the fluid. In regard to the electric field, we obtain $E_x(0) = E_x(d) = 0$. Because the uniform electric field is applied in the y -direction, inside and outside of the fluid, a tangential component of the electric field is continual at the boundary. In addition to this, $E_x = \text{constant}$ is obtained by Maxwell's equation $\nabla \times \mathbf{E} = 0$, so we find $E_x = 0$. Substituting this into Eq.(23), $H_y = \text{constant}$ is derived, and H_y must be zero throughout the cross section by the boundary condition. Thereby, $E_z = 0$ is obtained from Eq.(25). As to H_z , $H_z = \text{constant}$ is obtained from Eq.(11). However, it is the applied magnetic field H_0 . Generally, as to the electromagnetic field, its equations must be solved in the fluid, inside and outside of the walls. If there is no induced electric current in the walls and magnets, however, the boundary conditions of the electromagnetic field are obtained by the continuity of the tangential component of electromagnetic field at the boundary surface as mentioned above. The boundary conditions are not dependent on the dielectric constants and magnetic permeabilities of the substances of the walls and magnets. Moreover, the electric current induced outside of the fluid is inhibited by sandwiching the insulators between magnets. As dis-

cussed above, each component of the electromagnetic field can be described as follows:

$$\mathbf{H}^* = (H_x^*, 0, 1), \mathbf{E}^* = (0, 1, 0) \quad (26)$$

By substituting Eq.(26) into Eq.(9), the following equation is obtained.

$$\frac{dH_x^*}{dz^*} = Re_m j_y^*, \mathbf{j}^* = (0, j_y^*, 0) \quad (27)$$

Accordingly, the electric current flows in the y -direction only. Equation(13) is then reduced to

$$j_y^* = \gamma - v^* \quad (28)$$

4.2 Equation of motion

Component expressions of Eq.(15) are given as follows:

$$0 = \frac{d^2 v^*}{dz^{*2}} + (H_{a1}^2 - H_{a2}^2) j_y^* + \Theta^2 Re_m j_y^*$$

$$0 = \frac{dp^*}{dz^*} + (H_{a1}^2 - H_{a2}^2) j_y^* H_x^*$$

The y -component of the equation of motion expresses that there is no pressure gradient in the y -direction. By substituting Eq.(28) into the above equations, the following equations are obtained:

$$0 = \frac{d^2 v^*}{dz^{*2}} + H_{a1}^2 (\gamma - v^*) \quad (29)$$

$$0 = \frac{dp^*}{dz^*} + (H_{a1}^2 - H_{a2}^2) (\gamma - v^*) H_x^* \quad (30)$$

Hence, velocity distributions have nothing to do with the Hartmann's number of magnetization H_{a2} and magnetic field gradient parameter Θ . The pressure p^* , however, does depend on Hartmann's number of magnetization.

4.3 Equation of energy

Equation (19) combined with Eq.(28) leads to Eq. (31)

$$0 = \frac{d^2 T^*}{dz^{*2}} + \alpha^2 \left(\frac{dv^*}{dz^*} \right)^2 + \beta^2 (\gamma - v^*)^2 \quad (31)$$

5. Theoretical Analysis and Discussion

5.1 Velocity

By solving the governing equation of velocity (Eq.(29)) with the boundary condition, the following solution of velocity is obtained. (So far as there is no special note, the asterisks are hereafter omitted with regard to the nondimensional quantities.)

$$v(z) = \frac{\sinh(H_{a1}z)}{\sinh H_{a1}} + \gamma \left\{ 1 - \frac{\sinh H_{a1}(1-z) + \sinh H_{a1}z}{\sinh H_{a1}} \right\} \quad (32)$$

The function $v(z)$ which expresses the velocity profiles is governed by two independent nondimensional parameters γ and H_{a1} . The function $v(z)$ however, does not depend on Θ and H_{a2} which express the influences of magnetization and the magnetic field gradient, respectively. Equation (29) can be transformed into the following form:

$$v = \gamma + \frac{1}{H_{a1}^2} \frac{d^2 v}{dz^2}$$

In the above equation, as the applied magnetic field or H_{a1} increases, the second term of the right side becomes relatively small, compared to the first term which is constant. Hence, the velocity profiles have a tendency to be $v \simeq \gamma$ at the central region of the two

Fig. 2 Velocity profiles (dependency on electric field effect parameter)

