A Study of the Dynamic Damper with a Preview Action*

(Optimal Design of a Dynamic Damper with a Preview Action)

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This paper deals with a method of designing a dynamic damper with a preview action. It is the purpose of this investigation to achieve an optimal design of the dynamic damper with a preview action, which is applied to a vibratory system with damping. First, in order to attain a systematic design of the dynamic damper system, a modern control theory is used, and two kinds of design equations are derived; one for optimal design parameters and the other for optimal preview action. Then, from the viewpoint of nonlinear programming, these equations are solved, presenting design charts of the dynamic damper with a preview action. Furthermore, a dominant factor on the control effect is investigated. Finally, the design procedure proposed in the present paper is applied to the conventional dynamic damper, and the control effect for the suppression of the impact vibration is discussed.

Key Words: Shock, Vibration Control, Dynamic Damper, Preview Action, Transient Vibration

1. Introduction

The authors have already proposed⁽¹⁾ the principle of a dynamic damper with a preview action, which is capable of suppressing a transient vibration resulting from an impact force. As for the design of the dynamic damper attached to a one-degree-of-freedom undamped vibratory system, both the optimal tuning condition and the optimal preview condition of the damper system have been derived.

Then, in order to realize the principle of the newly proposed damper, an experiment has been carried out, demonstrating the control effect as well as the feasibility⁽²⁾ of the dynamic damper with a preview action.

It is known that an actual vibratory system contains damping, so that the optimal design conditions for the conventional dynamic damper must be adjusted in accordance with the amount of the damping⁽³⁾. In the same way, the optimal design conditions for the dynamic damper with a preview action would also be

dependent on the damping. However, the optimal design conditions presented in the reference⁽¹⁾ are for the case without damping. Accordingly, in order to make the dynamic damper with a preview action applicable to a general case, a damped vibratory system should be dealt with as a control object.

It is the purpose of this paper to present a systematic design procedure of the dynamic damper system with a preview action. To this end, the paper first applies an optimal control theory to the system and derives two kinds of design equations; one for optimal design parameters and the other for optimal preview action. Then, from the viewpoint of a nonlinear programming method, these equations are solved, presenting design charts of the dynamic damper with a preview action. Furthermore, dominant factors on the control effect are investigated. Finally, the design procedure proposed in the present paper is applied to the conventional dynamic damper, and the control effect for the suppression of the impact vibration is demonstrated.

Nomenclature

M : mass of a main vibratory system

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C : damping coefficient of a main vibratory system

K : spring constant of a main vibratory system

M_d : mass of a dynamic damper

C_d : damping coefficient of a dynamic damper

K_d : spring constant of a dynamic damper

ζ_t : damping ratio of a main vibratory system

ζ_d : damping ratio of a dynamic damper

ω_t : natural angular frequency of a main vibratory system

ω_d : natural angular frequency of a dynamic damper

f_{im} : impact force

f_t : transmitted force

μ : mass ratio

x : displacement of a main vibratory system

x_d : displacement of a dynamic damper

t_0 : preview time

2. Derivation of Optimal Design Conditions

2.1 Introduction of an optimal control theory

An equation of motion of a dynamic damper attached to a one-degree-of-freedom damped system is given by

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) + C_d\{\dot{x}(t) - \dot{x}_d(t)\} + K_d\{x(t) - x_d(t)\} = f_{im}(t) \quad (1)$$

$$M_d\ddot{x}_d(t) + C_d\{\dot{x}_d(t) - \dot{x}(t)\} + K_d\{x_d(t) - x(t)\} = 0 \quad (2)$$

Uniting Eqs. (1) and (2) leads to a state equation,

$$\dot{\mathbf{w}}(t) = \mathbf{M}\mathbf{w}(t) + \mathbf{b}_w f_{im}(t) \quad (3)$$

$$\mathbf{w}(0) = \mathbf{w}_0 \quad (4)$$

Transmitted force $f_t(t)$ in terms of a state vector $\mathbf{w}(t)$ is written as

$$f_t(t) = \mathbf{c}'_t \mathbf{w}(t) \quad (5)$$

where ' means a transpose of a vector and a matrix, and $\mathbf{M}$, $\mathbf{b}_w$ and $\mathbf{c}_t$ are defined as follows:

$$\mathbf{w}(t) = \text{col}\{x(t), \dot{x}(t), x_d(t), \dot{x}_d(t)\}$$

$$\mathbf{M} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -(K+K_d)/M & -(C+C_d)/M & K_d/M & C_d/M \\ 0 & 0 & 0 & 1 \\ K_d/M_d & C_d/M_d & -K_d/M_d & -C_d/M_d \end{bmatrix}$$

$$\mathbf{b}_w = \text{col}(0, 1/M, 0, 0)$$

$$\mathbf{c}_t = \text{col}(K, C, 0, 0) \quad (6)$$

Assuming that an initial displacement $x_d(0)$ of the dynamic damper is released at $t=0$, and an impact force $f_{im}(t)$ acts on a main vibratory system at $t=t_0$, then an initial state $\mathbf{w}_0$ and the impact force $f_{im}(t)$ are written as

$$\mathbf{w}_0 = \text{col}\{0, 0, x_d(0), 0\} \\ f_{im}(t) = f_0 \delta(t - t_0) \quad (7)$$

