Strain Concentration at Structural Discontinuities and Its Prediction Based on Characteristics of Compliance Change in Structures*

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Elevated temperature structural design codes pay attention to strain concentration at structural discontinuities due to creep and plasticity, since it causes an increase in creep-fatigue damage of materials. One of the difficulties in predicting strain concentration is its dependence on the magnitude of loading, the constitutive equations, and the duration of loading. In this study, the author investigated the fundamental mechanism of strain concentration and its main factors. The results revealed that strain concentration is caused by strain redistribution between elastic and inelastic regions, which can be quantified by the characteristics of structural compliance. The characteristics of structural compliance are controlled by elastic region in structures and are insensitive to constitutive equations. It means that inelastic analysis can be easily applied to obtain compliance characteristics. By utilizing this fact, a simplified inelastic analysis method was proposed based on the characteristics of compliance change for the prediction of strain concentration.

Key Words: Structural Analysis, Elastic-plastic, Creep, Stress Relaxation, Elastic Follow-up, Compliance

1. Introduction

In high-temperature and low-pressure structures such as fast breeder reactors, the initiation of creep-fatigue cracks by cyclic thermal stress is one of the main failure modes. In particular, consideration is paid to structural discontinuities, since strain concentration by plastic and creep deformation causes their strength degradation⁽¹⁾.

The elastic follow-up concept⁽²⁾ can explain the qualitative mechanism of strain concentration. When a high-rigidity and low-stress structure is connected to a low-rigidity and high-stress one and they are subjected to a displacement load, creep deformation in the low-rigidity portion unloads both portions. During the relaxation process, elastic recovery of the high-rigidity portion enhances the displacement of the low-

rigidity one since total displacement is constant. As a result, the low-rigidity portion exhibits a strain concentration, that does not occur in the elastic deformation.

For quantitative evaluation of strain concentration, the inelastic finite element method is expected to be applicable because prediction of this behavior becomes a statically indeterminate problem in which stress and strain couple. However, there are some difficulties in applying it to design, such as the dependence of their solutions on constitutive equations and load histories. Therefore, conventional designs have been based on elastic analysis methods with the help of conservative strain concentration factors⁽³⁾, and their rationalization is required from a demand on the cost reduction of plant constructions.

In this study, the quantitative mechanism of strain concentration is clarified through investigation of the factors affecting it by using the inelastic finite element method. This study provides a realistic design approach based on the inelastic finite element method and a simplified method with elastic analysis.

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Fig. 1 Y-piece structure subjected to thermal loading

2. Strain Concentration Mechanism of Structural Discontinuities

2.1 Plastic behavior

Elastic-plastic strain concentration behavior in a Y-piece (Fig. 1) was examined, since the Y-piece can represent strain behavior in structural discontinuities. Load conditions were assumed to be quasi-static thermal transients by heating the inner surface of the cylinder to 550°C while the skirt edge was kept at 50°C.

The Y-piece is composed of a high-rigidity portion (junction) and a low-rigidity part (skirt part). The interaction between both portions produces a secondary stress that overlaps with a peak stress induced by the local shape discontinuity of the junction. The mechanism of stress generation is common among typical structures in plant components such as nozzles and tubesheets.

The material was assumed to be 304SS and various constitutive equations were applied to investigate sensitivity of strain concentration to them. The adopted equations are an elastic equation, the Ramberg-Osgood type equation that can simply express non-linearity of stress-strain relationship

$$\epsilon = \sigma/E + A\sigma^n \quad (E=159\,000, A=1.0 \times 10^{-8}, n=3, 5, 7), \quad (1)$$

the Ludwik type equation that can consider yield stress

$$\epsilon = \frac{\sigma}{E} + \left\{ \frac{\sigma - \sigma_p}{K} \right\}^n \quad (\sigma > \sigma_p, \sigma_p = 75, n=3, 5, 7), \quad (2)$$

the perfectly plastic equation and the bilinear equations with two different expected strain ranges. The temperature dependence of the material was ignored for clear understanding of phenomena. The strain behavior was calculated with the above constitutive equations according to the analysis cases in Table 1. The analysis code is the nonlinear structural