Fig. 3 Velocity profiles (dependency on Hartmann's number) (a)

Fig. 4 Velocity profiles (dependency on Hartmann's number) (b)

plates. Figure 2 shows this effect in the case of $H_{a1} = 15$. Lorentz forces acting upon fluid elements are able to be expressed in nondimensional form as follows:

$$\mathbf{F} = j_y (H_z \mathbf{i}_x - H_x \mathbf{i}_z) \quad (33)$$

where j_y contains the applied electric current and induced electric current. The "applied electric current" means the current which is caused by the applied electric field. The induced electric current is opposite in direction to the applied electric current. Hence, the Lorentz force originating in the applied electric current accelerates the fluid, but, the Lorentz force originating in the induced electric current decelerates the fluid. Corresponding to the Lorentz forces originated in the induced electric currents, the condition of flow has three patterns, as shown in Figs. 3-5. When the applied electric current is relatively small (i.e. $\gamma < 0.5$), the applied magnetic field acts upon the fluid to decelerate it (see Fig. 3). In the case of $\gamma = 0.5$, the Lorentz forces produced by applied electric current and by induced electric current balance each other (see Fig. 4). Moreover, when $\gamma > 0.5$, the flow is ac-

Fig. 5 Velocity profiles (dependency on Hartmann's number) (c)

Fig. 6 Volume flow rate (dependency on Hartmann's number)

celerated by the Lorentz force originating in both the applied electric field and applied magnetic field (see Fig. 5). Figure 6 shows that volume flow rate variations depend on H_{a1} . The volume flow rate increases linearly with increasing the applied electric field, as shown in Fig. 7. The characteristics of the velocity profiles are reflected well in these figures.

5.2 Electric current density

By substituting the solution $v(z)$ into Eq.(28), the following equation is obtained:

$$j_y = \frac{1}{\sinh H_{a1}} [\gamma \sinh\{H_{a1}(1-z)\} + (\gamma-1)\sinh H_{a1}z] \quad (34)$$

The function $j_y(z)$ which expresses the electric current density distributions is governed by two independent nondimensional parameters γ and H_{a1} . The dependencies on Hartmann's number of electric current density distribution are easy to understand by Figs. 3-5 and Eq.(28). According to these three patterns of velocity profiles, the electric current density

distribution has three patterns, too. Particularly, in the case of $\gamma=0$ (i.e. there is no applied electric current), the electric current flows in the $-y$ -direction throughout the whole range of channel. The dependencies of electric current density distributions on the electric field effect parameter are shown in Figs. 8-10. As shown in the three figures, corresponding to an increase of applied magnetic field, the curves approach the center axis (i.e. $j_y=0$). In other words, increase of the applied magnetic field reduces the electric current.

5.3 Induced magnetic field

By substituting Eq.(34) into Eq.(27) and applying the boundary condition shown above, the solution is given by

$$H_x = \frac{R_{em}}{H_{a1}} [(\gamma-1-\gamma \cosh H_{a1}) \times (\cosh H_{a1}z - \cosh H_{a1}z_0) + \gamma \sinh H_{a1}(\sinh H_{a1}z - \sinh H_{a1}z_0)] \quad (35)$$

In the above equation, z_0 is expressed as follows by

Fig. 7 Volume flow rate (dependency on electric field effect parameter)

Fig. 9 Electric current density distributions (dependency on electric field effect parameter) (b)

Fig. 8 Electric current density distributions (dependency on electric field effect parameter) (a)

Fig. 10 Electric current density distributions (dependency on electric field effect parameter) (c)

defining $A = 1 - \gamma + \gamma \cosh H_{a1}$ and $B = \gamma \sinh H_{a1}$. In the case of $A > B$:

$$z_0 = \left\{ \operatorname{arctanh} \frac{B}{A} - \operatorname{arccosh} \frac{1 + \cosh H_{a1}}{2\sqrt{A^2 - B^2}} \right\} / H_{a1}$$

In the case of $A < B$:

$$z_0 = \left\{ \operatorname{arctanh} \frac{A}{B} - \operatorname{arcsinh} \frac{1 + \cosh H_{a1}}{2\sqrt{B^2 - A^2}} \right\} / H_{a1}$$

where

$$\operatorname{arccosh} \frac{1 + \cosh H_{a1}}{2\sqrt{A^2 - B^2}}$$

can be a positive or a negative value. The sign is determined by the physical requirement that z_0 must be a positive value. The function $H_x(z)$ which expresses induced magnetic field distributions is governed by three independent nondimensional parameters, Re_m , H_{a1} and γ . Figures 11-13 show the dependency on Hartmann's number of H_x in the case that the magnetic Reynolds number is 30. The magnetic field induced on the surface of the parallel plates (i.e. $H_x(0)$ and $H_x(1)$) have the same magnitude but opposite sense. $H_{a1} = 0$ means that a very small magnetic field

is applied. The gradient of the induced magnetic field gives induced electric current.