By using a transition matrix $\Phi(t)$, the state Eq. (3) can be solved as follows:

$$\mathbf{w}(t) = \begin{cases} \Phi(t) \mathbf{w}_0 & 0 \leq t < t_0 \\ \Phi(t) \mathbf{w}_0 + f_0 \Phi(t - t_0) \mathbf{b}_w & t \geq t_0 \end{cases} \quad (8)$$

where δ in Eq. (7) means Dirac's δ function, and $\Phi(t)$ in Eq. (8) is defined as

$$\Phi(t) = \exp[\mathbf{M}t] \quad (9)$$

Next, in order to design the damper system quantitatively, a performance index is to be introduced, and it is defined as

$$f_{ip} = \int_0^\infty f_t^2(t) dt \quad (10)$$

Moreover, the performance index f_{ip} is expanded by Eq. (8) into

$$\begin{aligned} f_{ip} &= \int_0^{t_0} \dot{\mathbf{w}}_0' \Phi'(t) \mathbf{c}_t \mathbf{c}_t' \Phi(t) \mathbf{w}_0 dt + \int_{t_0}^\infty \{ \mathbf{w}_0' \Phi'(t) \\ &\quad + f_0 \mathbf{b}_w' \Phi'(t - t_0) \} \mathbf{c}_t \mathbf{c}_t' \{ \Phi(t) \mathbf{w}_0 + f_0 \Phi(t - t_0) \mathbf{b}_w \} dt \\ &= \mathbf{w}_0' \mathbf{p} \mathbf{w}_0 + f_0 \mathbf{b}_w' \mathbf{p} \Phi(t_0) \mathbf{w}_0 \\ &\quad + f_0 \mathbf{w}_0' \Phi'(t_0) \mathbf{p} \mathbf{b}_w + f_0^2 \mathbf{b}_w' \mathbf{p} \mathbf{b}_w \\ &= \text{tr}[\mathbf{p} \mathbf{w}_0 \mathbf{w}_0'] + 2f_0 \text{tr}[\mathbf{p} \Phi(t_0) \mathbf{w}_0 \mathbf{b}_w'] \\ &\quad + f_0^2 \text{tr}[\mathbf{p} \mathbf{b}_w \mathbf{b}_w'] \end{aligned} \quad (11)$$

where tr means a trace, and the matrix $\mathbf{p}$ is defined as

$$\begin{aligned} \mathbf{p} &= \int_0^\infty \Phi'(t) \mathbf{c}_t \mathbf{c}_t' \Phi(t) dt \\ &= \int_{t_0}^\infty \Phi'(t - t_0) \mathbf{c}_t \mathbf{c}_t' \Phi(t - t_0) dt \end{aligned} \quad (12)$$

Furthermore, the matrix $\mathbf{p}$ satisfies the following matrix equation,

$$\mathbf{M}' \mathbf{p} + \mathbf{p} \mathbf{M} + \mathbf{c}_t \mathbf{c}_t' = 0 \quad (13)$$

The next step is to derive an optimal design equation which minimizes the performance index. To this end, an assumption is made such that a mass M_d of a dynamic damper is given beforehand, and the optimal tuning parameters such as C_d and K_d , and the optimal preview parameters such as $x_d(0)$ and t_0 are determined. First, a coefficient matrix $\mathbf{M}$ is expanded into the following equation:

$$\mathbf{M} = \hat{\mathbf{A}} + \hat{\mathbf{B}} \mathbf{G} \hat{\mathbf{C}} \quad (14)$$

where

$$\begin{aligned} \hat{\mathbf{A}} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -K/M & -C/M & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \hat{\mathbf{B}} = \begin{bmatrix} 0 \\ 1/M \\ 0 \\ -1/M_d \end{bmatrix} \\ \mathbf{G} &= (K_d, C_d) \\ \hat{\mathbf{C}} &= \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \end{aligned} \quad (15)$$

As seen from Eqs. (14) and (15), parameters K_d and C_d pertaining to the optimal tuning condition are all confined in a gain matrix $\mathbf{G}$, so that it can be said that the design of the dynamic damper with a preview action is dependent on the output feedback control problem of finding the optimal feedback gain matrix. Of the optimal design conditions, the optimal tuning condition is first derived in the following.

