Shielding Effect of Emitted Dislocations on Fatigue Crack Propagation*

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A computer simulation for fatigue crack growth is described. The simulation is based upon the crack-tip opening process which is induced by the emission and movement of dislocations from the crack-tip. The behavior of the dislocations is determined using the dislocation dynamics method. The basic analysis for the elastic interaction between a Mode I crack and discrete edge dislocations shows that the existence of dislocations in the vicinity of the crack-tip suppresses the emission of new dislocations. This result predicts that the shielding effect which results from the residual dislocations around a fatigue crack is an important factor affecting crack propagation rate. Further, the results of the simulation show that the temporary variation in crack propagation rate with the sharp increase or decrease in the stress intensity factor range can be caused by the change in the shielding effect of the residual dislocations which are left behind the moving crack front.

Key Words : Micromechanics, Dislocation, Fatigue, Crack Propagation, Shielding Effect, Dislocation Dynamics

1. Introduction

Fatigue crack growth occurs through the crack-tip opening process induced by the emission and movement of dislocations from the crack-tip⁽¹⁾. According to this process, a residual plastic zone which is created around a fatigue crack⁽²⁾ can be regarded as a cloud of the residual dislocations left behind the moving crack front cycle by cycle. Since the dislocations are stress sources, they alter the stress field of the crack. Accordingly, the existence of residual dislocations remarkably affects fatigue crack propagation rate, as several analyses have shown⁽³⁾⁻⁽⁶⁾.

On the other hand, it is well known that the temporary variation in fatigue crack propagation rate occurs with the sharp increase or decrease in the

stress intensity factor range. Such transient propagation is phenomenally understood from the change in the residual compressive stress that is developed within the residual plastic zone enveloping the crack⁽⁷⁾⁻⁽¹¹⁾. However, if considering the residual plastic zone as a region where a number of dislocations are left behind, such phenomenon can be discussed on the basis of the change in the arrangement of residual dislocations.

From the above viewpoint, this study aims to obtain basic information concerning the influence that the residual dislocations have on fatigue crack propagation rate. Especially, we direct attention to the case where the stress intensity factor range is suddenly changed. For this purpose, a simulation for fatigue crack growth is conducted through computations to predict the behavior of the dislocations emitted from the crack-tip in the presence of residual dislocations. The simulation model is based upon a Mode I crack and discrete edge dislocations on parallel slip planes inclined from the crack line. We transform a rigorous equation for the elastic interaction between a crack and dislocations⁽¹²⁾ to a suitable form for the simulation model. The derived equation is applied to the

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Fig. 1 Schematic figure to explain the fundamental equation⁽¹²⁾

dislocation dynamics method⁽¹³⁾⁻⁽¹⁹⁾ to determine the motion of each dislocation.

2. Simulation Method

2.1 Fundamental equation

Figure 1 shows a schematic figure to explain a fundamental equation for the elastic interaction between a crack and dislocations. The crack is assumed to be a semi-infinite crack in an infinite homogeneous elastic body. The crack-tip is situated at the origin of the complex coordinates. The x_1 and x_2 axes are the real and image axes, respectively. The crack line lies along the negative x_1 axis. The stress intensity factor of the crack is given by the sum of Modes I and II components, that is, $K = K_I + iK_{II}$. In addition to the crack, a number of discrete edge dislocations are supposed to exist in the elastic body. The dislocation positions are represented by ζ_j . Each dislocation possesses an arbitrary Burgers vector, $b_e(j)$.

On the dislocation located at ζ with the Burgers vector b_e , the following three forces are applied: (i) force by the crack; (ii) self image force; and (iii) force applied by the other real dislocations and their images. The three forces are represented by f_{kd} , f_{d^2} and f_{djd} , respectively. The total force acting on the dislocation is

$$f_d = f_{kd} + f_{d^2} + \sum_j f_{djd}, \quad (1)$$

where f_d is force per unit length of a dislocation line. For reference, the full specifications of the three forces in plane strain condition are shown below⁽¹²⁾:

$$\bar{f}_{kd} = \frac{1}{2i\sqrt{2\pi\zeta}} \left\{ Kb_e - \bar{K}b_e + \frac{\bar{K}b_e(\zeta - \bar{\zeta})}{2\zeta} - \bar{K}\bar{b}_e \right\} - \frac{K\bar{b}_e}{2i\sqrt{2\pi\bar{\zeta}}}, \quad (2)$$

