Analysis of a Loop-flow Type of Automated Manufacturing System*

Tadayoshi SHIOYAMA** and Hiroshi KISE**

This paper deals with an automated manufacturing system which consists of identical machines located along a loop type of conveyor with a loading-unloading station. The system is described by a Markov chain model and the production rate is analytically derived. The relations between the production rate and the arrival rate of parts and the average processing time of the machine are studied by numerical results. Furthermore, the optimal control problem of dispatching parts into the system is formulated as a Markov decision process. Some properties of the optimal control are considered, and the production rate under the optimal control is compared with that under non-control.

Key Words : Automated Manufacturing System, Loop Type of Conveyor, Markov Chain, Production Rate, Markov Decision Process, Optimal Dispatch Control

1. Introduction

The importance of the concept of the flexible manufacturing system (FMS)⁽¹⁾ has been recognized by those industries which have to manufacture or assemble products in small- to medium- sized batches. Thus, many kinds of FMS models have been intensively investigated^{(2)~(5)}. There are, however, a few works that have analyzed FMS's taking into account the interaction between transfer devices and machines. Nakamura et al⁽⁶⁾ have analyzed the production rate of an automated manufacturing system consisting of identical machines connected by automated transfer devices. Their analysis is, however, on the basis of approximating the system by an open type of system⁽⁶⁾.

In this paper, the loop-flow type of automated manufacturing system is exactly analyzed. The system is described by a Markov chain model and the

production rate is analyzed. Furthermore, the optimal control problem of dispatching workpieces into the conveyor of the system is considered. The problem is formulated as a Markov decision process, and properties of the optimal control policy are studied.

2. Model

Figure 1 illustrates the analytical model of the automated manufacturing system considered in this paper. A total of n identical machines are located along the loop-flow type of conveyor. A workpiece is processed exactly once by one of the available machines. Each machine can process one workpiece at a time. The workpiece, which has been finished processing, is called the product. Workpieces set on pallets at loading-unloading station are transferred in one direction by the conveyor at constant speed. In order to prevent a collision between a pallet on the conveyor and a pallet being released from a machine upon completion of processing, the conveyor is divided into slots, each of equal length with one pallet allocated per slot. Let Δt be the time during which one slot passes a fixed point. There is a checkpoint in front of each machine which checks whether or not the current

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** Department of Mechanical Engineering, Kyoto Institute of Technology, Matsugasaki, Sakyo-ku, Kyoto, 806, Japan

slot is empty and whether the pallet includes the workpiece or the product.

Workpieces arrive at the system according to Bernoulli trials, with probability λ of arrival in each time length Δt . If no machine is available, the workpiece in the system recirculates on the conveyor. The processing time at each machine is assumed to be exponentially distributed with the mean $1/\theta$. After processing, the product is released from the machine into the conveyor, if the slot in front of the machine is empty. Otherwise, it must wait there until an empty slot is available. The state such that a product is waiting to be released from a machine is called a blocked state. Products are transferred from the conveyor to the loading-unloading station, where they are removed from the pallet. It is assumed that the distance between machines and the loading-unloading station is equal to one slot. Since whether or not the processing is completed is observed whenever the front end of each slot passes a checkpoint, the system is controlled as if the processing time has a discrete value of $m\Delta t$ when the continuous processing time T satisfies the relation $m\Delta t \leq T < (m+1)\Delta t$ ($m=0, 1, \dots$). The mean of this discrete processing time T_P is given by⁽⁶⁾

$$T_P = \{\sigma/(1-\sigma)\}\Delta t$$

where σ is defined as $\sigma = \exp(-\theta\Delta t)$. (In the sequel of this paper, T_P is called the mean processing time.) It should be noted that σ satisfies the following:

$$\sigma = \Pr\{T \geq (m+1)\Delta t | T \geq m\Delta t\}$$

which implies that σ is the conditional probability that the processing is not completed at $(m+1)\Delta t$, given that the processing is not completed at $m\Delta t$.

