Simulation System for the Design of Link Mechanisms* (Topological and Kinematic Analyses of Mechanisms)

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The computer-based analysis presented in this paper provides an automated generation of kinematic loops and equations of motion necessary for the kinematic analysis of a complex mechanism, and is applicable to an arbitrary mechanism composed of open loops, closed loops, or both. First, all of the links in a mechanism are topologically classified into several types according to the number of joints adjacent to the link. Next, based on graph theory, the topological structure of the whole mechanism is automatically analyzed by using local relations given as input data between the joint and adjacent links. Then, a method for the analysis of displacement, velocity and acceleration of an arbitrary mechanism is presented by using kinematic loops. Finally, the forces and torques acting on joints are analyzed numerically. The proposed method is applied to an industrial robot including a closed loop, and is proven to give a high-accuracy analysis.

Key Words: Mechanism, Kinematics, Robot, Manipulators, CAE, Simulation, Topology, Equation of Motion

1. Introduction

In recent years, it has become very common to design various kinds of link mechanisms, such as robots and manipulators. For such purposes, it is very important to carry out not only kinematic and static analyses, but also dynamic analyses to include the actuator and its controller.

Generally, the characteristic equation of a link mechanism is expressed as simultaneous differential equations, with strong nonlinearity and interference among the variables. Therefore, it is necessary to develop a computer-aided engineering (CAE) system providing automated generation of equations and including functions from modeling to evaluation.

The link structures to be analyzed are classified into the following four types: an open loop mechanism with multidegrees of freedom, such as industrial robots; a closed loop mechanism, such as a four-bar link; a tree structure mechanism, such as hands; and

a mechanism of combined types. Therefore, it is essential to understand how the links and joints are connected to one another.

The IMP (Integrated Mechanical Program)⁽¹⁾⁽²⁾ developed by Uicker, et al., is taken as an example of a general-purpose computer program for analyzing link mechanisms. It is possible to treat a closed loop mechanism by the IMP code; however, it is impossible to deal directly with an open loop mechanism. It was reported that Nagamatsu⁽³⁾ et al. improved the IMP code to analyze the open loop mechanism; however, they did not describe the topological structure of the link mechanism. Kawabe et al.⁽⁴⁾ referred to the link structure in terms of a description of a robot model. The method for analyzing the link structure by means of a structural diagram or a characteristic polynomial was also reported⁽⁵⁾. On the other hand, various simulation systems for simple open loop mechanisms without a branch have been developed⁽⁶⁾⁽⁷⁾. Masuda et al. referred to an analytical method for industrial robots of open and closed loop dynamics introducing Lagrange multipliers to Appel's method⁽⁸⁾.

The new simulation system described in this paper is applicable to an arbitrary link mechanism

* Received 20th May, 1987. Paper No. 86-0378A, -0379A

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composed of open loops, closed loops, or both. First, it is shown that the definition of a kinematic loop consisting of a closed and an open loop enables us to express the topological structure of an arbitrary link mechanism. Next, an approach to analyze the above structure is described. The complex structure of the whole link mechanism is automatically analyzed from only the local relations between the joints and adjacent links given as input data. This approach is based on graph theory. Then, referring to the above kinematic loops, it is shown that the kinematics of an arbitrary mechanism composed of open and/or closed loops are able to be analyzed. Furthermore, by applying this to an industrial robot, the effectiveness of the proposed method is proved.

2. Structure of Link Mechanisms

2.1 Classification of link mechanism

A link mechanism is composed of the following two components: a link, which is the minimum unit of a link mechanism; and a joint, where two adjacent links move relatively. The topological structure composed of several joints connected together is referred to as a kinematic loop. As described later in Section 2.3, a kinematic loop consists of an open and a closed loop.

As shown in Fig. 1, an arbitrary-type link mechanism is classified into three types: an open kinematic loop consisting only of open loops; a closed kinematic loop consisting only of closed loops; and a combined kinematic loop consisting of both.

Such kinematic loops as these are, moreover, divided into two types: a kinematic loop with a tree structure, and that without a tree structure. Let a link and joint be a node and branch, respectively, for topological convenience. Then, a link and a joint are denoted by $\bigcirc$ and $-$ signs, respectively.