$$\bar{f}_{d^2} = -\frac{\mu}{4\pi(1-\nu)} \left[\frac{b_e^2\sqrt{\bar{\zeta}}}{4\zeta^{3/2}} \left(\sqrt{\frac{\bar{\zeta}}{\zeta}} + 1 \right) + \frac{b_e\bar{b}_e}{4} \left\{ \frac{3}{\zeta} + \frac{1}{2\sqrt{\zeta\bar{\zeta}}} \left(1 - \frac{\bar{\zeta}}{\zeta} \right) + \frac{3 - \sqrt{\bar{\zeta}/\zeta}}{\sqrt{\zeta}(\sqrt{\zeta} + \sqrt{\bar{\zeta}})} \right\} \right]$$

Fig. 2 Basic model for the simulation

$$-\frac{\bar{b}_e^2}{4\bar{\zeta}} \left(1 + \sqrt{\frac{\bar{\zeta}}{\zeta}} \right), \quad (3)$$

$$\begin{aligned} \bar{f}_{djd} = & \frac{\mu}{8\pi(1-\nu)} \left[(b_e b_e(j) + \bar{b}_e b_e(j) + b_e \bar{b}_e(j)) \right. \\ & \times \left\{ \frac{1}{\xi_j} \left(\sqrt{\frac{\bar{\zeta}_j}{\zeta}} + 1 \right) + \frac{1}{\xi_j^*} \left(\sqrt{\frac{\bar{\zeta}_j}{\zeta}} - 1 \right) \right\} \\ & - \bar{b}_e \bar{b}_e(j) \left\{ \frac{1}{\bar{\xi}_j} \left(\sqrt{\frac{\bar{\zeta}_j}{\zeta}} + 1 \right) + \frac{1}{\bar{\xi}_j^*} \left(\sqrt{\frac{\bar{\zeta}_j}{\zeta}} - 1 \right) \right\} \\ & - 2b_e(j)b_e i y_j \left\{ \frac{1}{\xi_j^2} \left(\sqrt{\frac{\bar{\zeta}_j}{\zeta}} + 1 \right) + \frac{1}{\xi_j^{*2}} \left(\sqrt{\frac{\bar{\zeta}_j}{\zeta}} - 1 \right) \right\} \\ & + \frac{b_e b_e(j) i y_j}{\xi_j^2} \left\{ \sqrt{\frac{\bar{\zeta}_j}{\zeta}} + \sqrt{\frac{\zeta}{\bar{\zeta}_j}} + 2 \right\} \\ & - \frac{\bar{b}_e b_e(j) i y_j}{\bar{\xi}_j^{*2}} \left\{ \sqrt{\frac{\bar{\zeta}_j}{\zeta}} + \sqrt{\frac{\zeta}{\bar{\zeta}_j}} - 2 \right\} \\ & - \left(\frac{\bar{b}_e \bar{b}_e(j)}{\bar{\xi}_j^{*2}} i y_j + 4 \frac{b_e \bar{b}_e(j) y y_j}{\xi_j^{*3}} \right) \left\{ \sqrt{\frac{\bar{\zeta}_j}{\zeta}} + \sqrt{\frac{\zeta}{\bar{\zeta}_j}} - 2 \right\} \\ & - \frac{b_e b_e(j) i y_j}{\zeta^{3/2}} \left(\frac{\sqrt{\bar{\zeta}_j}}{\xi_j} + \frac{\sqrt{\bar{\zeta}_j}}{\xi_j^*} \right) \\ & \left. + \frac{b_e \bar{b}_e(j) y y_j}{\xi_j^{*2} \zeta} \left(\sqrt{\frac{\bar{\zeta}_j}{\zeta}} - \sqrt{\frac{\bar{\zeta}_j}{\zeta}} \right) \right], \quad (4) \end{aligned}$$

where $\xi_j = \zeta - \zeta_j$, $\bar{\xi}_j^* = \bar{\zeta} - \bar{\zeta}_j$, $y = (\zeta - \bar{\zeta})/2i$ and $y_j = (\zeta_j - \bar{\zeta}_j)/2i$. The bar notation in the equations shows complex conjugate operator. μ and ν are the shear modulus and Poisson's ratio of the infinite elastic body.