3. Transition of State

The state of the system is observed at every instant when the front end of each slot passes a

Fig. 1 Model of automated manufacturing system

checkpoint. The state denoted M_i of the i -th machine takes one of the following:

processing ($M_i=0$), idle ($M_i=-1$), blocked ($M_i=-2$). The slot takes the following three states:

empty (0), including workpiece (1), including product (2). Let C_i ($i=1, \dots, n$) be the state of the slot immediately behind the i -th machine and C_0 the state of the slot immediately before the first machine. Then, the state of the system denoted i is described as

$$i = (C_0 M_1 C_1 M_2 \dots C_{n-1} M_n C_n).$$

The family of states constitutes a Markov chain. Table 1 shows the conditional probability $\Pr\{M'_i C'_i | C_{i-1} M_i\}$ that the state of the i -th machine and the slot immediately behind the i -th machine is $(M'_i C'_i)$ after the transition time Δt , given that the current state of the i -th machine and the slot immediately before the i -th machine is $(C_{i-1} M_i)$. For preventing the system from a deadlocked state, even if C_n is 0 or 2, the workpiece is not dispatched into the first slot when $C_i=1$ ($i=0, 1, \dots, n-1$) and $M_i \neq -1$ ($i=1, \dots, n$). (This action is called D -control in the sequel of this paper). Table 2 shows the conditional probability $\Pr\{C'_0 | C_0 M_1 C_1 M_2 \dots C_{n-1} M_n C_n\}$ that the state of the first slot is C'_0 after the transition time Δt , given that the current state of the system is $(C_0 M_1 C_1 M_2 \dots C_{n-1} M_n C_n)$. The transition probabilities of the system are obtained by the product of the conditional proba-

Table 1 $\Pr\{M'_i C'_i | C_{i-1} M_i\}$

current	after Δt	conditional
$C_{i-1} M_i$	$M'_i C'_i$	probability
0 -2	-1 2	1
0 0	0 0	σ
	-1 2	$1-\sigma$
0 -1	-1 0	1
1 -2	-2 1	1
1 0	0 1	σ
	-2 1	$1-\sigma$
1 -1	0 0	1
2 -2	-2 2	1
2 0	0 2	σ
	-2 2	$1-\sigma$
2 -1	-1 2	1

Table 2 $\Pr\{C'_0 | C_0 M_1 C_1 M_2 \dots M_n C_n\}$

	current	after Δt	conditional
	C_n	C'_0	probability
non-control	1	1	1
	0 or 2	0	$1-\lambda$
		1	λ
D-control		0	1

bilities given in Tables 1 and 2. Denote by S_R the set of recurrent states. Since the subchain constituted by states in S_R is irreducible and aperiodic, the steady-state probability $\pi(i)$ for state $i \in S_R$ exists⁽⁷⁾ and satisfies the following equations:

$$\pi(i) = \sum_{j \in S_R} \pi(j)P(ji) \quad (i \in S_R), \quad (1)$$

and

$$\sum_{i \in S_R} \pi(i) = 1, \quad (2)$$

where $P(ji)$ ($i, j \in S_R$) represents the transition probability from the state j to the state i . For instance, the transition probability $P(ij)$ from the state $i=(0, -1, 0, 0, 0)$ to the state j in case of 2 machines ($n=2$) are illustrated as follows:

j	$P(ij)$
$(0, -1, 0, -1, 2)$	$(1-\sigma)(1-\lambda)$
$(1, -1, 0, -1, 2)$	$(1-\sigma)\lambda$
$(0, -1, 0, 0, 0)$	$\sigma(1-\lambda)$
$(1, -1, 0, 0, 0)$	$\sigma\lambda$

4. Production Rate

4.1 Definition of production rate

Define the production rate as the number of products per unit time. Moreover, define R_i as the production rate of the i -th machine. In steady state, R_i is equal to the number of parts transferred into the i -th machine per unit time. The states corresponding to the event of transferring a workpiece into the i -th machine are the states in which $C_{i-1}=1$ and $M_i=-1$. Let S_i be the set of these states. Then, R_i ($i=1, \dots, n$) are given by

$$R_i = \sum_{i \in S_i} \pi(i)/\Delta t \quad (i=1, \dots, n). \quad (3)$$

The production rate R of the system is given by

$$R = \sum_{i=1}^n R_i \quad (4)$$

4.2 Analytical solution in case of 2 machines ($n=2$)

The partition $\{a_i; i=1, 2, \dots, N\}$ of the set S_R of recurrent states is called "lumpable"⁽⁸⁾ if the partition satisfies the following relations:

$$\sum_{j \in a_m} P(ij) = C_{km} \text{ (constant)} \quad (i \in a_k). \quad (5)$$