Table 1 Elastic-plastic analysis cases with various constitutive equations

Case	Analysis Type	Constitutive Equation	Power
TE	Elastic	Elastic	$n=1$
P3	Elastic-Plastic	Ramberg-Osgood	$n=3$
P5	Elastic-Plastic	Ramberg-Osgood	$n=5$
P7	Elastic-Plastic	Ramberg-Osgood	$n=7$
PL3	Elastic-Plastic	Ludwik	$n=3$
PL5	Elastic-Plastic	Ludwik	$n=5$
PL7	Elastic-Plastic	Ludwik	$n=7$
PP	Elastic-Plastic	Perfectly plastic	∞
PB5	Elastic-Plastic	Bi-linear	$(\Delta \epsilon_t = 0.5\%)$
PB10	Elastic-Plastic	Bi-linear	$(\Delta \epsilon_t = 1.0\%)$

Fig. 2 Elastic-plastic behavior under various constitutive equations

analysis system 'FINAS'⁽⁴⁾, and the mesh model is an axisymmetric one as shown in Fig. 1.

The calculated stress-strain behavior at the maximum strain portion (see Fig. 1) was compared as shown in Fig. 2⁽⁵⁾. The results indicate dependence of strain on constitutive equations and the increase in the strain concentration factor K_ϵ , which is defined as the ratio of elastic-plastic strain ϵ_{in} to elastically calculated ones ϵ_e :

$$K_\epsilon = \epsilon_{in}/\epsilon_e, \quad (3)$$

when non-linearity of stress-strain relationship is strong.

Next, the sensitivity to load level was investigated. With the fixed constitutive equation being the Ramberg-Osgood type ($n=5$), the strain behavior was calculated for temperatures increasing to 217°C ($1/3 \sigma_{max}$), to 383°C ($2/3 \sigma_{max}$) and to 550°C (σ_{max}), while the temperature of the edge of the skirt was kept at 50°C. The analysis cases are shown in Table 2 and the obtained results are shown on the stress-strain plot of Fig. 3⁽⁶⁾.

The elastic-plastic solutions (case P5) that corresponded to a load of $1/3 \sigma_{max}$, $2/3 \sigma_{max}$ or σ_{max} were compared with elastic ones (case TE, indicated by symbol $\square$). The results proved that the strain factor K_ϵ increases as the load level is increased.

Table 2 Elastic-plastic analysis cases under different load levels

Case	Analysis Type	Constitutive Equation	Power	Loading
TE	Elastic	Elastic	$n=1$	$1/3\sigma_{\max}, 2/3\sigma_{\max}, \sigma_{\max}$
P5	Elastic-Plastic	Ramberg-Osgood	$n=5$	$1/3\sigma_{\max}, 2/3\sigma_{\max}, \sigma_{\max}$

Fig. 3 Elastic-plastic behavior under different load levels

2.2 Creep behavior

The strain behavior at the maximum strain portion of the Y-piece was examined during stress relaxation when the surface of the Y-piece was kept at 550°C and the edge of the skirt was fixed at 50°C, as in Fig. 1. For investigation of sensitivity to creep strain equation, various kinds of creep strain equations were utilized, such as Norton's law that can simply express non-linearity of materials

$$\dot{\epsilon}_c = B\sigma^m \quad (B = 5.86 \times 10^{-15}, m = 3, 5, 7), \quad (4)$$

and the Blackburn type equation⁽⁷⁾,

$$\epsilon_c = C_1(1 - e^{-r_1 t}) + C_2(1 - e^{-r_2 t}) + \epsilon_m t, \quad (5)$$

which can describe detailed characteristics.

The list of analysis cases is shown in Table 3. The case of P5-C7 is a series of elastic-plastic analyses by plastic equation P5 and elastic-creep analysis by creep equation C7. The calculated stress relaxation behavior for 210 000 hours was compared on the stress-strain plot in Fig. 4⁽⁵⁾. Only a relaxation locus is shown for the case P5-C7. From this figure, it was found that strain concentration factor K_e increases with the progress of the stress relaxation hour, and it is proven that the magnitude of the strain concentration factor after 210 000 hours depends on the creep strain equations⁽⁸⁾.

In the meantime, all of the relaxation processes exist on an identical relaxation locus, regardless of the creep strain equations. Consequently, it was proven that the structures have the unique characteristic of being insensitive to the creep strain equations.