2.2 Derivation of the optimal tuning condition

Partially differentiating f_{ip} in Eq. (11) with respect to an element G_i of the gain matrix $\mathbf{G}$, we

obtain

$$\begin{aligned} \frac{\partial f_{ip}}{\partial G_i} = & \text{tr} \left[\frac{\partial \mathbf{p}}{\partial G_i} \mathbf{w}_0 \mathbf{w}_0' \right] + 2f_0 \cdot \text{tr} \left[\frac{\partial \mathbf{p}}{\partial G_i} \boldsymbol{\Phi}(t_0) \right. \\ & \times \mathbf{w}_0 \mathbf{b}_w' + \mathbf{p} \frac{\partial \boldsymbol{\Phi}(t_0)}{\partial G_i} \mathbf{w}_0 \mathbf{b}_w' \left. \right] + f_0^2 \text{tr} \left[\frac{\partial \mathbf{p}}{\partial G_i} \mathbf{b}_w \mathbf{b}_w' \right] \end{aligned} \quad (16)$$

Likewise, partially differentiating Eq.(13) with respect to G_i leads to

$$\hat{\mathbf{C}}' \mathbf{e}_i \hat{\mathbf{B}}' \mathbf{p} + \mathbf{M}' \frac{\partial \mathbf{p}}{\partial G_i} + \frac{\partial \mathbf{p}}{\partial G_i} \mathbf{M} + \mathbf{p} \hat{\mathbf{B}} \mathbf{e}_i \hat{\mathbf{C}} = 0 \quad (17)$$

where $\mathbf{e}_i$ means a vector with all entries zero except the i -th entry, which is unity. Equation(17) is further written as the following integral equation,

$$\frac{\partial \mathbf{p}}{\partial G_i} = \int_0^\infty \boldsymbol{\Phi}'(t) \{ \mathbf{p} \hat{\mathbf{B}} \mathbf{e}_i \hat{\mathbf{C}} + \hat{\mathbf{C}}' \mathbf{e}_i \hat{\mathbf{B}} \mathbf{p} \} \boldsymbol{\Phi}(t) dt \quad (18)$$

Substituting Eq. (18) into the first, the second and the fourth terms in the right-hand side of Eq. (16), we obtain the following expansion. As for the first term, it is expanded into

$$\begin{aligned} & \text{tr} \left[\frac{\partial \mathbf{p}}{\partial G_i} \mathbf{w}_0 \mathbf{w}_0' \right] \\ &= \int_0^\infty \text{tr} [\boldsymbol{\Phi}'(t) \{ \mathbf{p} \hat{\mathbf{B}}' \mathbf{e}_i' \hat{\mathbf{C}} + \hat{\mathbf{C}}' \mathbf{e}_i \hat{\mathbf{B}} \mathbf{p} \} \\ & \times \boldsymbol{\Phi}(t) \mathbf{w}_0 \mathbf{w}_0'] dt \\ &= 2 \int_0^\infty \text{tr} [\hat{\mathbf{C}}' \mathbf{e}_i \hat{\mathbf{B}}' \mathbf{p} \boldsymbol{\Phi}(t) \mathbf{w}_0 \mathbf{w}_0' \boldsymbol{\Phi}'(t)] dt \\ &= 2 \hat{\mathbf{B}}' \mathbf{p} \int_0^\infty \boldsymbol{\Phi}(t) \mathbf{w}_0 \mathbf{w}_0' \boldsymbol{\Phi}'(t) dt \hat{\mathbf{C}}' \mathbf{e}_i \\ &= 2 \hat{\mathbf{B}}' \mathbf{p} \mathbf{L}_1 \hat{\mathbf{C}}' \mathbf{e}_i \end{aligned} \quad (19)$$

where the matrix $\mathbf{L}_1$ is defined as

$$\mathbf{L}_1 = \int_0^\infty \boldsymbol{\Phi}(t) \mathbf{w}_0 \mathbf{w}_0' \boldsymbol{\Phi}'(t) dt \quad (20)$$

$\mathbf{L}_1$ also satisfies the next matrix equation,

$$\mathbf{M} \mathbf{L}_1 + \mathbf{L}_1 \mathbf{M}' + \mathbf{w}_0 \mathbf{w}_0' = 0 \quad (21)$$