2.2 Force applied to dislocation on inclined slip plane

Figure 2 shows the basic model used in this study. In this model, we consider a semi-infinite crack with a Mode I component and two parallel slip planes inclined by the angle of θ from the crack line. One slip plane is extended from the crack-tip and another slip plane is shifted from it by d in the negative x_1 direction. On each slip plane, a dislocation with the same Burgers vector b which is parallel to the slip plane exists. The dislocation positions, r and r_j , are distance measured from the x_1 axis.

For the simulation, the force which is applied to the dislocation on the slip plane extended from the crack-tip, f_d^{sp} , is essential. To obtain this force, we rotate the original coordinates shown in Fig. 1 counter-clockwise by the angle of θ . The rotation of the coordinates is made by substituting the following equations into Eqs. (2) - (4).

$$\left. \begin{aligned} \bar{f}^{sp} &= e^{i\theta} \bar{f}, \quad b_e = b_e(j) = b e^{i\theta} \\ \zeta &= r_j e^{i\theta} - d \quad (\text{for Eqs. (2), (3)}) \\ \zeta &= r e^{i\theta}, \quad \zeta_j = r_j e^{i\theta} - d \quad (\text{for Eq. (4)}) \end{aligned} \right\}. \quad (5)$$

The real parts in the derived equations give the forces inducing the glide motion of dislocations on the inclined slip plane. The imaginary parts give the forces inducing the climb motion from the slip plane. Since this study treats the glide motion of dislocations, the real parts of the forces, $\text{Re}(\bar{f}_{kd}^{sp})$, $\text{Re}(\bar{f}_{d_j}^{sp})$ and $\text{Re}(\bar{f}_{d_j,d}^{sp})$, are used. The forces are expressed in non-dimensional form using the parameters defined by

$$\left. \begin{aligned} \delta &= d/b, \quad \rho = r/b, \quad \rho_j = r_j/b \\ R &= \rho/\rho_j, \quad S = \delta/\rho_j \end{aligned} \right\}. \quad (6)$$

The derived equations are shown below: The force applied by the crack to the dislocation which is located at ρ_j on the slip plane shifted from the crack-tip is

$$F_{kd_j} = \frac{1-\nu}{\mu b} \text{Re}(\bar{f}_{kd_j}^{sp}) = -\frac{\kappa_1}{\sqrt{\rho_j}} g_1(S, \theta), \quad (7)$$

$$\begin{aligned} g_1(S, \theta) &= \frac{\sin \theta}{A\sqrt{A}} \{ m(\cos \theta - S \cos 2\theta) \\ &\quad + m^*(\sin \theta - S \sin 2\theta) \}, \end{aligned} \quad (8)$$

where

$$\left. \begin{aligned} A &= S^2 - 2S \cos \theta + 1 \\ A^* &= S^2 - 2S \cos \theta + \cos 2\theta \\ B &= 2 \sin \theta (\cos \theta - S) \\ m &= \sqrt{\{\sqrt{A} + (\cos \theta - S)\}/2} \\ m^* &= \sqrt{\{\sqrt{A} - (\cos \theta - S)\}/2} \end{aligned} \right\}. \quad (9)$$

In Eq. (7), the non-dimensional applied stress intensity factor is defined by

$$\kappa_1 = \frac{K_I(1-\nu)}{2\mu\sqrt{2\pi b}}. \quad (10)$$

Substituting $S=0$ (that is, $d=0$) into Eqs. (7) - (9), the force applied to the dislocation which is situated on the slip plane extended from the crack-tip is given by a simple form:

$$F_{kd} = -\frac{\kappa_1}{\sqrt{\rho}} \sin \theta \cos \frac{\theta}{2}, \quad (11)$$

where ρ_j has been rewritten to ρ .

