Shear Correction Factors in Creep-Damage Analysis of Beams, Plates and Shells*

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Modern design rules for thin-walled structures which operate at elevated temperatures are based on the demand that the creep and may be the damage behaviour should be taken into account. In the last four decades various models including the scalar or tensor valued hardening and damage variables are established. These models reflect the influence of the deformation or the damage induced anisotropy on the creep response. One problem in creep analysis of thin-walled structures is the selection of the structural mechanics model which has to be adequate to the choice of the constitutive equations. Considering complex loading conditions the structural mechanics model has to reflect for instance the different constitutive behaviour in tension and compression. Below the applicability of classical engineering models for beams, plates and shells to the creep-damage analysis is discussed. It will be shown that a first improvement of the classical approach can be given within the assumptions of the first order shear deformation theory. Based on the beam equations we demonstrate that the shear correction factors have to be modified within the time-step analysis.

Key Words : Creep-Damage, Beams, Shear Correction

1. Introduction

At elevated temperatures thin-walled structures (e.g. beams, plates, tubes, shells and discs) used in chemical plants, electrical power plants, etc. show a significant time-dependent behaviour accompanied by the development of creep and damage processes in the material. With respect to the enforced safety requirements the long-term behaviour of thin-walled structures under creep-damage conditions should be predicted in a more accurate manner. During the last 100 years various phenomenological models for primary, secondary and tertiary creep have been worked out. The accuracy of the analysis depends on the choice of a suitable material model (phenomenological, mechanism-based or micro-mechanical), of a suitable structural mechanics model (classical shell theory, theory with refined cross-section assumptions or three-dimensional analysis), and of a suitable numerics (e.g. finite element based or using direct variational methods). Within these items there are different possibilities for the improvement of creep-

damage predictions.

Let us focus our attention to the second item. Up to now the majority of published papers is based on the classical approach: to the formulation of the structural mechanics equations for beams the Bernoulli's assumptions are used, for plates and shells the governing equations are formulated with the help of Kirchhoff's hypotheses or the Kirchhoff-Love assumptions. All theories starting from these assumptions contain kinematical constraints that means the deformability of the cross-section is restricted. Therefore, the classical theories do not take into account the transverse shear strain (the thin-walled structure is rigid with respect to transverse shear), the thickness changes, the warping, etc. Hereafter, the effects connected with the transverse shear strains are discussed. Within the framework of the first order shear deformation theory we will derive the beam equations under consideration of creep strains and damage evolution. On an example of a clamped beam we will estimate the shear correction factors in the cases of steady-state creep and creep coupled with damage. Finally, the conclusions will be made with respect to creep-damage analyses of plates and shells.

For the description of creep-damage behaviour, phenomenological and mechanism-based models are in use. For the sake of simplicity, we limit our

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considerations to a Norton-type creep law modified by introducing the Kachanov-Rabotnov scalar damage parameter and the Leckie-Hayhurst description of the damage evolution. This model is in some situations incomplete and does not reflect the real damage behaviour. The reason for this unaccuracy is that we deal, for example, only with one scalar internal variable. The description of isotropic material behaviour is only possible. In contrast to other models, the identification of the material parameters is much easier. Due to the aim of this paper we will use the simplest material model and focus our attention to the improvement of the structural mechanics equations. A comparison of this type of the material model with the mechanism based approach is given in Ref.(1).

2. Creep Behaviour and Cross-Section Assumptions

The mechanical state of thin-walled structures can be described on the base of various structural mechanics models formulated with the help of different assumptions. In the case of plates the principles of formulation are discussed in Ref.(2). The classical plate and shell equations are grounded on assumptions presented, for instance, in Ref.(3). The equations following from these assumptions correspond to Kirchhoff's plate theory or Kirchhoff-Love's shell theory. With respect to the creep and damage behaviour the assumptions should be modified. There exist two possibilities to improve the plate or the shell equations.

Firstly, time-dependent material behaviour can result in finite deflections even in the case of infinitesimal strains. As was shown in Ref.(4) the simulations of creep behaviour for a plate in bending based on the Kirchhoff's plate theory lead to a significant underestimation of the time to fracture. Furthermore, the geometrically linear approach leads to unacceptable differences with results of experimental study on rectangular plates⁽⁵⁾. Taking into account the non-linear terms in the strain-displacement equations in the sense of the von Kármán plate theory a satisfactory agreement between the results of calculation and tests has been obtained.

Secondly, the question of the influence of the transverse shear strains behaviour is not enough investigated particularly if damage evolution is taken into account. A number of investigations show that the classical Kirchhoff-Love and the first order shear deformation shell theories can accurately predict the creep deformation and creep buckling of shells considering material models of primary and secondary creep⁽⁵⁾⁻⁽⁸⁾. The introduction of damage requires the

consideration of non-classical effects in the material behaviour, e.g. different tertiary creep rates by tension and compression or anisotropic behaviour induced by damage. Therefore, the question of the applicability of the first order shear deformation models of beams, plates and shells needs a special consideration.