In the section that follows, the definition of a kinematic loop enables us to discuss the structure of an arbitrary link mechanism. Furthermore, the properties of joints and links are stated which are necessary for their treatment.

2.2 Properties of joints and links

When a joint connects a link L_{i1} with another link L_{i2} , the joint J_j is divided into the following two

Fig. 1 Classification of mechanisms

kinds:

- Active joint :
for cases in which there is only one path from L_{i1} to L_{i2} , i. e., joint J_j , as shown in Fig. 2 (a).
- Passive joint :
for cases in which another path from L_{i1} to L_{i2} other than Joint J_j exists.

On the other hand, an active and a passive joint correspond to an open and a closed loop mechanism, respectively. All of the joints belong to either an open or a closed loop.

One link in a link mechanism is fixed to the base, which is referred to as a base link. Since, in kinematic analysis, the relative motion between links is dealt with, the portion fixed to the base including its surroundings, is considered as a base link. Links other than the base link are classified into three groups:

- End link,
which is connected to a single link.
- Intermediate link,
which is connected to two links.
- Branching link,
which is connected to three or more links.

The intermediate and the branching link, moreover, are classified by the property of the joints connected to them. An intermediate link is divided into the following two types:

- Intermediate link in an open loop,
which is connected to two active joints.
- Intermediate link in a closed loop,
which is connected to two passive joints.

Similarly, a branching link is divided into the following two types:

- Branching link in an open loop,

(a) Active joint

(b) Passive joint

Fig. 2 Classification of joints

in which the joint on the side of the base link is connected to a single active joint.

• Branching link in a closed loop,

in which the joint on the side of the base link is not connected to an active joint.

The link classification based on the above properties is tabulated in Table 1.

2.3 Definition of a kinematic loop

A topological network containing links and joints is referred to as a kinematic loop. A kinematic loop is composed of an open and a closed loop as defined below.

(i) Open loop

- An open loop starts from the base or a branching link and ends at a branching link in an open loop or an end link. When an open loop starts from a branching link and ends at a branching link in the open loop, an open link starts from a link close to the base link and ends at a link far from the base link.
- A link never repeats itself in a loop.
- Each link making up a kinematic loop is a different intermediate link in an open loop.
- Each joint making up a kinematic loop is a different active joint.

(ii) Closed loop

- A closed loop starts from the base or a branching link and ends at the same link as

the first one.

- A link never repeats itself in a closed loop except for the first and the final link.
- Each link making up a kinematic loop is an intermediate link in a closed loop or a branching link. However, when the kinematic loop includes a branching link in a closed loop, the link should be far from the base link compared with the first link in the closed loop.
- Each joint making up the kinematic loop is a passive joint different from one another.
- The number of independent closed loops, L_c , is expressed as

$$L_c = N_j - N_L + 1, \quad (1)$$
 where the number of links and joints is N_L and N_j , respectively.
- An independent closed loop is selected so as to minimize the number of joints included in each closed loop.
- The direction of a closed loop is arbitrarily oriented. The definition of a kinematic loop enables us to express the topological structure of an arbitrary-type link mechanism.

3. An Approach to the Selection of a Kinematic Loop

In this system, the definition of a link mechanism is given by using the local relation as to which link a joint is connected. Therefore, with this information, it is necessary to automatically select the kinematic loop required to carry out the kinematic analysis. In the section that follows, an approach to an automatic selection of a kinematic loop is described which is done by interpretive structural modeling (ISM), one of the methods clarifying the relation between the elements of a system⁽⁹⁾.

3.1 Definition of an adjacency matrix

To simplify expressions, the links and joints in a link mechanism are indicated by $L_i (i=1, 2, \dots, N_L)$ and $J_j (j=1, 2, \dots, N_j)$, respectively, and the base link is indicated by L_B . To express the topological relation as to which link a joint is connected, an adjacency matrix $A = (a_{i_1, i_2}) (i_1, i_2 = 1, 2, \dots, N_L)$, is defined as follows:

$$a_{i_1, i_2} = \begin{cases} 1 & \text{if } L_{i_1} \text{ is directly joined with } L_{i_2}, \text{ or if } i_1 = i_2 \\ 0 & \text{for all other cases.} \end{cases} \quad (2)$$

By definition, the adjacency matrix A results in a symmetric matrix with binary values.