5.4 Pressure

When the magnetic field H_x is induced, the pressure is changed in the z -direction (see Eq.(39)). But if $H_{a1} = H_{a2}$ there is no pressure gradient. As Eq.(17) shows, however, we should note that it cannot be $H_{a1} = H_{a2}$ physically, except $H_{a1} = H_{a2} = 0$ (i.e. there is no applied magnetic field). By considering Eq.(27) and Eq.(28) with the boundary condition, Eq.(30) is solved as follows:

$$p = \frac{Re_m(H_{a2}^2 - H_{a1}^2)}{2H_{a1}^2} [(\gamma - 1 - \gamma \cosh H_{a1}) \times (\cosh H_{a1}z - \cosh H_{a1}z_0) + \gamma \sinh H_{a1} \times (\sinh H_{a1}z - \sinh H_{a1}z_0)]^2 \quad (36)$$

The function $p(z)$ which expresses pressure distributions is governed by four independent nondimensional parameters H_{a1} , H_{a2} , Re_m and γ . Here, Hartmann's number of magnetization H_{a2} governs the pressure amplitude only; therefore, the pressure distributions are not affected by H_{a2} . Figures 14 and 15 show Hartmann's number dependency of pressure distributions in the case of $Re_m = 30$. Furthermore, electric field effect parameter dependencies are shown in Fig. 16 and Fig. 17. As is obvious from Eq.(33), Lorentz forces driven by the induced magnetic field decide the pressure distributions.

5.5 Temperature

By substituting Eq.(32) into energy equation Eq.(31) and applying the boundary condition, the solution is given as Eq.(37). Here, the first term of the right side shows the effect of viscosity, the second term shows the effect of Joule heating and the third term shows the effect of thermal conduction, respectively. Generally the viscosity term $T_a(z)$ can be assumed to

Fig. 11 Induced magnetic field distributions (dependency on Hartmann's number) (a)

Fig. 12 Induced magnetic field distributions (dependency on Hartmann's number) (b)

Fig. 13 Induced magnetic field distributions (dependency on Hartmann's number) (c)

Fig. 14 Pressure distributions (dependency on Hartmann's number) (a)

Fig. 16 Pressure distributions (dependency on electric field effect parameter) (a)

Fig. 15 Pressure distributions (dependency on Hartmann's number) (b)

Fig. 17 Pressure distributions (dependency on electric field effect parameter) (b)

be sufficiently smaller than the Joule heating term $T_\beta(z)$.

$$T(z) = T_a(z) + T_\beta(z) + z \quad (37)$$

where

$$T_a(z) = -\frac{\beta^2}{4H_{a1}^2 \sinh^2 H_{a1}} [(\gamma-1)^2 \{ \sinh^2(H_{a1}z) + H_{a1}^2 z^2 \} + 2\gamma(\gamma-1) \{ \sinh H_{a1} \times \{ \sinh(H_{a1}z) \cosh(H_{a1}z) - H_{a1}z \} - \cosh H_{a1} \{ \sinh^2(H_{a1}z) + H_{a1}^2 z^2 \} \} + \gamma^2 \{ \sinh^2 \{ H_{a1}(1-z) \} - \sinh^2 H_{a1} - 2zH_{a1}^2 + z^2 H_{a1}^2 \} - (\gamma-1)^2 \times \{ \sinh^2 H_{a1} + H_{a1}^2 \} z - 2\gamma(\gamma-1) \times \{ \sinh H_{a1} (\sinh H_{a1} \cosh H_{a1} - H_{a1}) - \cosh H_{a1} (\sinh^2 H_{a1} + H_{a1}^2) \} z + \gamma^2 (H_{a1}^2 + \sinh^2 H_{a1}) z] \quad (38)$$