Similarly, the second and the fourth terms are written as

$$2f_0 \text{tr} \left[\frac{\partial \mathbf{p}}{\partial G_i} \boldsymbol{\Phi}(t_0) \mathbf{w}_0 \mathbf{b}_w' \right] = 4f_0 \hat{\mathbf{B}}' \mathbf{p} \mathbf{L}_2 \hat{\mathbf{C}}' \mathbf{e}_i \quad (22)$$

$$f_0^2 \text{tr} \left[\frac{\partial \mathbf{p}}{\partial G_i} \mathbf{b}_w \mathbf{b}_w' \right] = 2f_0^2 \hat{\mathbf{B}}' \mathbf{p} \mathbf{L}_3 \hat{\mathbf{C}}' \mathbf{e}_i \quad (23)$$

where $\mathbf{L}_2$ and $\mathbf{L}_3$ satisfy the following matrix equation:

$$\mathbf{M} \mathbf{L}_2 + \mathbf{L}_2 \mathbf{M}' + \boldsymbol{\Phi}(t_0) \mathbf{w}_0 \mathbf{b}_w' = 0 \quad (24)$$

$$\mathbf{M} \mathbf{L}_3 + \mathbf{L}_3 \mathbf{M}' + \mathbf{b}_w \mathbf{b}_w' = 0 \quad (25)$$

Before proceeding with the expansion of the third term in the right-hand side of Eq. (16), we now discuss the calculation of the transition matrix $\boldsymbol{\Phi}(t_0)$. For the sake of simplicity, let us assume that all the eigenvalues $\lambda_i (i=0 \sim 3)$ are different. Then, $\boldsymbol{\Phi}(t_0)$ is expanded into

$$\boldsymbol{\Phi}(t_0) = \alpha_0 \mathbf{I} + \alpha_1 \mathbf{M} + \alpha_2 \mathbf{M}^2 + \alpha_3 \mathbf{M}^3 \quad (26)$$

where $\mathbf{I}$ means a unity matrix. Moreover, $\alpha_i (i=0 \sim 3)$ are obtained by means of Sylvester's interpolation formula⁽⁴⁾, and these are given numerically by solving the following simultaneous linear equations:

$$\begin{bmatrix} 1 & \lambda_1 & \lambda_1^2 & \lambda_1^3 \\ 1 & \lambda_2 & \lambda_2^2 & \lambda_2^3 \\ 1 & \lambda_3 & \lambda_3^2 & \lambda_3^3 \\ 1 & \lambda_4 & \lambda_4^2 & \lambda_4^3 \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} \exp[\lambda_1 t_0] \\ \exp[\lambda_2 t_0] \\ \exp[\lambda_3 t_0] \\ \exp[\lambda_4 t_0] \end{bmatrix} \quad (27)$$

Thus, substituting $\boldsymbol{\Phi}(t_0)$ given by Eq.(26) into Eq.(24), we can obtain the solution of the matrix Eq. (24).

Substituting Eq.(26) into the third term in the right-hand side of Eq.(16), we obtain

$$\begin{aligned} & 2f_0 \text{tr} \left[\mathbf{p} \frac{\partial \boldsymbol{\Phi}(t_0)}{\partial G_i} \mathbf{w}_0 \mathbf{b}_w' \right] \\ &= 2f_0 [\hat{\mathbf{B}}' (\alpha_1 \mathbf{I} + \alpha_2 \mathbf{M}' + \alpha_3 \mathbf{M}'^2) \mathbf{Q} \mathbf{C}' \\ & + \hat{\mathbf{B}}' \{ \alpha_2 \mathbf{Q} \mathbf{M}' + \alpha_3 (\mathbf{M}' \mathbf{Q} \mathbf{M}' + \mathbf{Q} \mathbf{M}'^2) \} \hat{\mathbf{C}}'] \mathbf{e}_i \end{aligned} \quad (28)$$

where

$$\mathbf{Q} = \mathbf{p} \mathbf{b}_w \mathbf{w}_0' \quad (29)$$

By substituting Eqs.(19), (22), (23) and (28) into Eq. (16), and setting this equation to zero, we have the optimal tuning condition, that is,

$$\begin{aligned} \frac{\partial f_{ip}}{\partial G} = & 2 \hat{\mathbf{B}}' \mathbf{p} \mathbf{L}_1 \hat{\mathbf{C}}' + 2f_0 \hat{\mathbf{B}}' \{ 2 \mathbf{p} \mathbf{L}_2 + (\alpha_1 \mathbf{I} + \alpha_2 \mathbf{M}' \\ & + \alpha_3 \mathbf{M}'^2) \mathbf{Q} + \alpha_2 \mathbf{Q} \mathbf{M}' + \alpha_3 (\mathbf{M}' \mathbf{Q} \mathbf{M}' + \mathbf{Q} \mathbf{M}'^2) \} \hat{\mathbf{C}}' \\ & + 2f_0^2 \hat{\mathbf{B}}' \mathbf{p} \mathbf{L}_3 \hat{\mathbf{C}}' \end{aligned} \quad (30)$$