Next, the sensitivity of strain concentration

Table 3 Creep analysis cases with various creep strain equations

Case	Analysis Type	Constitutive Equation	Power
TE	Elastic	Elastic	$n=1$
C3	Elastic-Creep	Norton's Law	$m=3$
C5	Elastic-Creep	Norton's Law	$m=5$
C7	Elastic-Creep	Norton's Law	$m=7$
P5-C7	Elastic-Plastic-Creep	Norton's Law Ramberg-Osgood	$m=7$ $n=5$
CBL	Elastic-Creep	Blackburn	—

Table 4 Creep analysis cases under different load levels

Case	Analysis Type	Constitutive Equation	Power	Loading
TE	Elastic	Elastic	$n=1$	$1/3\sigma_{\max}, 2/3\sigma_{\max}, \sigma_{\max}$
C5-1/3	Elastic-Creep	Norton's Law	$m=5$	$1/3\sigma_{\max}$
C5-2/3	Elastic-Creep	Norton's Law	$m=5$	$2/3\sigma_{\max}$
C5	Elastic-Creep	Norton's Law	$m=5$	$\sigma_{\max}$

Fig. 4 Stress relaxation behavior with various creep strain equations

behavior to load level was investigated. With the fixed constitutive equation being Norton's law ($m=5$), stress relaxation behavior for 210 000 hours was calculated when the inner surface of the cylinder was 217°C ($1/3\sigma_{\max}$), 383°C ($2/3\sigma_{\max}$) or 550°C ($\sigma_{\max}$) under a constant temperature 50°C of the edge of the skirt. The analysis cases are shown in Table 4, and the calculated stress relaxation locus is shown in Fig. 5⁽⁶⁾. This figure reveals that the magnitude of stress relaxation and strain concentration factor K_e increase as the load level increases.

3. Unified Characterization of Inelastic Deformation by Relaxation Locus

To understand the basic mechanism of strain concentration, common factors are examined among the influences of constitutive equation, load level and

Fig. 5 Stress relaxation behavior during 210 000 hours under different load levels

stress relaxation period on the strain concentration behavior due to plasticity and creep.

First, it is noted that the stress relaxation processes of the same structures are not dependent on the creep strain equations and exist on an identical relaxation locus, as shown in Fig. 4. Since the locus of the P5-C7 case is equal to that of the other cases calculated by creep analysis, there is a possibility of a common mechanism between plastic and creep behavior. Then, the author tried to plot elastic-plastic stress-strain curves in Fig. 2 with the stress relaxation curves of Fig. 4. The result in Fig. 6 indicates that the elastic-plastic solutions exist on an almost equal stress relaxation locus. From these results, it can be estimated that the stress relaxation locus from a certain load means a common strain concentration characteristic between plasticity and creep, which does not depend on constitutive equations.

Next, the effect of load level is examined. When comparing elastic-plastic solutions for three kinds of load levels described in Fig. 3 with ones for creep of Fig. 5, the results become those in Fig. 7. Stress relaxation locus at each load level intersects with the elastic-plastic solution for the equal load level, and strain concentration factor K_ϵ is identical for both plasticity and creep. Therefore, the load level seems to exert the same effect on elastic-plastic and creep behavior.

4. Relaxation Locus Regulated by Characteristics of Compliance Change

The reason why the stress relaxation locus exhibits common strain concentration characteristics between plasticity and creep is investigated from a mechanical point of view. The elastic follow-up parameter q , defined by the gradient of the line that connects the elastic analysis result on the stress-strain plot of Fig. 8 with the inelastic solution⁽⁷⁾, is

Fig. 6 Comparison of elastic-plastic behavior and creep behavior with various constitutive equations

Fig. 7 Comparison of elastic-plastic behavior and creep behavior under different load levels

Fig. 8 Strain concentration behavior described by elastic follow-up parameter q

introduced here in order to quantitatively handle the stress relaxation locus. $q=1$ means that strain is constant and corresponds to a displacement-controlled condition. On the other hand, $q=\infty$ expresses a fixed stress and is called the stress controlled condition. Conventional structures have intermediate conditions with q value between 1 and ∞ , and this value becomes a parameter for quantifying the stress

Fig. 9 Two-bar model to explain elastic follow-up concept

relaxation locus.

Next, the characteristics of the q value are examined on a two-bar model (Fig. 9) composed of a low-rigidity high-stress bar ($i=1$) and high-rigidity low-stress one ($i=2$), which simply explains the elastic follow-up concept⁽²⁾.

When Norton's law (Eq.(4)) is assumed for a creep strain equation, the q value on the creep is deduced by an explicit equation as

$$q = \frac{1 + (l_2/l_1)(a_1/a_2)}{1 + (l_2/l_1)(a_1/a_2)^m} \text{ (creep)} \quad (6)$$

and it is proven to become a constant value, that depends on the shape and the non-linearity of the stress-strain relationship. The stress relaxation locus of the two-bar model becomes a straight line, because the q value indicates the gradient on the stress-strain plot.