The self-image force of the dislocation which is situated at ρ_j on the slip plane shifted from the crack-tip is

$$\begin{aligned} F_{d_j} &= \frac{1-\nu}{\mu b} \text{Re}(\bar{f}_{d_j}^{sp}) = -\frac{g_2(S, \theta)}{16\pi\rho_j}, \quad (12) \\ g_2(S, \theta) &= 2 + \frac{2S - \cos \theta}{\sqrt{A}} + \frac{1 - S \cos \theta}{A} + \frac{S \sin^2 \theta}{A\sqrt{A}} \end{aligned}$$

$$\begin{aligned} &+ \frac{\cos 3\theta}{A} \left\{ (\cos \theta - S) + \frac{A^*}{\sqrt{A}} - \frac{2B \sin \theta}{A} \right\} \\ &- \frac{\sin 3\theta}{A} \left\{ \sin \theta - \frac{B}{\sqrt{A}} - \frac{2B(\cos \theta - S)}{A} \right\}. \end{aligned} \quad (13)$$

When, $S=0$ we obtain $g_2=4$. Accordingly, the above equations become a very simple form:

$$F_{d^2} = -\frac{1}{4\pi\rho}, \quad (14)$$

where ρ_j has been rewritten to ρ .

The force applied to the dislocation which is located at ρ on the slip plane extended from the crack-tip by another dislocation which is situated at ρ_j on the shifted slip plane is given by

$$F_{d_j,d} = \frac{1-\nu}{\mu b} \text{Re}(\bar{f}_{d_j,d}^{sp}) = \frac{g_3(R, S, \theta)}{8\pi\rho} \quad (15)$$

$$\begin{aligned} g_3(R, S, \theta) &= 2R \cos \theta \left\{ \cos 2\theta \left(\frac{C}{F} - \frac{C}{H} \right) \right. \\ &\quad + \sin 2\theta \left(\frac{D}{F} - \frac{E}{H} \right) \left. \right\} + 2R \cos^3 \theta \left(\frac{CM - DN^*}{F} \right. \\ &\quad + \frac{CM^* - EN}{H} \left. \right) + R \sin \theta (2 \cos^2 \theta + 1) \\ &\quad \times \left(\frac{DM + CN^*}{F} + \frac{EM^* + CN}{H} \right) \\ &\quad + \frac{R \sin \theta}{F^2} \left[\sin 3\theta \left\{ F^* M \left(2R - 1 - \frac{1}{L} \right) \right. \right. \\ &\quad \left. \left. - GN^* \left(2R - 1 + \frac{1}{L} \right) \right\} \right. \\ &\quad \left. - \cos 3\theta \left\{ GM \left(2R - 1 - \frac{1}{L} \right) + F^* N^* \left(2R - 1 \right. \right. \right. \\ &\quad \left. \left. + \frac{1}{L} \right) \right\} + 2(R-1)(F^* \sin 3\theta - G \cos 3\theta) \right] \\ &\quad + \frac{2R^2 \sin \theta}{H^2} \{ \sin 3\theta (H^* M^* - IN - H^*) \\ &\quad - \cos 3\theta (IM^* + H^* N - 1) \} \\ &\quad + \frac{R \sin \theta}{H^2} \left[2 \sin \theta \left\{ \frac{H^* M^* + IN}{L} - H^* \right\} \right. \\ &\quad \left. - \cos \theta \left\{ \frac{IM^* + H^* N}{L} - (IM^* + H^* N) - 2I \right\} \right] \\ &\quad - \frac{4R^2 \sin^2 \theta}{H^3} \left[(C \cos \theta + E \sin \theta) \right. \\ &\quad \times \left\{ H^* M^* \left(1 + \frac{1}{L} \right) - IN \left(1 - \frac{1}{L} \right) - 2H^* \right\} \\ &\quad + (C \sin \theta - E \cos \theta) \left\{ IM^* \left(1 + \frac{1}{L} \right) \right. \\ &\quad \left. \left. + H^* N \left(1 - \frac{1}{L} \right) - 2I \right\} \right]. \end{aligned} \quad (16)$$

In the above equations, the following non-dimensional functions are used to relieve complicated algebra calculations.

Fig. 3 Force acting on the dislocation which is located on: (a) the inclined slip plane extended from the crack-tip; (b) the slip plane shifted by $-\delta$ from the crack-tip ($\theta = 90$ deg.)

$$\left. \begin{aligned}
 C &= (R-1)\cos\theta + S \\
 D &= (R-1)\sin\theta, \quad E = (R+1)\sin\theta \\
 F &= C^2 + D^2, \quad F^* = C^2 - D^2, \quad G = 2CD \\
 H &= C^2 + E^2, \quad H^* = C^2 - E^2, \quad I = 2CE \\
 L &= \sqrt{A}/R \\
 M &= \{m \cos(\theta/2) + m^* \sin(\theta/2)\} / \sqrt{R} \\
 M^* &= \{m \cos(\theta/2) - m^* \sin(\theta/2)\} / \sqrt{R} \\
 N &= \{m \sin(\theta/2) + m^* \cos(\theta/2)\} / \sqrt{R} \\
 N^* &= \{m \sin(\theta/2) - m^* \cos(\theta/2)\} / \sqrt{R}
 \end{aligned} \right\} \quad (17)$$

In addition, the functions shown in Eq.(17) have no physical meaning.