2.3 Derivation of the optimal preview condition

We now derive the optimal preview condition of the dynamic damper system with a preview action. For this purpose, we first describe the initial state vector $\mathbf{w}_0$ by using the unity vector $\mathbf{e}_3$, that is,

$$\mathbf{w}_0 = x_d(0) \mathbf{e}_3 \quad (31)$$

Substituting Eq. (31) into Eq. (11), the performance index f_{ip} can be written as an explicit function of $x_d(0)$.

$$\begin{aligned} f_{ip} = & x_d^2(0) \text{tr} [\mathbf{p} \mathbf{e}_3 \mathbf{e}_3'] + 2f_0 \cdot x_d(0) \text{tr} [\mathbf{p} \boldsymbol{\Phi}(t_0) \\ & \times \mathbf{e}_3 \mathbf{b}_w'] + f_0^2 \cdot \text{tr} [\mathbf{p} \mathbf{b}_w \mathbf{b}_w'] \end{aligned} \quad (32)$$

Partially differentiating Eq.(32) with respect to $x_d(0)$, we obtain the necessary condition,

$$\begin{aligned} \frac{\partial f_{ip}}{\partial x_d(0)} = & 2x_d(0) \text{tr} [\mathbf{p} \mathbf{e}_3 \mathbf{e}_3'] \\ & + 2f_0 \cdot \text{tr} [\mathbf{p} \boldsymbol{\Phi}(t_0) \mathbf{e}_3 \mathbf{b}_w'] \end{aligned} \quad (33)$$

Next, we derive the necessary condition for the preview time t_0 . As seen from Eq.(11), only $\boldsymbol{\Phi}(t_0)$ contains the preview time t_0 , so that the following equation is easily given by partially differentiating Eq. (11) with respect to t_0 .

$$\begin{aligned} \frac{\partial f_{ip}}{\partial t_0} = & 2f_0 \cdot \text{tr} \left[\mathbf{p} \frac{\partial \boldsymbol{\Phi}(t_0)}{\partial t_0} \mathbf{w}_0 \mathbf{b}_w' \right] \\ & = 2f_0 \cdot \text{tr} [\mathbf{p} \mathbf{M} \boldsymbol{\Phi}(t_0) \mathbf{w}_0 \mathbf{b}_w'] \end{aligned} \quad (34)$$

Setting Eqs.(33) and (34) to zero, we obtain the optimal preview condition.

3. Optimal Design of the Preview Action

Based upon the optimal design conditions obtained in the preceding chapter, design charts by means of which the optimal design of the damper system is achieved, are presented. With regard to the numerical solution, Powell's algorithm is applied

because of the nonlinearity of the design conditions. As for the design parameters of the main vibratory system, the following values are used for the numerical calculation: $\zeta_t = 0.005 \sim 0.3$, $\mu = 0.05 \sim 0.2$.

Figure 1 shows the relation between the control effects and ζ_t . In the figure, $\bar{f}_{tp}$ and $\bar{f}_{t\max}$ mean a nondimensional f_{tp} and a nondimensional maximum value $f_{t\max}$ of the transmitted force, respectively, and these are defined as

$$\bar{f}_{tp} = f_{tp} / (f_0^2 \cdot \omega_t) \quad (35)$$

$$\bar{f}_{t\max} = f_{t\max} / (f_0 \cdot \omega_t) \quad (36)$$

As seen from the figure, $\bar{f}_{tp}$ slightly decreases as ζ_t increases, and it takes the minimum value 0.281 at $\zeta_t = 0.08$. If ζ_t is further increased, then $\bar{f}_{tp}$ begins to deteriorate, and the tendency becomes conspicuous, especially when ζ_t is greater than 0.1. On the other hand, $\bar{f}_{t\max}$ decreases monotonously in proportion as

Fig. 1 Control effects versus the damping ratio ζ_t of a main vibratory system

Fig. 2 Optimal tuning conditions (ζ_t versus ζ_d)

ζ_t increases in the range where ζ_t is between 0.005 and 0.3. It is important to note that $\bar{f}_{tp}$ and $\bar{f}_{t\max}$ depend only on ζ_t and are independent of μ . In the design of the conventional dynamic damper, μ plays an important role in the control effect, however, in regard to the preview action, it is determined only by ζ_t . This property is observed in the case of the active damper as well⁽⁶⁾, and it is interesting that the dynamic damper with a preview action has the same tendency as the active damper. Figure 1 also illustrates the control effect of the elastic support corresponding to the case without the dynamic damper. In this case, $\bar{f}_{tp}$ decreases precipitously as ζ_t decreases, however, it does not become less than 1.0.

