A Modified Wheeler Model to Improve Predictions of Crack Growth Following a Single Overload*

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This paper modifies the Wheeler retardation concept to improve the accuracy of crack growth predictions after a single overload. This modification is primarily based on the number of delay cycles, N_d , and on the overload affected crack length, a_d . N_d is used to calibrate the Wheeler exponent, m , and a_d is employed to assess the effective overload plastic zone size. In addition, the regression equations for m and a_d are established in terms of the overload ratio OLR. The modified Wheeler model together with these equations performed well in reproduction of the actual crack growth behavior in 5083-O aluminum alloy. All predictions of the number of delay cycles were within a factor of 1.1 of the experimental observations.

Key Words: Shaping Exponent, Number of Delay Cycles, Overload Affected Crack Length, Effective Overload Plastic Zone Size

1. Introduction

Crack growth retardation after overloads is an important problem in life predictions for cracked members. A number of semi-empirical models have been developed to account for crack growth retardation due to tensile overloads, like those of Wheeler⁽¹⁾, Willenborg et al.⁽²⁾, Matsuoka et al.⁽³⁾, and Russell⁽⁴⁾. Although none of these models covers a wide range of crack propagation situations, the Wheeler model is attractive because of its simplicity.

A basic assumption of the Wheeler model is that the interaction of the crack-tip plastic zones is the most important source of load sequence effects on crack growth under variable amplitude loading. In the Wheeler model, crack retardation after overloads ceases as the current crack-tip plastic zone touches

the boundary of the overload plastic zone (Fig. 1(a)). However, examination of this model shows that the crack retardation has not yet been finished at this point because the crack tip remains contained within the overload plastic zone and is subjected to the influence of the overload. Furthermore, several researchers found that crack propagation was still influenced by the overload even though the crack tip had left the overload plastic zone⁽⁵⁾⁻⁽⁸⁾. These findings imply that the Wheeler model exhibits probably some conservatism in predictions of crack retardation following single overloads. In the Wheeler model, the retardation behavior is described by means of the shaping exponent, m . This exponent depends upon the material properties as well as the characteristics of the load history⁽⁹⁾⁻⁽¹²⁾.

Single tensile overload is not only the primitive building-block of variable amplitude loading, but also is one of the simplest loading patterns causing retardation effects. The single overload retardation behavior is characterized by the number of delay cycles, N_d , and the overload affected crack length, a_d . In the current study, the experimentally determined parameter N_d was utilized to determine the Wheeler exponent

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Fig. 1 Illustration of the plastic zones for: (a) the Wheeler model, and (b) the modified Wheeler model

m , and the parameter a_d was used to compute the effective overload plastic zone size. The regressions m and a_d were established to the overload ratio (OLR). These regression equations were combined with the modified Wheeler model to predict the crack growth behavior following a single overload. The computational results are compared here with experimental observations.

Nomenclature

- a : crack length
- a_i : current crack length
- a_d : overload affected crack length
- a_o : overload position
- da/dN : crack growth rate per cycle
- C : constant in the Paris equation
- C_{pi} : retardation parameter
- C'_{pi} : modified retardation parameter
- $K_{max,i}$: maximum stress intensity in the i th cycle
- K_{ol} : overload stress intensity
- ΔK : stress intensity factor range
- m : shaping exponent
- N : number of cycles
- N_d : number of delay cycles (observed delay cycles)
- N_p : predicted delay cycles
- n : exponent in the Paris equation

OLR: overload ratio (P_{ol}/P_{max})

P_{max} : maximum load

P_{min} : minimum load

P_{ol} : tensile overload

R : load ratio (P_{min}/P_{max})

r_{pi} : current plastic zone size for the i th cycle

r'_{pi} : plastic zone size at the crack size $a = a_o + a_d$

r_{po} : overload plastic zone size

r'_{po} : effective overload plastic zone size

σ_y : yield stress

α : plastic zone size constant

2. Crack Growth Analysis

2.1 Wheeler retardation model

Wheeler introduced a retardation parameter, C_{pi} , and suggested that the retarded crack growth rate could be represented by

$$(da/dN)_{retarded} = C_{pi} \times C(\Delta K)^n, \quad (1)$$

where ΔK is the stress intensity factor range, and C and n are material constants. The retardation parameter is defined as (Fig. 1(a)):

$$C_{pi} = \left(\frac{r_{pi}}{a_o + r_{po} - a_i} \right)^m \quad \text{as long as } a_i + r_{pi} < a_o + r_{po} \quad (2)$$

or

$$C_{pi} = 1 \quad \text{for } a_i + r_{pi} \geq a_o + r_{po}.$$

In the equation, a_o denotes the crack size where the overload was placed; a_i is the instantaneous crack length in the i th loading cycle; and the exponent m is empirically adjusted.