From (5), the lumped transition probability P^*_{ij} from a lumped state a_i to a lumped state a_j is given by $P^*_{ij} = C_{ij}$ ($i, j=1, \dots, N$). If the condition (5) is satisfied, then the lumped process is a Markov chain⁽⁸⁾. In the case of 2 machines, $N=32$ and the lumped transition probabilities P^*_{ij} ($i, j=1, \dots, 32$) are shown in Table 3, where $\begin{pmatrix} -1 \\ -2 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \end{pmatrix}$ represents that $M_2=-1$ or -2 and $C_2=0$ or 2 . The number of the first column represents the number i of the lumped state a_i ($i=1, \dots, 32$). Denote by π^*_i ($i=1, \dots, 32$) the steady-state probability of the lumped state a_i ($i=1, \dots, 32$). Then,

π^*_i ($i=1, \dots, 32$) is obtained by solving the following equations:

$$\pi^*_i = \sum_{j=1}^{32} \pi^*_j P^*_{ji} \quad (6)$$

and

$$\sum_{i=1}^{32} \pi^*_i = 1. \quad (7)$$

Define X_i ($i=1, \dots, 31$) as

$$X_i \equiv \pi^*_i / \pi^*_{32} \quad (8)$$

X_{28}, X_{29} and X_{30} are obtained as follows:

$$\begin{aligned} X_{28} &= \{(1-\lambda)/\lambda\} \{1-\sigma^6\} / \{1+(1-\lambda)\sigma^4\}, \\ X_{29} &= \sigma(1-\sigma) \end{aligned}$$

and

$$X_{30} = \sigma^2.$$

For simplicity we define the following:

$$\begin{aligned} a &= [\{1-\sigma(1-\lambda)\}/(\sigma\lambda)] \{1-\sigma^2(1-\lambda)\} \\ &\quad / \{\sigma(1-\lambda)\} - (1-\sigma) \\ b &= (1-\sigma)(1/\sigma+1)/\sigma + \{1-\sigma(1-\lambda)\}/(\sigma\lambda) \\ c_1 &= a(1-\lambda)\sigma^2/\lambda + \{1-\sigma(1-\lambda)\}(1-\sigma)/\lambda \\ c_2 &= a(1-\lambda)\sigma^2(1-\sigma)/\lambda \\ c_3 &= a\sigma + \{1-\sigma(1-\lambda)\}/\lambda \\ c_4 &= a(1-\lambda)\sigma^2/\lambda \\ c_5 &= \sigma\{c_3(1-\lambda)^2 + c_4\lambda\}(1-\sigma)/\{\sigma(1-\lambda)\} \\ p &= \sigma\{c_3(1-\lambda)^2 + c_4\lambda\}(1-\sigma) \\ q &= c_3\sigma(1-\lambda)^2(1-\sigma) \\ r &= c_3\sigma(1-\lambda)(1-\sigma) \\ s &= c_4\sigma(1-\sigma)(1/\sigma+1) \\ u &= a(1-\lambda)/\lambda - c_3\sigma^2(1-\lambda)^2(1-\sigma) \\ e &= (1-\sigma)(1+\sigma)\sigma(\lambda X_{28} + X_{30}) + \sigma X_{29} \\ v &= (c_5+b)(1-\lambda)/\lambda + q \\ w &= (c_5+b)\sigma(1-\lambda)/\lambda - p \\ f &= u/\sigma + v \\ g &= [u/\{\sigma^2(1-\lambda)\} - \{c_1\sigma^2(1-\lambda) \\ &\quad + r\}]X_{28} - c_2X_{30} + we - s \end{aligned}$$