Finally, using the obtained three forces, the total

force applied to the dislocation which is located at ρ on the slip plane extended from the crack-tip is given by

$$F_d = F_{kd} + F_{d^2} + \sum_j F_{djd}. \quad (18)$$

2.3 Criterion of dislocation emission

Figure 3(a) shows the force acting on the dislocation which is located at ρ on the slip plane extended from the crack-tip. The force is given by $F_d = F_{kd} + F_{d^2}$ as a function of the dislocation position ρ . The slip plane angle θ is changed between $10 \sim 90$ degrees.

For all slip plane angles, there is a region where strong negative force acts on the dislocation due to its image force. Since such barrier region exists in the vicinity of the crack-tip, dislocation emission seems impossible. However, as Lin et al. have explained in their review paper⁽¹²⁾, this contradiction can be overcome by remembering that the elastic limit breaks down in the dislocation core. If the stress intensity factor is high enough, the size of the barrier region becomes less than the dislocation core size. In that case, the dislocation core completely overlaps the barrier region, so that the strong negative force is canceled. As a result, the dislocation emission from the crack-tip spontaneously will occur. From this idea, we assume that a dislocation is emitted when $F_d > 0$ at the position corresponding to the dislocation core size, ρ_0 . For example, Fig. 3(a) shows that when $\rho_0 = 5$ and the slip plane angle is more than 50 degrees, a dislocation is emitted from the crack-tip. However, when the slip plane angle is less than 50 degrees, the crack is shielded and no dislocation emission occurs unless the stress intensity factor is increased.

Figure 3(b) shows the force applied to the dislocation which is situated at ρ_j on the slip plane shifted from the crack-tip by δ in the negative x_1 direction. The force is given by $F_{dj} = F_{kdj} + F_{dj^2}$. To ease comparing, the slip plane angle is assumed to be 90 degrees. The figure shows that as increasing the distance δ , the size of the barrier region increases and the dislocation emission becomes difficult. Considering this result, we assume that dislocations are emitted only from the crack-tip. In addition, the position of the dislocation emission is at $\rho_0 = 5$.

2.4 Force-velocity relation

Dislocation motion is predicted by the dislocation dynamics method. In this method, the change in the dislocation position, Δr , during time increment, Δt , is calculated using a force-velocity relation⁽²⁰⁾. In this study, the following linear relation is employed.

$$\Delta r / \Delta t = (f_d - f_f) / B_v, \quad (19)$$

where B_v is a material constant, f_d is the force acting

Fig. 4 Crack propagation process in the simulation

on a dislocation and f_f is friction force. Under the assumption of $f_f=0$, the above equation is rewritten to a simple non-dimensional form:

$$\Delta\rho/\Delta T = F_a, \quad (20)$$

where $T = \mu t / (1 - \nu) B_v$ and F_a is calculated by Eq. (18).

2.5 Crack propagation

Figure 4 shows the crack propagation process supposed in the simulation. Dislocations are emitted from the crack-tip when the criterion mentioned in section 2.3 is satisfied, as shown in Fig. 4(b). The emitted dislocations move on the inclined slip plane during a limited time ($T=4000$). After then, the same emission and movement of dislocations are assumed to occur on the slip plane which is below the crack line (Fig. 4(c)).

The slip plane angle θ is supposed to be 54.44 degrees. This angle is realistic one in fcc metals when the crack plane is (001) and the slip planes are (111) and (11 $\bar{1}$)⁽²¹⁾. Speaking rigorously, force acts between the dislocations on the upper and lower slip planes. However, if the angle between the slip planes is more than 60 degrees, the interaction between the dislocations on the two slip planes is negligibly small⁽²²⁾, so that this interaction is omitted in the simulation.