$$T_\beta(z) = -\frac{\beta^2}{4H_{a1}^2 \sinh^2 H_{a1}} [(\gamma-1)^2 \times \{ \sinh^2(H_{a1}z) - H_{a1}^2 z^2 \} + 2\gamma(\gamma-1) \times \{ \sinh H_{a1} \{ \sinh(H_{a1}z) \cosh(H_{a1}z) - H_{a1}z \} - \cosh H_{a1} \{ \sinh^2(H_{a1}z) - H_{a1}^2 z^2 \} \}]$$

$$+ \gamma^2 \{ \sinh^2 \{ H_{a1}(1-z) \} - \sinh^2 H_{a1} + 2zH_{a1}^2 - z^2 H_{a1}^2 \} - (\gamma-1)^2 \{ \sinh^2 H_{a1} - H_{a1}^2 \} z - 2\gamma(\gamma-1) \{ \sinh H_{a1} (\sinh H_{a1} \cosh H_{a1} - H_{a1}) - \cosh H_{a1} (\sinh^2 H_{a1} - H_{a1}^2) \} z - \gamma^2 (H_{a1}^2 - \sinh^2 H_{a1}) z] \quad (39)$$

The function $T(z)$ which expresses the temperature distributions is governed by three independent non-dimensional parameters β , γ and H_{a1} . Here, parameter α is eliminated by Eq. (21). Actually, if the electromagnetic field is sufficiently intense when the velocity profiles are controlled by the electromagnetic field, as in this case, the temperature T_a becomes negligibly small compared to T_β (i.e. $T_a \ll T_\beta$). Furthermore, β does not take part in the distributions, but controls the amplitude only. Here, the dependencies of the temperature profiles on H_{a1} are shown in Fig. 18 and Fig. 19, for the case of β fixed at 10. In the case of $\gamma=0$ (see Fig. 18), the gaps from the linear line mean self-heating caused by the induced electric current. The tem-

Fig. 18 Temperature distributions (dependency on Hartmann's number) (a)

Fig. 19 Temperature distributions (dependency on Hartmann's number) (b)

perature distributions reflect the ones of electric current density.

6. Phenomenological law of causation

Conducting magnetic fluids, in the electromagnetic field, induce electric field, by dynamical motions. Superimposing with the applied electric field, electric currents are induced in proportion to the electric conductivity, and then the magnetic field is induced corresponding to the electric current distribution. Really, the conductivity is given as function of temperature, so the temperature affects the velocity profiles, electromagnetic field distribution and so on. On one side, the motion of the fluid is affected by the Lorentz force caused by the electric currents. Generally, if conducting fluids move across a magnetic field, the motions are hindered by the Lorentz force. This Lorentz force is caused by the induced and applied electric currents. Therefore, the acceleration of the

Fig. 20 Phenomenological law of causation

flow caused by the applied electric current is automatically suppressed. So the volume flow rate converges to a constant value (see Fig. 6). The induced electric current distributions are determined by the velocity profiles and applied magnetic field. The induced electric current as described here, changes the magnetic field in the fluid as shown in Fig. 11. The result is shown in Fig. 20.

7. Conclusions

The authors have analyzed the Couette-like flow characteristics of conducting magnetic fluid between two parallel plates in the applied electromagnetic field and applied temperature. The results are summarized as follows.

(1) Some new nondimensional parameters caused by magnetization of fluid are defined by use of the strict basic equations system for nonpolar conducting magnetic fluids. The relations between the nondimensional parameters and the usual ones of

MHD are shown.

(2) Pressure distributions at right angles to flow direction are obtained theoretically, which do not exist in the nonconducting magnetic fluid.

(3) The solution of velocity for the fluid with magnetization is expressed in Eq.(32). In the equation of motion, the term with Hartmann's number and the one with magnetic field gradient parameter cancel out each other. The velocity solution appears not to be dependent on magnetization. Magnetization influences the flow through the medium of Hartmann's number, since Hartmann's number is defined so that it depends on magnetization in Eq.(16).

(4) The dependencies of induced magnetic field (Eq.(35)) and temperature field (Eq.(37)) on both Hartmann's number and electric field effect parameter are clarified. It is shown that the thermal effect parameter of Joule heating governs only the magnitude of temperature, and not the distributions.

(5) The boundary condition of the induced magnetic field is shown generally and analytically when

the electromagnetic field is applied.

(6) The pressure distributions of the magnetic fluid are shown in Eq.(36). It is clarified that Hartmann's number of magnetization does not affect the distributions.

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