3.2 Calculation of a reachability set

Based upon Boolean algebra, A^2, A^3 , etc. are calculated by using A defined by Eqn. (2). When this calculation is continued, all of the elements in the matrix become constant after a number of finite itera-

Table 1 Classification of Links

Type of Link	Type and Number of Joint			Illustration
	Active Joint		Passive Joint	
	to Base	to End		
End Link	1	0	0	
Open Loop Intermediate Link	1	1	0	
Open Loop Branching Link	1	0 or 2	0 or 2	
Closed Loop Intermediate Link	0	0	2	
Closed Loop Branching Link	0	0 or 2	2 or 4	
Base Link	0	0 or 2	0 or 2	

Notes { — : Active Joint
— : Passive Joint

tions $p(\max N_L - 1)$. Then, the following equation holds,

$$A \neq A^2 \cdots \neq A^{p-1} = A^p = A^*, \quad (3)$$

where A^* is referred to as the reachability matrix. The element a_{i_1, i_2}^a in which i_1 and i_2 represent, respectively, the indices of the row and column of the matrix $A^a (a=1, 2, \dots, p-1)$, has the following significance:

$$(a) \quad a_{i_1, i_2}^a = 1$$

Link L_{i_1} is joined with link L_{i_2} by a maximum of a joints.

$$(b) \quad a_{i_1, i_2}^a = 0$$

Link L_{i_1} is joined with link L_{i_2} by more than a joints.

Next, by definition of a kinematic loop, it is necessary to examine the connection of the base link L_B with each link L_i . Then, the set $N^a(L_B)$ containing links connected to the base link L_B by a joints will be obtained.

The set $N^a(L_B) (0 \leq a \leq p-1)$ is such that

$$\bullet \text{ when } a=0; N^0(L_B) = \{L_B\} \quad (4)$$

$$\bullet \text{ when } a=1; N^1(L_B) = \{\forall L_i; a_{B,i}^1=1, \text{ for } \forall i \neq B\} \quad (5)$$

$$\bullet \text{ when } 2 \leq a \leq p-1; N^a(L_B) = \{\forall L_i; a_{B,i}^a=1, \text{ and } a_{B,i}^{a-1}=0, \text{ for } \forall i \neq B\}. \quad (6)$$

3.3 The first link and joint of a kinematic loop

By definition of a kinematic loop, whether a certain joint is contained in an open loop or in a closed loop is determined by the property of the joint. On the other hand, the property of a joint contained in a certain kinematic loop is the same, so that it is sufficient to select and examine the property of one joint in a kinematic loop. Here, the property of the first link in a kinematic loop is examined. By definition, a kinematic loop starts from the base link or a branching link, which is named the start link. Start link L_{i_1} is assumed to be connected to joint J_j which is jointed with link L_{i_2} . Moreover, links L_{i_1} and L_{i_2} are each assumed to belong to set $N^a(L_B)$ and set $N^{a+1}(L_B)$, respectively.

The joint connecting the start link is taken as a candidate for the first joint in a kinematic loop; however, this candidate is omitted by definition of the kinematic loop if $a_2 < a_1$. Thus, joint J_j is taken as a candidate for the first joint in a kinematic loop if $a_2 > a_1$. A candidate is shown to satisfy the sufficient condition in the case of a closed loop. Here, this is referred to as the start joint.

3.4 Classification of joints

This section explains the method for classifying whether a start joint in Section 3.3 is an active joint or a passive joint, namely, whether a kinematic loop starting from its start joint is an open loop or a closed loop.

① The adjacency matrix A_j of the link mechanism except for start joint J_j is obtained by substituting the relation in Eq. (7) into the adjacency matrix A defined by Eq. (2).

$$a_{i_1, i_2} = a_{i_2, i_1} = 0 \quad (7)$$

② In the same way as stated in Section 3.2, the set $N_j^p(L_{i_1})$, containing links connected to $(\beta-1)$ joints from the second link in a kinematic loop, L_{i_2} , is obtained. Especially, when $\beta=0$ or 1, their sets are defined, respectively, as follows:

$$N_j^0(L_{i_1}) = \{L_{i_1}\} \quad (8)$$

$$N_j^1(L_{i_1}) = \{L_{i_2}\}. \quad (9)$$

③ The joint J_j is a passive joint, if the start link L_{i_1} belongs to the set $N_j^p(L_{i_1}) (\beta=2, 3, \dots)$; otherwise the joint J_j is an active joint.