Then, X_{31} and X_i ($i=1, \dots, 27$) are obtained as follows:

$$\begin{aligned} X_{31} &= g/f \\ X_{26} &= \sigma^2(\lambda X_{28} + X_{30}) \\ X_{25} &= \{(1-\sigma)/\sigma\} X_{26} + \sigma X_{29} \\ X_{22} &= \sigma^2(1-\lambda)X_{28} \\ X_{21} &= \sigma(1-\sigma)(1-\lambda)X_{28} \\ X_{24} &= [1/\{\sigma^2(1-\lambda)\}][X_{28} - \sigma(1-\lambda)X_{31}] \\ X_{23} &= \{1/(\sigma\lambda)\} \{X_{31} - \sigma(1-\sigma)\lambda X_{24} \\ &\quad - \sigma X_{25} - \sigma(1-\sigma)X_{26}\} \\ X_{27} &= \sigma(1-\lambda)\{X_{23} + (1-\sigma)X_{24}\} \\ X_{17} &= [(1-\sigma)\lambda/\{\sigma(1-\lambda)\}]X_{27} \\ &\quad + (1-\sigma)\{X_{25} + (1-\sigma)X_{26}\} \\ X_{18} &= \lambda X_{17} + (1-\sigma)/\sigma \\ X_{16} &= \sigma X_{18} + (1-\sigma)\sigma \\ X_{15} &= \{(1-\sigma)/\sigma\} X_{16} \\ X_{14} &= (1-\lambda)\{X_{17} + (1-\sigma)\sigma X_{24} + (1-\sigma)X_{31}\} \\ X_8 &= \sigma(1-\lambda)\{X_{14} + (1-\sigma)X_{28}\} \\ X_7 &= \{(1-\sigma)/\sigma\} X_8 \\ X_{13} &= \{(1-\sigma)/\sigma\} X_{27} \\ X_{12} &= \{\lambda/(1-\lambda)\} X_8 + \sigma\{X_{16} + (1-\sigma)X_{30}\} \end{aligned}$$

Table 3 Transition probabilities P_{ij}^* from a lumped state α_i to a lumped state α_j ($i, j=1, 2, \dots, 32$)