The size of the plastic zone is

$$r_{pi} = \frac{1}{\alpha\pi} \left(\frac{K_{max,i}}{\sigma_y} \right)^2; \quad (3)$$

$$r_{po} = \frac{1}{\alpha\pi} \left(\frac{K_{ol}}{\sigma_y} \right)^2, \quad (4)$$

where $K_{max,i}$ represents the maximum stress intensity in the i th cycle; K_{ol} is the overload stress intensity; the value of the constant α is taken as 1 or 3 for the plane stress or plane strain situation for Irwin plastic zone size correction⁽¹³⁾; and σ_y is the yield stress.

2.2 The modified Wheeler model

The crack growth behavior after the application of a tensile overload is shown in Fig. 2. The retardation phenomenon after the overload is normally characterized by two parameters, i.e., the number of delay cycles, N_d , and the overload affected crack length, a_d . N_d is the number of cycles needed for the Wheeler retardation parameter to reach unity and for the retarded crack to grow beyond the overload affected crack length, a_d . A modification of the Wheeler model is proposed as follow.

From the a vs. N curve of a single tensile overload test, the overload affected crack length, a_d , can be determined. As the crack propagates to a length of

Fig. 2 Schematic representation of the crack retardation after a single overload

$a = a_o + a_d$, the size r'_{pi} of the current crack tip plastic zone is added to a_d for an effective overload plastic zone r'_{po} (Fig. 1(b)). The size of the plastic zone is $r'_{po} = a_d + r'_{pi}$. (5)

Replacing r_{po} with r'_{po} in Eq. (2) leads to:

$$C'_{pi} = \left(\frac{r_{pi}}{a_o + r'_{po} - a_i} \right)^m \quad \text{for } a_i + r_{pi} < a_o + r'_{po}$$

or

$$C'_{pi} = 1 \quad \text{if } a_i + r_{pi} \geq a_o + r'_{po}. \quad (6)$$

With C'_p in place of C_{pi} , Eq. (1) can be rewritten as $(da/dN)_{retarded} = C'_p \times C(\Delta K)^n$. (7)

In this equation, the Wheeler exponent m is calibrated based on the number of delay cycles, N_d . C'_{pi} is computed using an assumed m -value, and then the corresponding crack growth is determined using Eq. (7). With the resulting N_p , the number of post-overload cycles necessary for C'_{pi} to approach unity is also given. If N_p compares unfavorably with N_d , then the assumed m -value is replaced by a new one. In this way, iterations are performed until a certain m -value is searched at which N_p approaches N_d . For quick assessment of the m -value, a bisection algorithm is applied.

3. Experimental Procedures

In the present study, a test program was applied to the 5083-O aluminum alloy. The chemical composition and mechanical properties of this alloy are given in Table 1. The test specimens were of the ASTM compact type of an L-T orientation. The thickness was 9.8 mm, and the width was 50.8 mm.

Crack growth experiments were performed on a closed-loop servohydraulic testing machine. In the tests, all the specimens were precracked according to

Table 1 Chemical composition and mechanical properties of the 5083-O aluminum alloy

Weight percent (%)							
Si	Fe	Cu	Mn	Mg	Cr	Zn	Ti
0.143	0.238	0.040	0.591	4.431	0.101	0.049	0.005
0.2% Yield stress (MPa)	Ultimate tensile stress (MPa)		Modulus of elasticity (GPa)		Elongation on a gauge length of 25mm (%)		No. of tests
157	312		72		25		4

Fig. 3 Crack length vs. applied cycles for various overload conditions

the ASTM-E647-88a standard⁽¹⁴⁾. Crack length measurements were made using a 20× traveling microscope. The constant amplitude cycles had a triangular waveform. The stress ratio was $R=0.15$ ($P_{max}=4415$ N), and the test frequency was 10 Hz. The parameters of the Paris equation were found to be $C=1.549 \times 10^{-11}$ and $n=2.88$. The units for da/dN and ΔK were mm/cycle and $\text{MPa}\sqrt{\text{m}}$, respectively.

In the overload test, a single overload was applied at a crack length of about $a_o=15$ mm, and the frequency for the overload cycle was 0.1 Hz. For the overload application, five overload ratios (OLR) of $P_{oi}/P_{max}=1.4, 1.6, 1.8, 2.0$, and 2.2 were applied. After each overload, constant amplitude loading was applied until the pre-overload crack growth rate was resumed. To compare the experimental and computational results, overload ratios of $P_{oi}/P_{max}=1.5, 1.7, 1.9$ and 2.1 were also used at $a_o=15$ mm.