According to the known deformation mechanisms the primary and secondary creep rates are dominantly controlled by the von Mises stress, e.g. Ref.(9). The accelerated creep is additionally influenced by the kind of the stress state. For example, different tertiary creep rates and fracture times can be obtained from creep tests performed under uniaxial tension with the stress σ and under torsion with the shear stress $\tau = \sigma/\sqrt{3}$ ⁽¹⁰⁾. Let us introduce the conventional creep-damage material model of Kachanov-Rabotnov-Hayhurst, which is based on the Norton-Bailey creep law and the scalar-valued phenomenological damage parameter⁽¹¹⁾

$$\dot{\epsilon}_{ij}^{cr} = \frac{3}{2} a \left(\frac{\sigma_v M}{1 - \omega} \right)^n \frac{s_{ij}}{\sigma_{vM}}, \quad \dot{\omega} = b \frac{[\alpha \sigma_I + (1 - \alpha) \sigma_{vM}]^k}{(1 - \omega)^l} \quad (1)$$

In this notation $\dot{\epsilon}_{ij}^{cr}$ are the components of the creep strain rate tensor, s_{ij} are the components of the stress deviator, σ_{vM} is the von Mises stress, σ_I is the maximum positive principal stress and ω is the damage parameter. α denotes a weighting coefficient allowing to consider the influence of principal damage mechanisms (σ_I -controlled or σ_{vM} -controlled). The material constants a , b , n , k and l are taken for the 316 stainless steel at 650°C from Ref.(12):

$$a = 2.13 \cdot 10^{-13} \text{ MPa}^{-n}/\text{h}, \quad b = 9 \cdot 10^{-10} \text{ MPa}^{-k}/\text{h}, \\ n = 3.5, \quad k = 2.8, \quad l = 2.8, \quad \alpha = 1$$

The elastic part of the strain tensor is determined by the generalised Hooke's law assuming the isotropic elasticity without influence of damage. The elastic material constants are $E = 1.44 \cdot 10^5 \text{ MPa}$ and $\nu = 0.314$, where E and ν are Young's modulus and Poisson's ratio, respectively.

Figure 1 shows creep curves for tensile, compressive and shear stresses simulated by the constitutive model (1). The corresponding stress values provide the same value of the von Mises stress. It is obvious that the tertiary creep rate is significantly dependent on the kind of loading. Figure 2 presents creep curves calculated by the combined action of both the normal and the shear stresses.

It is seen that even the small superposed shear stress can significantly influence the axial strain response and decrease the fracture time. On the other hand, if we consider the combination of the compression and shear, the shear creep strain rate remains constant. The change of the sign of the normal stress influences both the normal and the shear creep

Fig. 1 Creep responses by tension, torsion and compression computed using Eq. (1)

Fig. 2 Creep responses by combined tension (compression) and torsion computed using Eq. (1)

responses. The stress states with combined normal tensile (compressive) stress and the small shear stress are typical for the transversely loaded beams, plates and shells.

Based on the creep damage material response let us discuss the through-the-thickness assumptions for modeling of thin-walled structures. First, since even the small shear stress can significantly influence the material response, the transverse shear stress and the resulting transverse shear strain cannot be neglected. Thus at least, the first order shear deformation model has to be used for the creep-damage analysis. Second, the dependence of the creep response on the sign of the normal stress can lead to a non-symmetrical thickness distributions of the displacement, the strain and the stress fields. This has to be considered by specifying the through-the-thickness approximations for displacements or stresses.

3. Observations on Beam Equations

In what follows we introduce the beam equations and discuss the assumptions of the first order shear deformation theory in detail. The following simplified derivations will provide conclusions regarding cross-

Fig. 3 Straight beam with a rectangular cross section in cartesian coordinates

section assumptions in connection with the effect of the creep-damage. Let us consider a beam with a rectangular cross-section, Fig. 3. Assuming the plane stress conditions the principle of virtual displacements yields

$$\begin{aligned} -\delta W_i &= \frac{bh}{2} \int_0^l \int_{-1}^1 (\sigma_x \delta \epsilon_x + \tau_{xz} \delta \gamma_{xz} + \sigma_z \delta \epsilon_z) d\zeta dx \\ &= \int_0^l \bar{q}(x) \delta w(x, -h/2) dx = \delta W_a \end{aligned} \quad (2)$$

Here l denotes the beam length, σ_x , σ_y , τ_{xz} and ϵ_x , ϵ_y , γ_{xz} are the components of the stress and the strain tensors, respectively, w is the beam deflection and $\zeta = 2z/h$ is the thickness coordinate. W_i is the work of the internal forces and W_a is the work of the external forces. For the sake of simplicity we assume the absence of tractions on the edges $x=0$ and $x=l$. In the following derivations we use the abbreviations

$$\begin{aligned} \frac{\partial}{\partial x}(\dots) &\equiv (\dots)_{,x}, \quad \frac{\partial}{\partial z}(\dots) \equiv (\dots)_{,z}, \\ \frac{d}{dx}(\dots) &\equiv (\dots)', \quad \frac{d}{d\zeta}(\dots) \equiv (\dots)', \quad \frac{d}{dt}(\dots) \equiv (\dots)' \end{aligned}$$