The process shown in Fig. 4(a)-(c) is assumed as one cycle of fatigue. When the number of emitted dislocations on each slip plane is N , the crack will be advanced by $\Delta a = 2Nb \cos \theta$ from the point O_1 to the point O_2 . Accordingly, the non-dimensional crack propagation rate is defined by

Fig. 5 Force applied to the dislocation located on the slip plane extended from the crack-tip by another dislocation fixed on the shifted slip plane ($\theta=90$ deg.)

$$\Delta a = 2N \cos \theta. \quad (21)$$

All emitted dislocations during one cycle are left behind the crack front by Δa cycle by cycle, as illustrated in Fig. 4(d). Their influence is automatically reflected in the following computations. In real fatigue process, some of the emitted dislocations can be absorbed to the crack with load releasing. However, such revers motion of dislocations is not taken into account in the simulation for simplicity.

Just before starting the dislocation emission to a new slip plane, the minimum value of the stress intensity factor which is needed for the emission of one dislocation, κ_{op} , is calculated. The parameter κ_{op} indicates the magnitude of the shielding effect resulting from the residual dislocation arrays. Further, the effective stress intensity factor which is defined below as the difference between the applied stress intensity factor κ_I and the magnitude of the shielding effect κ_{op} is calculated.

$$\Delta \kappa_{eff} = \kappa_I - \kappa_{op}. \quad (22)$$

3. Results and Discussion

3.1 Interaction between dislocations in presence of crack

Figure 5 shows the force applied to the dislocation which is situated at ρ on the slip plane extended from the crack-tip by another dislocation which is fixed at $\rho_f=50$ on the shifted slip plane. The interval between the parallel slip planes is δ . When $\delta=0$, both dislocations exist on the same slip plane extended from the crack-tip. For all cases, the slip plane angle θ is 90 degrees.

Fig. 6 Changes in (a) crack propagation rate and (b) the effective stress intensity factor with crack propagation under the constant κ_I

The figure shows that negative force is applied to the dislocation in the vicinity of the crack-tip due to the existence of another dislocation which is fixed on the shifted slip plane. This negative force increases the size of the barrier region mentioned in section 2.3. Accordingly, when residual dislocations exist around the crack, the emission of new dislocations becomes difficult or the crack is shielded. Even if a new dislocation is emitted from the crack-tip, its free movement to a further position is interrupted by the stress field of residual dislocations. Furthermore, the interrupted dislocation strongly suppresses the following emission, as understood from the case of $\delta=0$ in Fig. 5.

The above shielding effect of residual dislocations controls the total number of dislocations emitted from the crack-tip. Therefore, such effect will be an important factor that determines crack propagation rate. This point is discussed in the next section.

3.2 Stable crack propagation

Figure 6 shows the change in crack propagation rate $\Delta\alpha$ and the change in the effective stress intensity factor $\Delta\kappa_{eff}$ with crack propagation under the con-

Fig. 7 Arrangement of residual dislocations left behind the moving crack front ($\kappa_I=2.5$)

stant applied stress intensity factor ($\kappa_I=1\sim 2.5$). The position of the crack-tip is shown by α which is normalized by Burgers vector b . The initial position of the crack-tip is located at $\alpha=0$. Figure 7 shows the arrangement of residual dislocations by the first 10 cycles when $\kappa_I=2.5$. In addition, the dislocation arrays under the crack line are left out.

From Fig. 6 (a), it is understood that although the applied stress intensity factor is constant, crack propagation rate rapidly decreases during the early stage. Especially, when $\kappa_I=1$, the crack stops propagating. This reduction in $\Delta\alpha$ occurs since the shielding effect becomes stronger and the effective stress intensity factor $\Delta\kappa_{eff}$ decreases with the total number of residual dislocations increasing, as shown in Fig. 6 (b). The above change in propagation rate clearly coincides with the arrangement of the residual dislocations shown in Fig. 7.

After then, although the total number of residual dislocations goes on increasing, crack propagation rate gradually recovers and approaches to a constant value. That is because the crack-tip goes away from the excessive residual dislocations left behind during the early stage. The shielding effect depends on not only the number of residual dislocations but also the average distance from the residual dislocations to the crack-tip. Both effects are balanced with crack propagation, so that $\Delta\kappa_{eff}$ approaches to a constant value. This change in $\Delta\kappa_{eff}$ results in the recovery of crack propagation rate. The above consideration suggests that the stable crack propagation occurs under the influence of residual dislocations.