α_i	α_j	P_{ij}^*	α_i	α_j	P_{ij}^*
1 $\begin{pmatrix} 0 & -1 & 0 \\ -2 & 2 \end{pmatrix}$	0 $\begin{pmatrix} -1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix}$	$1-\lambda$ λ	21 $\begin{pmatrix} 0 & 0 & 0 & -2 & 1 \end{pmatrix}$	1 $\begin{pmatrix} 0 & 0 & -1 & 2 \end{pmatrix}$	σ $1-\sigma$
2 $\begin{pmatrix} 0 & -1 & 0 & 0 \\ 2 \end{pmatrix}$	0 $\begin{pmatrix} -1 & 0 & -1 & 2 \end{pmatrix}$	$(1-\sigma)(1-\lambda)$ $(1-\sigma)\lambda$ $\sigma(1-\lambda)$ $\sigma\lambda$	22 $\begin{pmatrix} 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	1 $\begin{pmatrix} -1 & 2 & -1 & 2 \end{pmatrix}$	$(1-\sigma)^2$ $(1-\sigma)\sigma$ $(1-\sigma)\sigma$ σ^2
3 $\begin{pmatrix} 1 & -1 & 0 \\ -2 & 2 \end{pmatrix}$	0 $\begin{pmatrix} 0 & 0 & -1 & 0 \end{pmatrix}$	$1-\lambda$ λ	23 $\begin{pmatrix} 1 & 0 & 0 \\ -2 & 2 \end{pmatrix}$	0 $\begin{pmatrix} 0 & 1 & -1 & 0 \end{pmatrix}$	$(1-\lambda)\sigma$ $\lambda\sigma$ $(1-\lambda)(1-\sigma)$ $\lambda(1-\sigma)$
4 $\begin{pmatrix} 1 & -1 & 0 & 0 \\ 2 \end{pmatrix}$	0 $\begin{pmatrix} 0 & 0 & -1 & 2 \end{pmatrix}$	$(1-\sigma)(1-\lambda)$ $(1-\sigma)\lambda$ $\sigma(1-\lambda)$ $\sigma\lambda$	24 $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 \end{pmatrix}$	0 $\begin{pmatrix} -2 & 1 & -1 & 2 \end{pmatrix}$	$(1-\lambda)(1-\sigma)^2$ $\lambda(1-\sigma)^2$ $(1-\lambda)(1-\sigma)\sigma$ $\lambda(1-\sigma)\sigma$ $(1-\lambda)(1-\sigma)\sigma$ $\lambda(1-\sigma)\sigma$ $(1-\lambda)\sigma^2$ $\lambda\sigma^2$
5 $\begin{pmatrix} 0 & -1 & 2 \\ 2 \end{pmatrix}$	0 $\begin{pmatrix} -1 & 0 & -1 & 2 \end{pmatrix}$	$1-\lambda$ λ	25 $\begin{pmatrix} 1 & 0 & 0 & -2 & 1 \end{pmatrix}$	1 $\begin{pmatrix} 0 & 1 & -1 & 2 \end{pmatrix}$	σ $1-\sigma$ $(1-\sigma)^2$ $(1-\sigma)\sigma$ $(1-\sigma)\sigma$ σ^2
6 $\begin{pmatrix} 0 & -1 & 2 & 0 \\ 2 \end{pmatrix}$	0 $\begin{pmatrix} -1 & 0 & -2 & 2 \end{pmatrix}$	$(1-\sigma)(1-\lambda)$ $(1-\sigma)\lambda$ $\sigma(1-\lambda)$ $\sigma\lambda$	26 $\begin{pmatrix} 1 & 0 & 0 & 0 & 1 \end{pmatrix}$	1 $\begin{pmatrix} -2 & 1 & -1 & 2 \end{pmatrix}$	$(1-\sigma)^2$ $(1-\sigma)\sigma$ $(1-\sigma)\sigma$ σ^2
7 $\begin{pmatrix} 0 & -1 & 2 & -2 & 1 \end{pmatrix}$	1 $\begin{pmatrix} -1 & 0 & -2 & 2 \end{pmatrix}$	1	27 $\begin{pmatrix} 0 & 0 & 1 & -1 & 0 \\ 2 \end{pmatrix}$	0 $\begin{pmatrix} -1 & 2 & 0 & 0 \end{pmatrix}$	$(1-\lambda)(1-\sigma)$ $\lambda(1-\sigma)$ $(1-\lambda)\sigma$ $\lambda\sigma$ $(1-\lambda)(1-\sigma)^2$ $\lambda(1-\sigma)^2$ $(1-\lambda)(1-\sigma)\sigma$ $\lambda(1-\sigma)\sigma$ $(1-\lambda)(1-\sigma)\sigma$ $\lambda(1-\sigma)\sigma$ $(1-\lambda)\sigma^2$ $\lambda\sigma^2$
8 $\begin{pmatrix} 0 & -1 & 2 & 0 & 1 \end{pmatrix}$	1 $\begin{pmatrix} -1 & 0 & -2 & 2 \end{pmatrix}$	$1-\sigma$ σ	28 $\begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 2 \end{pmatrix}$	0 $\begin{pmatrix} -1 & 2 & -2 & 1 \end{pmatrix}$	$(1-\sigma)^2$ $(1-\sigma)\sigma$ $(1-\sigma)\sigma$ σ^2
9 $\begin{pmatrix} 1 & -1 & 2 \\ -2 & 2 \end{pmatrix}$	0 $\begin{pmatrix} 0 & 0 & -1 & 2 \end{pmatrix}$	$1-\lambda$ λ	29 $\begin{pmatrix} 0 & 0 & 1 & -2 & 1 \end{pmatrix}$	1 $\begin{pmatrix} -1 & 2 & -2 & 1 \end{pmatrix}$	$1-\sigma$ σ
