Creation Technique and its Applications to Functional Continua using Topological Optimization*

Juhachi ODA** and Yasuto SAKAI**

In this paper, we propose a new method to produce the functional continuum. It is a topological optimization technique based on a function of the continuum and composed of three steps. In the first step, an initial mechanism model, which conforms the given function is determined, using only some levers and supports. The mechanism model is changed to a framed structure as the Rahmen model in the second step. The cross section and length of members in the framed structure are optimized to maintain the original design requirement. In the final step, the framed structure is changed to the functional continuum using some well known shape optimization techniques. Two examples of application of this technique for the functional continuum are presented.

Key Words: Functional Continuum, Topological Continua, Topological Optimization, Shape Optimization, Microdevice, Micromachine

1. Introduction

The development of micromachines and micro-devices warrants the production of a microcontinuum structure, satisfied by a given function of the device⁽¹⁾⁻⁽³⁾. Such a continuum structure is called a "functional continuum". In this paper, we propose a new method to produce the functional continuum. The method consists of a topological optimization technique based on some function of the continuum. This technique is composed of three important steps. In the first step, an initial mechanism model which conforms the given function is determined using only some levers and supports. The mechanism model is changed to a framed structure as a Rahmen model in the second step. The cross section and length of members in the framed structure are then optimized to maintain the original design requirement. In the final step, the framed structure is changed into the functional continuum using some well known shape

optimization techniques.

The method presented above is applied to two problems of functional continuum. The first problem is to produce the continuum that has the deformation property of the output $\alpha\delta$ in the y axis on the input δ in the x axis, which is orthogonal to y axis. The obtained continuum can be applied to a microdevice of a displacement transducer. The second problem is to produce the continuum that has a bi-axial tensile stress condition in the central part, in spite of being subjected to the uniaxial tensile loading. The obtained continuum can be applied to a new bi-axial tensile test specimen for thin plates.

2. Functional Continuum and Function Based Topological Optimization Method

Figure 1 shows an example of the functional

(a) Function (b) Functional continua

Fig. 1 Functional continuum

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** Bionic Design Laboratory, Department of Human & Mechanical Systems Engineering, Kanazawa University, 2-40-20 Kodatsuno, Kanazawa, Ishikawa 920-8667, Japan. E-mail: oda18@t.kanazawa-u.ac.jp

Fig. 2 Flow chart to produce a functional continua

Fig. 3 Flow to produce a functional continua

continuum. The continuum has a deformation property for the output $\alpha\delta$ in the y axis on the input δ in the x axis. It can thus be used as an orthogonal displacement transducer. To produce such a continuum as a multiple connected body gets very difficult because it is a complex topological optimization problem.

However, to produce such a mechanism, is relatively easy. Therefore a method to produce the functional continuum is considered, using a mechanism model, which conforms the given function of a continuum. Figure 2 shows the developmental flow chart to produce the functional continuum. In this flow chart there are three important steps. The procedures at these steps can be described concretely using the schematic shown in Fig. 3. The goal is to produce a continuum. When a displacement δ is given in the x axis, a displacement $\alpha\delta$ in the y axis occurs. An initial mechanism model, which conforms the given function, must be determined in the first step. Figure 3(b) shows the mechanism model composed of only some levers and supports. By changing the length of levers and the position of supports, the optional displacement ratio α can be determined. In the second step, this mechanism model is changed to a framed structure (Rahmen model) as shown in Fig. 3(c). In the model the cross section and length of members must be optimized to maintain the relationship between input δ and output $\alpha\delta$ as the original design requirement. The framed structure is changed to the continuum model, as shown in Fig. 3(d), in the final step. In this model some well known shape optimization techniques have to be used to maintain the relationship between input δ and output $\alpha\delta$. The method presented above can be called a "function based topological optimization method".

Fig. 4 Lever model

3. Numerical Examples

3.1 Continuum as orthogonal displacement transducer

As a first example, a continuum as the orthogonal displacement transducer, shown in Fig. 1, is considered. If the mechanism model, shown in Fig 3(b), is used in the first step, the relationship between a displacement ratio α and lever parameters (a, l, θ) is given in the following equation,

$$\alpha = \frac{\delta y}{\delta x} = \frac{l-a}{a} \cdot \frac{1}{\tan \theta} \quad (1)$$

because $\delta x = \sin \theta \delta x'$ and $\delta y = \cos \theta \delta y'$ in 1/4 part of the mechanism model as shown in Fig. 4.