In addition, the q value can be defined for elastic plastic behavior in the same manner as that of creep (Fig. 8). When the Ramberg-Osgood expression is assumed (Eq.(1)), we can obtain an equation similar to Eq.(6) for plasticity as

$$q = \frac{1 + (l_2/l_1)(a_1/a_2)}{1 + (l_2/l_1)(a_1/a_2)^n} \text{ (Plasticity)}. \quad (7)$$

In the case of actual structural design, the elastic region (elastic core) remains in the wall thickness region, since full-section plasticity is not allowed. For this case, it is possible to consider the high-rigidity low-stress portion of Fig. 9 as an elastic body. When Bar 2 is made of an elastic body, Eqs.(6) and (7) become the following same equation.

$$q = 1 + (l_2/l_1)(a_1/a_2) \quad (8)$$

Equation (8) is a function only of the geometry that does not depend on constitutive equations. The q value has the physical meaning of the compliance characteristics of the system, because Eq.(8) can be described as follows by compliance C_i of Bar i , when Young's modulus of the material is E .

$$q = 1 + (l_2/l_1)(Ea_1/Ea_2) = 1 + C_2/C_1 \quad (9)$$

Next, the case of the Y-piece is examined. The progress of creep strain distribution during stress relaxation was observed in the case of $m=7$ Norton's law (C7). The upper stage of Fig. 11 shows creep strain distributions at the Time 1, the Time 2 and the Time 3 in the stress-strain plot of Fig. 10

Fig. 10 Stress relaxation locus and elastic-plastic curves

Fig. 11 Comparison of inelastic strain contours

The creep strain has been generated in low-rigidity high-stress portion in the juncture of a skirt, as shown in the upper stage of Fig. 11 and it is proven that the high-rigidity low-stress portion in the circumference remains in the elastic condition⁽⁸⁾. From this fact, the reason why the stress relaxation locus of the Y-piece is insensitive to constitutive equations seems to be for regulating the q value by the compliance characteristics, because the high-rigidity low-stress portion is the elasticity condition, which is the same mechanism as the two-bar model. When stress relaxation progresses from Time 1 to Time 3, the creep strain region expands. Redistribution of stress and strain can occur under the constant boundary condition, because the Y-piece is the statically indeterminate structure⁽⁹⁾. This progression of creep strain region leads to a change of the compliance ratio between the low-rigidity high-stress portion (corresponding to C_1 in Eq.(9)) and the high rigidity low-stress portion (corresponds to C_2 in Eq.(9)). In other words, the restraint force from the surrounded elastic region decreases. Variation of compliance character-

istics leads to a change of q value that is shown as a gradient on the stress-strain plot. Therefore the stress relaxation locus would draw the curve in the stress-strain plot. When over-plotting elastic-plastic solutions (Cases P3, P5, P7 in Table 1) with the relaxation locus of Case C7 on the stress-strain plot, P3, P5 and P7 stress-strain curves intersect with each point of the relaxation locus at Time 1, Time 2 and Time 3, as shown in Fig. 10. The bottom portion of Fig. 11 shows plastic-strain distributions of P3, P5 and P7, which are quite similar to the distribution of the creep strain at Time 1, Time 2 and Time 3 on the stress relaxation locus. The surrounding portions are the elastic cores that have the same configuration between plasticity and creep, which causes the same compliance in structures. From the above observations, the reason why strain concentration characteristic is common between plasticity and creep and is insensitive to constitutive equations is that its q value is regulated by the compliance characteristics of the system, when the elastic region has been left in the structures. In the meantime, the compliance of the Y-piece changes with the reduction of the elastic region (elastic core) during relaxation, which cannot be explained by the two-bar model. Therefore, stress relaxation locus in the structural discontinuity becomes a curve, while the relaxation locus of the two-bar model with a fixed q value becomes a straight line, as shown in Fig. 8.

The load level also has an influence on the size of the elastic core, which regulates the compliance characteristics. The effect of load level on strain concentration factor K_e is analyzed from this viewpoint.

The author attempted to rewrite the stress-strain curves and the stress relaxation locus of Fig. 7 for the relationship between inelastic strain and q value as in Fig. 12, because the size of the elastic core in the high-rigidity low-stress portion is proportional to the inelastic strain in the low-rigidity high-stress portion under monotonic loading. In this figure, the reason why the result of P5 fluctuates in a small strain region is the high sensitivity of the q value to calculation error in this regime. The q values of both plasticity and creep increase with an increase of the load level or the progress of the stress relaxation period. Quantitatively, the relationship between q value and elastic-plastic strain is similar to one of creep strain, and the characteristics after 210 000 hours' stress relaxation of all load levels agree with those of the elastic-plastic strain for identical load levels.