10 $\begin{pmatrix} 1 & -1 & 2 & 0 \\ 2 \end{pmatrix}$	0 $\begin{pmatrix} 0 & 0 & -2 & 2 \end{pmatrix}$	$(1-\sigma)(1-\lambda)$ $(1-\sigma)\lambda$ $\sigma(1-\lambda)$ $\sigma\lambda$	30 $\begin{pmatrix} 0 & 0 & 1 & 0 & 1 \end{pmatrix}$	1 $\begin{pmatrix} -1 & 2 & -2 & 1 \end{pmatrix}$	$(1-\sigma)^2$ $(1-\sigma)\sigma$ $(1-\sigma)\sigma$ σ^2
11 $\begin{pmatrix} 1 & -1 & 2 & -2 & 1 \end{pmatrix}$	1 $\begin{pmatrix} 0 & 0 & -2 & 2 \end{pmatrix}$	1	31 $\begin{pmatrix} 1 & 0 & 1 & -1 & 0 \\ 2 \end{pmatrix}$	0 $\begin{pmatrix} -2 & 1 & 0 & 0 \end{pmatrix}$	$(1-\lambda)(1-\sigma)$ $\lambda(1-\sigma)$ $(1-\lambda)\sigma$ $\lambda\sigma$
12 $\begin{pmatrix} 1 & -1 & 2 & 0 & 1 \end{pmatrix}$	1 $\begin{pmatrix} 0 & 0 & -2 & 2 \end{pmatrix}$	$1-\sigma$ σ	32 $\begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 2 \end{pmatrix}$	0 $\begin{pmatrix} -2 & 1 & -2 & 1 \end{pmatrix}$	$(1-\sigma)^2$ $(1-\sigma)\sigma$ $(1-\sigma)\sigma$ σ^2
13 $\begin{pmatrix} 0 & -2 & 1 & -1 & 0 \\ 2 \end{pmatrix}$	0 $\begin{pmatrix} -1 & 2 & 0 & 0 \end{pmatrix}$	$1-\lambda$ λ		0 $\begin{pmatrix} -2 & 1 & 0 & 1 \end{pmatrix}$	$(1-\sigma)\sigma$ $(1-\sigma)\sigma$ σ^2
14 $\begin{pmatrix} 0 & -2 & 1 & 0 & 0 \\ 2 \end{pmatrix}$	0 $\begin{pmatrix} -1 & 2 & -2 & 1 \end{pmatrix}$	$(1-\sigma)(1-\lambda)$ $(1-\sigma)\lambda$ $\sigma(1-\lambda)$ $\sigma\lambda$		1 $\begin{pmatrix} 0 & 0 & -2 & 1 \end{pmatrix}$	$(1-\lambda)(1-\sigma)$ $\lambda(1-\sigma)$ $(1-\lambda)\sigma$ $\lambda\sigma$
15 $\begin{pmatrix} 0 & -2 & 1 & -2 & 1 \end{pmatrix}$	1 $\begin{pmatrix} -1 & 2 & -2 & 1 \end{pmatrix}$	1		0 $\begin{pmatrix} -2 & 1 & 0 & 1 \end{pmatrix}$	$(1-\sigma)^2$ $(1-\sigma)\sigma$ $(1-\sigma)\sigma$ σ^2
16 $\begin{pmatrix} 0 & -2 & 1 & 0 & 1 \end{pmatrix}$	1 $\begin{pmatrix} -1 & 2 & -2 & 1 \end{pmatrix}$	$1-\sigma$ σ		0 $\begin{pmatrix} -2 & 1 & 0 & 1 \end{pmatrix}$	$(1-\sigma)^2$ $(1-\sigma)\sigma$ $(1-\sigma)\sigma$ σ^2
17 $\begin{pmatrix} 1 & -2 & 1 & -1 & 0 \\ 2 \end{pmatrix}$	0 $\begin{pmatrix} -2 & 1 & 0 & 0 \end{pmatrix}$	$1-\lambda$ λ		0 $\begin{pmatrix} -2 & 1 & 0 & 1 \end{pmatrix}$	$(1-\sigma)^2$ $(1-\sigma)\sigma$ $(1-\sigma)\sigma$ σ^2
18 $\begin{pmatrix} 1 & -2 & 1 & 0 & 0 \\ 2 \end{pmatrix}$	0 $\begin{pmatrix} -2 & 1 & -2 & 1 \end{pmatrix}$	$1-\sigma$ σ		0 $\begin{pmatrix} -2 & 1 & 0 & 1 \end{pmatrix}$	$(1-\sigma)^2$ $(1-\sigma)\sigma$ $(1-\sigma)\sigma$ σ^2
19 $\begin{pmatrix} 0 & 0 & 0 \\ -2 & 2 \end{pmatrix}$	0 $\begin{pmatrix} 0 & 0 & -1 & 0 \end{pmatrix}$	$\sigma(1-\lambda)$ $\sigma\lambda$ $(1-\sigma)(1-\lambda)$ $(1-\sigma)\lambda$		0 $\begin{pmatrix} -2 & 1 & 0 & 1 \end{pmatrix}$	$(1-\sigma)^2$ $(1-\sigma)\sigma$ $(1-\sigma)\sigma$ σ^2
20 $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 2 \end{pmatrix}$	0 $\begin{pmatrix} -1 & 2 & -1 & 2 \end{pmatrix}$	$(1-\lambda)(1-\sigma)^2$ $\lambda(1-\sigma)^2$ $(1-\lambda)(1-\sigma)\sigma$ $\lambda(1-\sigma)\sigma$ $(1-\lambda)(1-\sigma)\sigma$ $\lambda(1-\sigma)\sigma$ $(1-\lambda)\sigma^2$ $\lambda\sigma^2$		0 $\begin{pmatrix} -2 & 1 & 0 & 1 \end{pmatrix}$	$(1-\sigma)^2$ $(1-\sigma)\sigma$ $(1-\sigma)\sigma$ σ^2