4. Results and Discussion

Figure 3 shows the observed crack length as the number of applied cycles for the various OLR values. N_d and a_d were assessed using the method shown in Fig. 2. The retardation results are listed in Table 2. The tabulated data show the correlation between OLR and N_d . The higher the OLR value became, the

Table 2 Retardation results for the investigated 5083-O aluminum alloy

OLR	K_{ol} MPa $\sqrt{m}$	N_d (cycles)	r_{po} (mm)	a_d (mm)	m -value
1.4	15.54	19000	1.05	3.18	0.33
1.6	17.76	33800	1.37	4.73	0.43
1.8	19.97	45800	1.73	5.58	0.51
2.0	22.19	87700	2.14	8.98	0.58
2.2	24.42	129500	2.59	9.38	0.72

Fig. 4 Dependence of m on OLR

greater was the N_d value. This is probably due to the larger compressive residual stresses introduced by a higher overload in the near crack tip region. As a result, many more post-overload fatigue cycles were required to exhaust the retardation effect.

The m -values for the various overload conditions are listed in Table 2. These values varied from 0.33 to 0.72 and increased as OLR increased. The correlation between OLR and m is depicted in Fig. 4 in a plot as a double log. All the data points follow an approximately straight line. This correlation can be represented by the power law with a correlation coefficient of $R=0.995$ in the following form:

$$m=0.19(OLR)^{1.65} \quad (8)$$

The data for the overload affected crack length, a_d , are also presented in Table 2. The values of a_d also increased with OLR. The relationship between a_d and OLR can also be represented by an approximately straight line as shown in Fig. 5. This relationship may be expressed in an exponential manner with a correlation coefficient of $R=0.981$. The resulting expression is

$$a_d=1.40(OLR)^{2.49} \quad (9)$$

The predictive ability of the Wheeler model can be checked by comparing the overload affected crack length, a_d , with the calculated overload plastic zone size, r_{po} . The values of a_d and r_{po} are listed in Table 2. The tabulated values of r_{po} was calculated with a

Fig. 5 Relationship between a_d and OLR

Fig. 6 Actual and predicted crack growth at OLR=1.5 and $a_o=15$ mm for the 5083-O specimen

constant α taken as 3 because all the fracture surfaces in the overloaded specimens exhibited a plane strain behavior with almost no shear lips on the specimen side surfaces, and also because the ratio of the specimen thickness, t , to the plane strain overload plastic zone size, r_{po} , ranged between 3.8 and 9.3. Such a range points to what would normally be plane strain behavior⁽¹⁵⁾. As seen in Table 2, the values of a_d were higher than those of r_{po} at various overload ratios. This observation indicates that the Wheeler model gives conservative crack growth predictions. Similar conclusions have previously been drawn by Robin and Pelloux⁽¹⁶⁾, and Sheu et al.⁽¹⁷⁾.

The modified Wheeler model was combined with Eqs. (8) and (9), and predictions were then made for an OLR of 1.5 at a crack length of $a_o=15$ mm. A comparison of the predicted and observed crack growth is presented in Fig. 6. The two crack growth curves follow each other closely, indicating a small difference between the observed and predicted number of delay cycles. Figures (7) through (9) show the predicted and measured crack growth behavior for the OLR values of 1.7, 1.9 and 2.1 at a crack length of $a_o=15$ mm. As seen in these figures, the modified

Fig. 7 Actual and predicted crack growth at OLR=1.7 and $a_o=15$ mm for the 5083-O specimen

Fig. 8 Actual and predicted crack growth at OLR=1.9 and $a_o=15$ mm for the 5083-O specimen

Fig. 9 Actual and predicted crack growth at OLR=2.1 and $a_o=15$ mm for the 5083-O specimen

Wheeler model adequately reflects the actual growth trends. Figure 10 shows a satisfactory correlation between the predicted and actual results, which differ by a factor of 1.1.

5. Conclusions

The following conclusions can be reached from

Fig. 10 Comparison between actual and predicted delay cycles

the results of the present study.

(1) A calibration process for the exponent m of the Wheeler model based on the basis of the number of delay cycles following a single overload has been proposed. The exponent m has been found to be strongly dependent on OLR, and the dependence can be expressed by a power law.

(2) The plastic zone size is an important factor in Wheeler predictions. Use of the Irwin plastic zone size in the Wheeler model resulted here in conservatism in crack growth predictions. On the other hand, the application of an effective overload plastic zone size in the Wheeler model led to better crack growth predictions.

(3) The combination of the modified Wheeler model with the m and a_d regression equations confirmed the potential to accurately predict the growth behavior after a single overload. In the case of 5083-O aluminum alloy, the resulting growth predictions followed closely the experimental results.

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