Figures 2 and 3 show the optimal tuning conditions versus ζ_t . In the figure, the natural angular frequency $\bar{\omega}_d$ of the dynamic damper is described nondimensionally, and it is defined as

$$\bar{\omega}_d = \omega_d / \omega_t \quad (37)$$

It is seen from the figure that a change rate of ζ_d against ζ_t is insignificant in the case of ζ_t less than 0.05, however, if ζ_t becomes larger than 0.05, ζ_d decreases rapidly and this tendency becomes especially outstanding when ζ_t becomes greater than 0.1. It is also noted that ζ_d becomes smaller when μ grows larger. On the contrary, $\bar{\omega}_d$ remains constant in the case where ζ_t is less than 0.1. However, it varies sharply when ζ_t becomes larger than 0.1. If μ increases, then $\bar{\omega}_d$ decreases.

The optimal preview conditions are shown in Figs. 4 and 5. Here, the nondimensional $\bar{t}_0$ and $\bar{x}_d(0)$ are defined as

$$\bar{t}_0 = t_0 \cdot \omega_t \quad (38)$$

$$\bar{x}_d(0) = Kx_d(0) / (f_0 \cdot \omega_t) \quad (39)$$

As seen from the figure, $\bar{t}_0$ is constant when ζ_t is less than 0.1, and $\bar{t}_0$ becomes large when μ increases. However, when ζ_t becomes larger than 0.1, $\bar{t}_0$ varies

Fig. 3 Optimal tuning conditions (ζ_t versus $\bar{\omega}_d$)

linearly with ζ_t . With regard to $\bar{x}_d(0)$, it is a maximum at around $\zeta_t = 0.15$, and this is quite opposite to any of the cases shown so far. However, in the case where ζ_t is less than 0.1, $\bar{x}_d(0)$ varies little, which is similar to previous cases.

There exists an inverse proportional relationship between $\bar{x}_d(0)$ and μ , so that $\bar{x}_d(0)$ becomes half if μ is doubled. Therefore, the control effect $\bar{f}_{tp}$ is determined independently from μ ; however, it also turns out that $\bar{x}_d(0)$ needs to be increased if μ becomes smaller. Accordingly, an answer to the question of how to determine the value of the mass ratio could exist in Fig. 5, that is, the allowance for the initial stroke is a key to the determination of the mass ratio μ . If μ is given, then ζ_d , $\bar{\omega}_d$ and $\bar{t}_0$ can be read by Figs. 2~4. Arranging the design procedure of the dynamic damper with a preview action, Fig. 6 shows a flow chart for designing the system.

Fig. 4 Optimal preview conditions (ζ_t versus $\bar{t}_0$)

Fig. 5 Optimal preview conditions (ζ_t versus $\bar{x}_d(0)$)

4. Numerical Example

4.1 Example of the optimal design

We now apply the design procedure obtained in the preceding chapter to a numerical example. Table 1 shows design parameters for the calculation. As for the damping ratio of the main vibratory system, it is taken as 0.01~0.1. The results of the calculation are shown in Table 2. In the calculation, an impulse f_0 is set to 127.324 N·s, which is equivalent, for instance, to a half-sine impact force with a maximum value of 100 kN and a duration time of 2 ms. Figure 7 shows the transmitted force of the damper system obtained under the optimal conditions.

Ikeda⁽³⁾ presented an experimental equation for the design of a dynamic damper applicable to a damped vibratory system. The transmitted force controlled by the dynamic damper is also shown in Fig. 7. For example, in the case of $\zeta_t = 0.01$, the maximum value $f_{t \max}$ of the transmitted force without

Table 1 Specifications of the proposed damper system

M kg	C kN·s/m	K MN/m	M _d kg	f ₀ N·s
400	2.51~25.13	39.48	40	127.324

Table 2 Optimal parameters and control effects versus ζ_t

ζ_t	f_{tp} kN ² ·s	ζ_d	$\omega_d/2\pi$ Hz	$\bar{x}_d(0)$ cm	t_0 ms
0.01	1.47	0.330	49.86	0.723	9.97
0.05	1.44	0.326	49.87	0.734	9.93
0.10	1.43	0.320	49.89	0.742	9.87

Fig. 6 Flow chart for the design of the preview action

a damper is 39.4 kN, and the residual vibration is conspicuous. This case corresponds to the conventional spring support method because a mass is supported by a spring and a dashpot. As a result of applying the conventional dynamic damper, the residual vibration is suppressed, so that the damping effect is gained fairly well. However, with regard to $f_{t \max}$, it is still as large as 37.6 kN, which translates to a 4.6% improvement. f_{tp} in this case becomes 20.4 kNs, and it is 16.0% of f_{tp} without the damper.