④ The procedure ①~③ is followed for all of the start joints obtained in Section 3.3.

3.5 Selection of an open loop

When a start joint J_j is an active joint, the kinematic loop starting from joint J_j is an open loop. In this case, the open loop starting from J_j is obtained by the adjacent relation between the link and the joint. The open loop is selected in the order starting from the start link with a smaller value a , defined by Eqs. (4)~(6). By definition, the open loop ends at the end of the link or the branching link in the open loop.

3.6 Selection of a closed loop

When a start joint J_j is a passive joint, the kinematic loop starting from J_j is taken as a candidate for a closed loop. A new selection of a closed loop is determined so as to be independent of the closed loops selected above and to minimize the number of loop joints.

① As for the set $N_j^p(L_{i_1}) (\beta \geq 2)$ of a passive start joint J_j , let k_j be the minimum value β for the set containing the start link L_{i_1} . The value k_j equals the number of loop joints minimized for the closed loops which start from the start link L_{i_1} , pass by the joint J_j and link L_{i_2} , and come back to the link L_{i_1} .

② For every passive start joint J_j , let $J_{j'}$ be the joint for which k_j has a minimum and positive value. At the same time, the start link L_{i_1} , L_{i_2} and value k_j are defined as $L_{i_1'}$, $L_{i_2'}$, and $k_{j'}$, respectively.

③ The links and joints contained in a closed loop starting from the start joint $J_{j'}$ are obtained in the order from the back of a closed loop. The links connected to γ joints from the start link $L_{i_1'}$ belong to the set L^{γ} . The last link of a closed loop connects to $k_{j'}$ joints from the start link, and is identical with the link $L_{i_1'}$ itself. The results are as follows:

$$\gamma = k_{j'} \quad (10)$$

$$L^{\gamma} = L_{i_1'}. \quad (11)$$

④ For the link contained in the set $N_j^{\gamma-1}(L_{i_1'})$, one of the links connecting link L^{γ} is selected. Then, the γ -

th joint in the closed loop is obtained represented by J' .

⑤ The new γ and L' are replaced by $(\gamma-1)$ and the link selected in ④, respectively.

⑥ When the new γ is positive, processes ④ and ⑤ are iterated. When $\gamma=0$, process ⑦ is followed.

⑦ A closed loop starting from start joint $J_{j'}$ is determined as follows:

$$\begin{aligned} L^0 &= L_{i_1'} - J^1 (= J_{j'}) \\ &- L^1 (= L_{i_2'}) - J^2 - L^2 - \dots \\ &\dots - L^{k_{j'}-1} - J^{k_{j'}} - L^{k_{j'}} (= L_{i_1'}). \end{aligned} \quad (12)$$

⑧ If other closed loops exist which start from start joint $J_{j'}$, pass by $k_{j'}$ joints and return again to the start link L_{i_1} , then such loops are similarly selected. This is the case in which, regarding a certain γ , a multiple number of links exists which are connected to L' and belongs to the set $N_j^{-1}(L_{i_1'})$.

⑨ When the joint J_j selected in process ④ belongs to passive start joints such as obtained in Section 3.4, the equation $k_i=0$ is set and the selected joint is omitted as a candidate for the first joint in a closed loop.

⑩ As for every passive start joint J_j obtained in Section 3.4, a closed loop is selected by the following processes till ②~⑨ $k_j=0$.

⑪ When the number of selected closed loops is equal to that of Eq. (1), the procedure for selecting the closed loop terminates. If the number of selected closed loops is less than that of Eq. (1), the procedure for selecting the open loop goes back again to process ①, letting k_j be the next smallest value β to the previous β . In this case, when a newly obtained loop is not independent of the previously selected closed loops, this loop should be omitted.

3.7 Multiple joints between adjacent links

When a link L_{i_1} is connected to link L_{i_2} with n joints, $J_m (m=1, 2, \dots, n), (n-1)$ closed loops exist between them. Therefore, it is necessary that these closed loops are, in advance, selected before the calculation mentioned above, and the equation is set as follows:

$$k_m=0 \quad (m=1, \dots, n). \quad (13)$$

3.8 The order to solve the kinematic loop

Generally, it is more efficient if the displacement, velocity, acceleration, angular velocity and angular acceleration of each link constituting the link mechanism are calculated from the base link to the end link, and conversely, the force and torque acting on each joint, from the end link to the base link^{(7),(10)}.