$$\begin{aligned}
X_{11} &= X_{15} - \{(1-\sigma)/\sigma\}X_{12} + (1-\sigma)X_{29} \\
X_{20} &= \{(1-\lambda)/\lambda\}(X_{24} - \sigma X_{12} - \sigma^2 X_{22}) \\
X_{10} &= \lambda X_{13} + (1-\sigma)\sigma\lambda X_{20} + (1-\sigma)(\sigma X_{22} + \lambda X_{27}) \\
X_4 &= [1/\{\sigma(1-\lambda)\}]\{[1-\sigma^2(1-\lambda)]X_{20} \\
&\quad - \sigma(1-\lambda)X_{10} - \sigma(1-\lambda)X_{27}\} \\
X_2 &= \{(1-\lambda)/\lambda\}\{X_4 - \sigma X_8\} \\
X_6 &= [1/\{\sigma(1-\lambda)\}]\{[1-\sigma(1-\lambda)]X_2 \\
X_{19} &= \{(1-\lambda)/\lambda\}\{X_{23} - X_{11} - (1-\sigma)X_{12} \\
&\quad - \sigma X_{21} - \sigma(1-\sigma)X_{22}\} \\
X_5 &= (1-\sigma)(1-\lambda)\{X_{19} + (1-\sigma)X_{20}\} \\
X_9 &= \{\lambda/(1-\lambda)\}X_5 + (1-\sigma)\{X_{21} + (1-\sigma)X_{22}\} \\
X_1 &= \{(1-\lambda)/\lambda\}\{X_5 + (1-\sigma)(X_2 + X_6)\} \\
X_3 &= \{\lambda/(1-\lambda)\}X_1 + X_7 + (1-\sigma)X_8
\end{aligned}$$