3.3 Transient crack propagation

Figure 8 shows the change in crack propagation rate $\Delta\alpha$ with the sudden reduction in the applied stress intensity factor from $\kappa_I=2.5$ (the first step) to $\kappa_I=1.75\sim 2.25$ (the second step). The simulation of the first step was continued until $\Delta\alpha$ approached a constant value. Then, κ_I was sharply reduced at the zero position of α shown in the figure.

Fig. 8 Change in crack propagation rate with the decrease in κ_I from 2.5 to 1.75~2.25

Fig. 10 Change in crack propagation rate with the increase in κ_I from 1.75 to 2~2.5

Fig. 9 Arrangement of residual dislocations when κ_I is decreased from 2.5 to: (a) 1.75; (b) 2; and (c) 2.25

As understood from Fig. 8, crack propagation rate temporarily decreases in all cases after the sharp change in κ_I . This transient crack propagation is due to the shielding effect resulting from the greater number of residual dislocations which are left behind during the first step. Further, as the reduction range

in κ_I from the first step to the second step is greater, the influence of the residual dislocations is stronger. Accordingly, the temporary decrease in $\Delta\alpha$ becomes remarkable.

Such phenomenon is clearly reflected to the arrangement of residual dislocations, as shown in Fig. 9. After the sudden reduction in κ_I , not only the number of the dislocations emitted for one cycle temporarily decreases but also their movement to the farther position is suppressed. This tendency is remarkable as the decrease range in κ_I is greater.

After the above transient propagation, crack propagation rate gradually recovers to a constant value. This recovery of $\Delta\alpha$ is because the crack tip moves to a further position from the residual dislocations left behind during the first step and their influence is weakened.

On the other hand, Fig. 10 shows the result of the reverse case, that is, the change in crack propagation rate with the sudden increase in the stress intensity factor from $\kappa_I = 1.75$ (the first step) to $\kappa_I = 2 \sim 2.5$ (the second step). As the same to the previous case, the simulation of the first step was continued until $\Delta\alpha$ approached a constant value. Then, κ_I was sharply increased at the zero position of α .

The figure shows that the crack propagation rate temporarily increases after the sudden change in κ_I in all cases. This transient increase in $\Delta\alpha$ is because the influence of the residual dislocations which is left behind during the first step is weaker than that of the residual dislocations during the second step. Further, as the increase range in κ_I is greater, the influence of the residual dislocations left behind during the first step is relatively weakened. Accordingly, the temporary increase in $\Delta\alpha$ becomes remarkable. After the above

Fig. 11 Arrangement of residual dislocations when κ_I is increased from 1.75 to: (a) 2; (b) 2.25; and (c) 2.5

transient propagation, Δa gradually approaches a constant value. This recovery of Δa results from that the number of the emitted dislocations temporarily increases at the early portion of the second step and the shielding effect becomes stronger.

As shown in Fig. 11, the above phenomenon is reflected to the arrangement of residual dislocations. After the sudden reduction in κ_I , the number of the emitted dislocations temporarily increases and the dislocations reaches to further positions. This tendency is remarkable as the increase range in κ_I is greater.

The results of the simulation mentioned above quantitatively agrees with the experimental results concerning the temporary variation in fatigue crack propagation rate with the sudden change in the stress intensity factor range⁽⁷⁾⁻⁽¹¹⁾. From this coincide, it can be concluded that the change in the shielding effect of residual dislocations is one of the reasons for the transient propagation of fatigue cracks.

4. Conclusions

(1) The residual dislocations which are left behind the moving crack front suppress the emission of new dislocations from the crack-tip. This shielding

effect can cause non-propagating crack when the stress intensity factor is less than a limiting value. Even if the crack propagates stably, crack propagation rate will be reduced by the shielding effect of residual dislocations.

(2) The tendency of crack propagation obtained from the simulation coincides with the experimental results concerning the temporary decrease or increase in fatigue crack propagation rate with the sharp change in the stress intensity factor range⁽⁷⁾⁻⁽¹¹⁾. This coincidence suggests that one of the reasons for the transient propagation is the change in the shielding effect resulting from the residual dislocations.

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