In an open loop, the order in which the kinematic loop is solved, as stated in Section 3.5, is equal to the order in which the displacement of each link is solved. On the other hand, in a closed loop, the kinematic loop is selected in the order of the smaller number of joints,

so the order should be rearranged in the order described above.

① Let L be the set in which the displacement is given. First, L contains only the base link.

② All of the closed loops, which connect links contained in L and are connected only to several closed loops, are searched, and the links are added to L .

③ All of the open loops connected to the links contained in L are obtained, and the links are added to L .

④ This procedure for determining the kinematic loops terminates when the number of kinematic loops in which the already determined order of solving is equal to the number of kinematic loops included in the link mechanism as described in Sections 3.5~3.7. Then, all of the links are contained in L . If not equal, processes ② and ③ should be iterated.

4. Kinematics

Two kinds of kinematic problems can be worked out in this system. One problem, the forward problem, is applied to find the position, velocity, and acceleration of links in the work space Σ_w , when the displacement, velocity, and acceleration of joints are known. The other, the backward problem, is applied to find the displacement, velocity, and acceleration of joints, when the trajectory of links are given in the work space Σ_w . Force analysis evaluates the force and torque acting on joints.

In this paper, coordinates are defined as follows:

Σ_w = Work space fixed Cartesian coordinates,

Σ_m = Local Cartesian coordinates for each link

L_m ,

Σ_{L_m, J_j} = Local Cartesian coordinates at each joint J_j for link L_m ,

Σ_{G_m} = Local Cartesian coordinates at each center of mass G_m of link L_m , of which axes are parallel to Σ_m .

The following transformation matrices represent a homogeneous transformation describing the relative translation and rotation between two coordinates^{(1),(11)} as follows:

T_m = Transformation matrix from Σ_m to Σ_w ,

B_{L_m, J_j} = Transformation matrix from Σ_{L_m, J_j} to Σ_m ,

P_m = Transformation matrix at joint J_j .

4.1 Forward problem in an open loop

In an open loop, the transformation matrix T_m is recursively computed in the order from the base link to the end link, for the purpose of decreasing computing time⁽⁷⁾.

$$T_m = T_{m-1} B_{L_{m-1}, J_m} P_m B_{L_m, J_m}^{-1} \quad (14)$$

Where index m is numbered increasing outward to the end link. The first and second time derivatives of the

T matrix, $\dot{T}_m(=\partial T_m/\partial t)$ and $\ddot{T}_m(=\partial^2 T_m/\partial t^2)$, are similarly obtained from equations by time differentiating Eq. (14).

4.2 Forward problem in a closed loop

In a closed loop consisting of a sequence of λ links, L_1-L_λ , the following constraint equation must be satisfied concerning the displacement of joints,

$$B_{L1,J1}P_1B_{L2,J1}^{-1}B_{L2,J2}P_2\cdots\cdots B_{L\lambda,J\lambda}P_\lambda B_{L1,J\lambda}^{-1}=I, \quad (15)$$

where I denotes a 4×4 unit matrix. As to the velocity and acceleration of joints, equations obtained by differentiating Eq. (14) must be similarly satisfied.

As the displacement, velocity, and acceleration for joints with a driving device are already known, the unknown values for the remaining joints are calculated to satisfy the above constraint conditions. In this system, Eq. (15) is linearized by using the Taylor expansion and ignoring the higher-than-second-order terms. Using this, the unknown displacements, etc., are numerically solved by an iterative method similar to that in an IMP code⁽²⁾.

4.3 Backward problem

The backward problem is solved similarly to the forward problem in the case of a closed loop. A dummy joint is automatically inserted between the work space Σ_w and the link of which trajectory is constrained.