Then, the creep strain distributions after 210 000 hours' stress relaxation from three levels of initial stresses were compared with the corresponding plastic strain distributions, as shown in Fig. 13.

Fig. 12 History of elastic follow-up parameter

Fig. 13 Equivalent creep and plastic strain contour

In Fig. 13, the proportions of the creep strain region and the plastic strain one are almost the same, and it is proven that these tendencies are similar to those in Fig. 11 that shows the change of the inelastic region according to constitutive equations and relaxation time.

From the above study, the change of strain concentration factor with the load level is found to be caused by the variation of the compliance ratio between high- and low-rigidity portions. Characteristics of compliance change are determined by reducing the elastic core in structures, as well as the effects of constitutive equations and relaxation time.

These characteristics can be evaluated using the relaxation locus from each load level and by elastic-plastic analysis with the same inelastic strain as that after relaxation.

5. Inelastic Design Approach Using Relaxation Locus

When the elasticity region (elastic core) is left in

the structure and the inelastic regime is limited, the stress relaxation locus shows characteristics of compliance change, which are insensitive to constitutive equations.

Therefore, any engineer can obtain the same relaxation locus (characteristics of compliance change) by stress relaxation analysis using simple creep strain equations like Norton's law.

It is also possible to estimate the rough characteristics from the elastic analysis results, if the elasticity region can be predicted by using yield stress⁽¹⁰⁾.

However, attention should be given to full inelastic deformation without an elastic core, the q values of which are characterized by Eq.(6) or Eq.(7) and strongly depend on constitutive equations.

It is possible to predict the strain concentration behavior using characteristics of compliance change obtained by the next graphical solution (Fig. 14), when the elastic core has been left.

(Step 1)

When the load level is regulated, calculate the stress relaxation locus (characteristics of compliance change) by relaxation creep analysis of this load.

If the load level is unknown, evaluate the characteristics of every load level by connecting relaxation locus from multiple initial stress or multiple elastic plastic solution for an identical load level.

(Step 2)

Obtain the inelastic solution from the point of intersection between stress relaxation locus, which shows compliance characteristics, and material stress-strain curve.

The elastic-plastic solution at a certain temperature is obtained, if a monotonic curve of the same temperature is used as the stress-strain curve. It is also possible to evaluate strain range and initial stress of relaxation required for creep-fatigue evaluation, if the relationship of stress range-strain range is utilized.

Furthermore, it may be possible to simply predict the time history of stress relaxation by using isochronous stress-strain curves⁽¹¹⁾, which are obtained from the creep test results of various stresses under constant temperature and fixed time, using the procedure of Fig. 15.

It is possible to predict the effect of load conditions and material properties by this method, even if analytical work is not repeated. Moreover, the characteristics of compliance change can explain structural stabilities (constraint conditions in structures)⁽¹²⁾ by them. Its characteristics can be utilized for stress categorization⁽¹³⁾ and for optimum inelastic design by restricting structural compliance (Fig. (16)).

Step 1 : Evaluate characteristics of compliance change

Step 2 : Get cross point of relaxation locus and stress-strain curves

Fig. 14 Simplified inelastic analysis based on compliance characteristics

Fig. 15 Isochronous stress-strain curves⁽¹¹⁾

6. Conclusions

Regarding strain concentration at the structural discontinuities due to plasticity and creep, the following was clarified.

(1) Though the strain concentration factor is dependent on the constitutive equation, the load level and the stress relaxation time, stress and strain exist on a unique relaxation locus of the structure.

(2) The stress relaxation locus is insensitive to

Fig. 16 Optimum design considering inelastic behavior

the constitutive equations, since compliance characteristics of the elastic region restrict stress and strain redistribution.

(3) The compliance characteristics of the structure change by reducing the elastic region with the progress of stress relaxation. As a result, the stress relaxation locus in the structure discontinuity becomes a curve.

(4) By utilizing the insensitivity of the compliance characteristics to the constitutive equations, the evaluation method of strain concentration was proposed using its characteristics and the stress-strain curves of materials. Any engineer can obtain the same solution and can easily evaluate the effects of loads and materials by this method.

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