By choosing an optional, a, l and θ , we can obtain several values of α . However, for a practical model Eq.(1) is not exact, because the lever is an elastic beam on an elastic support. To obtain the models of $\alpha=1.0$ and 2.0 , we can therefore consider the framed structure model, shown in Fig. 5. In the model, the

lever length l and the cross sectional area A_2 of the support member are constant, α , θ and the cross sectional area A_1 of the lever beam are changed to maintain $\alpha=1.0$ and 2.0 . To obtain the optimum α , θ and A_1 , the genetic algorithm is used. As a consequence, α , θ and A_1/A_2 are 48 mm, 35° and 0.84 for $\alpha=1$ model, and 35 mm, 21.25° and 0.92 for $\alpha=2$ model, respectively. Using these results, the models as the two-dimensional continuum can be obtained as shown in Fig. 6. Moreover the shapes are changed as shown in Fig. 7 to decrease the concentrated deformation in the input part A and to increase the rotation of the lever at the support part B. The values of α on the continuums, shown in Fig. 7, are 0.972 and 1.97. These values are almost equal to the objective values 1.0 and 2.0, respectively.

Fig. 5 Framed structure

(a) $\alpha=1.0$ model

(b) $\alpha=2.0$ model

Fig. 6 $\alpha=1.0$ & 2.0 models and it's element subdivisions

3.2 Continuum having bi-axial stress field under uni-axial loading

The measurement of the bi-axial mechanical properties of thin plates is important for designing such structural elements. However, the bi-axial mechanical test is difficult, because maintaining exact bi-axial load conditions is a technical challenge. In this section, a continuum for the bi-axial mechanical test as shown in Fig. 8 is created. To create such a continuum, we consider a mechanism model as shown in Fig. 9. If this model is loaded in the Y direction, by a tensile load P , the tensile loads T_1 and T_2 will be produced in the members I and II, as points A and B in the member III move in the directions of the arrows as shown (Fig. 9). In the crossing part M of the members I and II, a bi-axial stress field will be produced. If the cross section and length of the members composing the mechanism model are changed, an optional value of T_1/T_2 will be obtained. That is, we can obtain the optional bi-axial stress field.

To clarify the idea illustrated above, the Rahmen model shown in Fig. 10(a), that simulates the mechanism model shown in Fig. 9, is used in the second step. In this model, the material is the same for all the members and the cross section is also the same for the members except for the members near the part M. The objective is to determine the lengths of all the

(a) $\alpha=1.0$ model

(b) $\alpha=2.0$ model

Fig. 7 Improved models and it's element subdivisions

Fig. 8 Bi-axial tensile stress condition

Fig. 9 Mechanism model

(a) Member No. (b) Optimum structure

Fig. 10 Rahmen model

members, such that the tensile stresses produced in member 13(14) and 15 are equal. Figure 10(b) shows the optimum Rahmen structure obtained. In the structure, σ_1/σ_2 , in which σ_1 or σ_2 is the axial stress of member 13(14) or 15 respectively, is 0.995 which is approximately the bi-axial tensile load condition.

In the next step, an original shape as a functional

(a) Original model (b) Improved model

Fig. 11 Continua model

Fig. 12 σ_y/σ_x in test part

continuum can be derived using the optimum Rahmen structure. Figure 11(a) shows the model. In this shape, the cross section of the uniform thickness plate is realized from the cross section of an optimum Rahmen structure by adjusting and manipulating the uniform thickness plate. Moreover, the final shape (Fig. 11(b)) of the functional continuum can be obtained by decreasing the stress concentration in the original shape. Figure 12 shows the stress distribution in the part M of the final shape. In this analysis, the element subdivision for the finite element method, shown in Fig. 11, is used. From Fig. 12, it is obvious that the stress condition is almost that of bi-axial tensile.

Using this functional continuum, we can obtain the bi-axial mechanical properties of a thin plate such as a film⁽⁴⁾. Figure 13 shows the testing method. The

Fig. 13 Testing method of thin plates

matrix plate is made of a rigid material in the shape of the functional continuum proposed in this section. The thin plate to measure the mechanical properties is pasted on the drilled middle part of the matrix plate.

4. Conclusions

In this paper, we propose a new method to produce the functional continuum. The method consists of a topological optimization technique based on the function of a continuum and is composed of three steps. First, an initial mechanism model, which conforms to the given function, is determined using only some levers and supports. The mechanism model is changed to a framed structure as the Rahmen model in the second step. The cross section and length of members in the framed structure are optimized to

maintain the original design requirement. In the final step, the framed structure is changed to the functional continuum using some well known shape optimization techniques. This method is applied to two problems of functional continuum. The first problem is to produce the continuum as an orthogonal displacement transducer. The second problem is to produce the continuum having a bi-axial stress field under a uni-axial loading. From the results of these problems, it is obvious that the proposed method is very useful for producing the functional continuum.

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