4.4 Force analysis

Figure 3 illustrates the relations among all of the forces and moments for link L_m . The formulation of the equations of motion for link L_m is based on the Newton-Euler approach to achieve a short computing time⁽⁷⁾. Each link's equations are referenced to its own local coordinates at the center of mass Σ_{G_m} as follows:

$$m_m T_m^{-1}(\ddot{\vec{r}}_{G_m}^w - \vec{g}^w) - \vec{f}_{G_m} = \sum_{j \text{ for } L_m} \{B_{L_m,J_j}(\vec{F}_{J_j} + \vec{F}_{K_j} + \vec{F}_{C_j})\}, \quad (16)$$

$$I_{G_m} \ddot{\vec{\omega}}_m + \vec{\omega}_m \times (I_{G_m} \vec{\omega}_m) - \vec{n}_{G_m} = \sum_{j \text{ for } L_m} [B_{L_m,J_j}(\vec{M}_{J_j} + \vec{M}_{K_j} + \vec{M}_{C_j}) + \vec{s}_{G_m,J_j} \times \{B_{L_m,J_j}(\vec{F}_{J_j} + \vec{F}_{K_j} + \vec{F}_{C_j})\}], \quad (17)$$

Fig. 3 Forces and moments acting on link

where m_m denotes the mass of link L_m . I_{G_m} denotes the inertia matrix of link L_m about G_m , $\vec{g}^w$ denotes gravity with respect to Σ_w , and $\vec{f}_{G_m}$ and $\vec{n}_{G_m}$ denote the external force and moment acting on G_m with respect to Σ_{G_m} , which are equivalent to the sum of those acting on arbitrary points of link L_m , respectively.

$\ddot{\vec{r}}_{G_m}^w$ denotes the acceleration of G_m with respect to Σ_w , which is calculated from T_m . $\vec{\omega}_m$ and $\ddot{\vec{\beta}}_m$ denote the angular velocity and acceleration of link L_m with respect to Σ_w , which is recursively calculated in the order from the base link. $\vec{F}_{K_j}$, $\vec{M}_{K_j}$, $\vec{F}_{C_j}$, and $\vec{M}_{C_j}$ denote the force and moment acting on the spring of joint J_j , and those acting on the damper of J_j , respectively. Finally $\vec{s}_{G_m,J_j}$ denotes the location of joint J_j with respect to Σ_{G_m} .

In the force analysis, $\vec{F}_{J_j}$ and $\vec{M}_{J_j}$ are the unknown variables to be solved. If joint J_j has a driving device, the driving force (if joint J_j is translational) or torque (if J_j is rotational) is the projection of $\vec{F}_{J_j}$ and $\vec{M}_{J_j}$ onto the axis of translation or rotation of J_j .

In the case of an open loop, from Eqs. (16) and (17), the recursive relations are obtained about $\vec{F}_{J_j}$ and $\vec{M}_{J_j}$ ⁽⁷⁾.

In the case of a closed loop, simultaneous equations are made up of Eq. (16), Eq. (17) and the relations which represent that no driving force or torque occurs in joints without a driving device. The unknown variables, $\vec{F}_{J_j}$ and $\vec{M}_{J_j}$, are numerically solved from the above equations.

5. Example

A numerical example was carried out to ascertain the validity of the proposed method. An industrial robot including a closed loop is shown in Fig. 4. Figure 5 shows the output of the topological analysis. In this adjacency figure, the link and the joint are denoted by

Fig. 4 Industrial robot

Fig. 5 Results of topological analysis

(a) Joint displacement

(b) Driving torque

Fig. 6 Results of kinematic analysis

○ and — signs, respectively, and the selected kinematic loops are shown in the order to be solved. Figure 6 shows the displacement and required driving torque of the joints, when the hand L_7 moves at a sine-curved rate from the top to the bottom while holding its direction horizontally. Figure 7 displays the configuration of the robot. The above kinematic analysis was numerically calculated after a maximum of 3 iterations for each time step, for the case where the criteria of convergence was equal to 10^{-5} . It is concluded that sufficient accuracy was obtained for practical use.

Fig. 7 Configuration of mechanism

6. Conclusions

The methods for topological and kinematic analyses necessary to design a link mechanism composed of a closed and/or open kinematic loop have been proposed. In the topological analysis, the complex structure of a whole link mechanism is automatically described in the form of kinematic loops. In kinematic analysis, when the displacement, velocity, and acceleration of joints, or the trajectory of the link in a work space are given, the driving force and torque for the joints are numerically calculated with high accuracy. In addition, this system affords animated representation to help designers understand the results of simulation visually.

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