By using $X_i (i=1, \dots, 31)$, π^*_{32} is given by

$$\pi^*_{32} = 1 / \left(1 + \sum_{i=1}^{31} X_i \right) \quad (9)$$

Fig. 2 Production rate R of system

Fig. 3 Production rate R_i of machine i ($i=1, 2, 3$)

Combining equations (8) and (9), $\pi^*_i (i=1, \dots, 31)$ is obtained by

$$\pi^*_i = \pi^*_{32} X_i \quad (i=1, \dots, 31). \quad (10)$$

Production rates R_1 , R_2 and R are calculated by

$$R_1 = (\pi^*_3 + \pi^*_4 + \pi^*_9 + \pi^*_{10} + \pi^*_{11} + \pi^*_{12}) / \Delta t, \quad (11)$$

$$R_2 = (\pi^*_{13} + \pi^*_{17} + \pi^*_{27} + \pi^*_{31}) / \Delta t \quad (12)$$

and

$$R = R_1 + R_2. \quad (13)$$

4.3 Numerical results

The production rate R of the system is illustrated in Fig. 2 in the case of the mean processing time $T_p = 5\Delta t$, with the number n of machines as a parameter. The result for $n=3$ is obtained by solving Eqs. (1) and (2) with the Gauss-Seidel method. It is found that the production rate R takes the maximum value at some λ . The reason for this is that the increase of λ leads to the increase of the blocking probability.

Figure 3 shows $R_i (i=1, 2, 3)$ in the case of 3 machines ($n=3$). The value of λ corresponding to the maximum of R_i increases as i increases. The reason for this is that the i -th machine processes the workpiece which has not been processed by the previous machines. The differences between $R_i (i=1, 2, 3)$ decrease with λ because the arrival rate of a workpiece at the rear machines increases with λ .

Figure 4 illustrates the relation between the maximum value $R_{\max}$ of the production rate R and T_p , with the number n of machines as a parameter. It is found that the effect of increasing the number of machines is superior to the effect of increasing the processing speed of a machine.

5. Optimal Dispatch Control

In this section, we deal with the dispatch control problem in which it must be decided whether or not

Fig. 4 Maximum production rate $R_{\max}$ of system vs mean processing time T_p

Fig. 5 Production rate R of system under the optimal control

the workpiece is dispatched into the conveyor, depending on the state of the system, in order to maximize the production rate of the system. The optimal control problem is formulated as a Markov decision process. In each state, action k is chosen from $\{0, 1\}$ where an action 1(0) represents the acceptance(rejection) of dispatching the workpiece into the conveyor. Denote by $q^k(i)$ the expected amount of immediate production during Δt , given that an action k is chosen in state i . Let $P^k(ij)$ be the conditional probability that the system moves to state j after Δt , given that an action k is chosen in state i . Since the transition probability $[P^k(ij)]$ has a single ergodic chain, the maximum average production during Δt satisfies the following optimality equation⁽⁹⁾⁽¹⁰⁾

$$g + v(i) = \max_k [q^k(i) + \sum_{j \in S} P^k(ij)v(j)], \quad (i \in S) \quad (14)$$

where $v(i)$ ($i \in S$) represents the relative values⁽⁹⁾ of the production.

Figure 5 illustrates the production rate R of the system under the optimal control. In the figure, solid and dotted lines show the production rates under the optimal and D -control, respectively. The optimal solution of (14) is solved by Howard's policy iteration method. Table 4 shows the optimal control action for each state in the case of $T_p = 5\Delta t$. The value in the table implies that the optimal action in the corresponding state is to accept(reject) dispatching the workpiece into the conveyor if λ is less(greater) than the value. The symbol D represents that the D -con-

Table 4 Optimal control action

1, 0	0.75	D	D
1, -2		D	D
1, -1		0.44	0.48
0, 0		/	0.49
0, -2		/	0.52
0, -1		/	0.52
$C_1 M_2$ / $C_2 M_1$	0, 0	1, -2	1, 0

trol must be adopted, and the blank (no value) implies that the optimal action is to accept dispatching the workpiece into the conveyor for all λ . The symbol / represents that the corresponding states are not recurrent states. The value in the table decreases as C_1 moves from 0 to 1. The value monotonically decreases when M_2 moves as $-1 \rightarrow -2 \rightarrow 0$.

6. Conclusions

With a Markov chain model, we analyzed the automated manufacturing system in which identical machines were located along the loop flow-type of conveyor. We studied the dependence of the production rate on the arrival rate of workpieces and on the processing time. We formulated the optimal dispatch control problem as a Markov decision process. It was found that the production rate was much improved by adopting the optimal control.

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