As is obvious from Fig. 7, the control effect of the preview action is superior. For instance, $f_{t \max}$ hardly suppressed by the conventional dynamic damper is controlled to 15.89 kN, which is about 40% of $f_{t \max}$ without the damper. The residual vibration is also eliminated completely. In regard to f_{tp} , it is suppressed to as small as 1.15% if f_{tp} in the case of no damper is set to 100%, and the effectiveness is superior to the conventional dynamic damper by as much as 13.9 times.

In the reference⁽¹⁾, the criterion of the optimality for the design of the dynamic damper with a preview action is defined such that the maximum value of the first wave of the transmitted force is equal to that of the second wave. However, in this paper, the criterion is defined as the minimum of f_{tp} , so that the first wave

results in a larger value than the second one. The difference tends to be less as ζ_t becomes larger, and it finally becomes null when ζ_t is about 0.3. If ζ_t grows larger than 0.3, then contrarily the second wave becomes larger than the first one.

4.2 Discussions on the mass ratio

By using a numerical example, we now discuss one of the characteristics of the preview action, that is, the fact that the mass ratio does not influence on the control effect. For this purpose, the damping ratio is set to 0.1, and the amount of the mass ratio is varied from 0.05 to 0.2. The optimal design parameters under these conditions are tabulated in Table 3. The transmitted forces under these conditions are shown in Fig. 8. It is already known from Fig. 1 that the control effect f_{tp} is 1.43 kN²s in any case; however, it shall now be proved that the waveform of the transmitted force is almost the same.

Figure 9 gives a force transmissibility obtained by the optimal design parameters as shown in Table 3. According to the optimal force transmissibility of the conventional dynamic damper, a resonant peak is divided in two, each of equal levels. However, in the case of the preview action, the optimal damping ratio is a fair degree larger than that of the conventional one, so that the resonant peak is not divided, with the result that the peak level in the case of $\mu=0.1$, for instance, is larger than the conventional one by 3.5 dB.

It is a matter of course that eigenvalues of a system are subject to a mass ratio μ , so that the force transmissibility as shown in Fig. 9 also changes. However, as seen from Fig. 8, there seems little difference in the transmitted forces which are filtered by the characteristics of the force transmissibility. In regard of this, let us consider the reasons. Figure 10

Fig. 7 Control effect for suppressing the transmitted force

Table 3 Optimal parameters and control effects ($\zeta_t=0.1$, $\mu=0.05\sim 0.2$)

μ	f_{tp} kN ² s	$f_{t \max}$ kN	ζ_d	$\omega_d/2\pi$ Hz	$X_d(0)$ cm	t_0 ms
0.05	1.43	15.0	0.321	50.13	1.444	9.84
0.10	1.43	15.0	0.320	49.89	0.742	9.87
0.20	1.43	15.0	0.313	49.12	0.388	9.98

Fig. 8 Control effect for suppressing the transmitted force with a damping ratio μ

shows waveforms of the transmitted force f_{td} and f_{tim} in case of $\mu=0.05$ and 0.2 . f_{td} means the transmitted force resulting from the control force obtained by releasing the damper mass from $x_d(0)$, and f_{tim} is the transmitted force by impact force. Obviously, the waveforms of $\mu=0.05$ and 0.2 are completely different, so that up to this stage, the influence of the eigenvalues on f_{td} and f_{tim} are seen. It is also noted that the waveform of f_{tim} after the first one and one-half wave and that of f_{td} after the second wave are almost the same, with the result that these waves are canceled to produce the transmitted force f_t where the first wave which is not effaced remains. The first wave shown in the figure is subject to the eigenvalues, so that it should differ in accordance with the difference of mass ratios. However, as seen from the figure, there seems to be little difference between the first waves. The reason is discussed in the following.

Figure 11 shows the control force f_d and the transmitted force f_{td} of the control force. As for the control force, it is given by releasing the damper mass, which acts on the main mass. Figure 11 also contains the control force depicted by a dotted line, which is obtained under the condition that a main mass is assumed to be infinite. These two control forces coin-

cide with each other in the period of 0 to 5.4 ms. When the time exceeds 5.4 ms, the characteristics of the main vibratory system effect on the control forces. Namely, these waveforms are no longer the same. Therefore, it is concluded that the control force acts on the main vibratory system without being influenced by the main system during the period between 0 and 5.4 ms. It is also expressed that the transmitted force is a response of the main vibratory system to the control force, where the properties of the dynamic damper are not included during this time. In addition, the maximum transmitted force of the first wave is produced at 5 ms, one-fourth the natural period of the main vibratory system. Furthermore, there seems little difference between the control forces in the cases of $\mu=0.05$ and 0.2 , as shown in Fig. 12. As a result, the shapes of the first wave of the transmitted forces in these cases coincide with each other during the time between 0 and 5.4 ms.

4.3 Design of the conventional dynamic damper for impact vibration control

Taking into consideration the suppression of the impact vibration, the conventional dynamic damper (without a preview action) is now designed. With regard to the solution of the optimal design conditions, it is obtained similarly as before by adding the

Fig. 9 Force transmissibility of the optimally tuned system

Fig. 10 Components of the transmitted force with a damping ratio μ

Fig. 11 Control force f_d and the transmitted force f_{td} of the control force

Fig. 12 Control force with a mass ratio μ

Fig. 13 Optimal tuning values of the dynamic damper by the conventional design method and by the proposed method

following constraint :

$$t_0=0, x_d(0)=0 \quad (40)$$

Results are shown in Fig. 13, where the control effect and the optimal tuning parameters versus ζ_i are illustrated for the case of $\mu=0.1$. As far as ζ_d is concerned, the value of the proposed method is, as a whole, smaller than that of the conventional one. In the case of $\mu=0.1$, for instance, an approximately 20% difference exists between them. As for $\bar{\omega}_d$, it is larger in the proposed case than in the conventional one, and its tendency becomes conspicuous as μ increases. However, there exists little difference between the control effects, the proposed case being only 2% superior. Therefore it is concluded that the optimal design of the conventional dynamic damper is quite close to the optimal design for the impact vibration control. Figure 14 shows the transmitted forces calculated under the conditions that ζ_i is 0.1 and μ is 0.1, demonstrating that there is little difference between the proposed method and the conventional one.

5. Conclusions

For the purpose of suppressing the impact vibration of a damped vibratory system, the design procedure of the dynamic damper with a preview action is presented, and the control effect for the suppression of the impact vibration is shown.

A summary of the results of this study are as follows.

1. With a view to suppressing the transient vibration of a damped vibratory system, the optimal tuning conditions as well as the optimal preview conditions were derived by using a modern control theory.
2. Design charts were shown, by means of which the optimal parameters of the dynamic damper with a preview action were systematically obtained.
3. Based upon the presented design procedure, some examples were solved numerically, demonstrat-

Fig. 14 Control effect for the suppression of the transmitted force ($\zeta_i=0.1, \mu=0.1$)

ing its effectiveness for the suppression of the impact vibration.

4. In the design of the conventional dynamic damper, a mass ratio is an important factor, dominating the control effect. However, it turns out that the influential factor on the design of the dynamic damper with a preview action is not the mass ratio, but the damping ratio of the main vibratory system.

5. It was shown that there exists an inverse proportional relationship between the mass ratio and the initial displacement of the damper mass.

6. In the case where ζ_i is less than 0.1, the optimal tuning parameters and the optimal preview parameters were regarded as constant, however, if ζ_i becomes more than 0.1, these parameters vary, so that adjustment is needed in such a case.

References

- (1) Tanaka, N. and Kikushima, Y., A Study on the Dynamic Damper with a Preview action (1st Report ; A Principle of the Dynamic Damper with a Preview action). Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 52, No. 484, C(1986), p. 3175.
- (2) Tanaka, N. and Kikushima, Y., A Study on the Dynamic Damper with a Preview Action (2nd Report ; Experiment of the Dynamic Damper with a Preview action). Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 53, No. 487, C(1987), p. 650.
- (3) Ikeda, K. and Imoi, T., On the Dynamic Damper for the Damped Vibratory System, Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 43, No. 369 (1977), p. 1707.
- (4) Ogata, K., State Space Analysis of Control Systems, (1967), p. 257, McGrawHill.
- (5) Konno, H., Nonlinear Programming, (1987), 206, Nikkagiren.
- (6) Tanaka, N. and Miyashita, M., Design of the Servodamper Control System Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 43, No. 366, (1977), p. 550.
- (7) Tanaka, N. and Kikushima, Y., A Study on the Vibrationless Forge Hammer (2nd Report ; Optimal Design of Feedforward Element), Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 52, No. 476, C (1986), p. 1297.