Specifying the through-the-thickness approximations of axial displacement u and deflection w , various engineering displacement based beam theories presented, for instance, in Ref.(13) can be suggested. For example, a refined displacement based beam model can be obtained with

$$\begin{aligned} u(x, \zeta) &= u_0(x) + \varphi(x) \frac{h}{2} \zeta + u_1(x) \Phi(\zeta), \\ w(x, \zeta) &= w_0(x) + w_1(x) \Omega(\zeta), \end{aligned} \quad (3)$$

where u_0 and w_0 are the displacements of the beam centreline, φ is the cross section rotation, $\Phi(\zeta)$ and $\Omega(\zeta)$ are distribution functions, which should be specified, and $u_1(x)$ and $w_1(x)$ are unknown functions of the x -coordinate.

Another possibility is the use of stress based approximations, for example, following from the elasticity solution of the Bernoulli-Euler beam equations

$$\begin{aligned}\sigma_x &= \frac{6M(x)}{bh^2}\zeta, \quad \tau_{xz} = \frac{3Q(x)}{2bh}(1-\zeta^2), \\ \sigma_z &= \frac{3\bar{q}(x)}{4b}\left(-\frac{2}{3} + \zeta - \frac{1}{3}\zeta^3\right),\end{aligned}\quad (4)$$

where Q and M are the shear force and the bending moment, respectively. Applying the stress approximations E . Reissner derived the elastic plate equations by means of the mixed variational equation⁽¹⁴⁾. The displacement approximations (3) neglecting the terms $u_1\Phi$ and $w_1\Omega$ or the stress approximations (4) lead to a first order shear deformation beam theory. By generalisation the corresponding models of plates and shells can be obtained. The stress approximations (4) are not suitable for creep problems because even for the steady state creep solution of a beam the normal stress σ_x is non-linearly distributed along the thickness coordinate⁽¹⁵⁾.

Let us derive the first order shear deformation beam equations without approximation of the stress σ_x . The transverse shear and the transverse normal stresses are then specified as follows

$$\begin{aligned}\tau_{xz} &= \frac{2Q(x)}{bh} \frac{\psi'(\zeta)}{\psi_0}, \quad \sigma_z = \frac{\bar{q}(x)}{b} \frac{\psi(\zeta) - \psi(1)}{\psi_0}, \\ \psi_0 &= \psi(1) - \psi(-1)\end{aligned}\quad (5)$$

$\psi(\zeta)$ is a given function satisfying the boundary conditions $\psi'(\pm 1) = 0$. The variation of W_i in Eq.(2) can be written as

$$\begin{aligned}-\delta W_i &= \frac{bh}{2} \int_0^1 \int_{-1}^1 [\delta(\tau_{xz}\gamma_{xz} + \sigma_z \varepsilon_z) \\ &= (\gamma_{xz}\delta\tau_{xz} + \varepsilon_z\delta\sigma_z - \sigma_x\delta\varepsilon_x)] d\zeta dx\end{aligned}\quad (6)$$

With the approximations (5) and the linear strain-displacement equations

$$\varepsilon_x = u_{,x}, \quad \varepsilon_z = w_{,z}, \quad \gamma_{xz} = u_{,z} + w_{,x}$$

we obtain

$$\begin{aligned}\frac{bh}{2} \int_0^1 \int_{-1}^1 \delta(\tau_{xz}\gamma_{xz} + \sigma_z \varepsilon_z) d\zeta dx \\ = \int_0^1 [\delta(Q\bar{w}' - Q\bar{u}) + \bar{q}\delta w(x, -1) - \bar{q}\delta\bar{w}] dx\end{aligned}\quad (7)$$

with

$$\begin{aligned}\bar{w}(x) &= \frac{1}{\psi_0} \int_{-1}^1 w(x, \zeta) \psi'(\zeta) d\zeta, \\ \bar{u}(x) &= \frac{2}{h} \frac{1}{\psi_0} \int_{-1}^1 u(x, \zeta) \psi''(\zeta) d\zeta\end{aligned}\quad (8)$$

Let us assume the additive split of the total strain tensor into an elastic and a creep part

$$\varepsilon_{ij} = \varepsilon_{ij}^e + \varepsilon_{ij}^{cr},$$

and ε_{ij}^{cr} to be known functions of the coordinates x, ζ for the fixed time variable. Further we will use the linear through-the-thickness approximation of the axial displacement

$$u(x, \zeta) = u_0(x) + \varphi(x)\zeta \frac{h}{2}$$

With these assumptions the underlined term in Eq.(6) can be transformed into

$$\int_0^1 \left[N' \delta u_0 + M' \delta \varphi + \frac{1}{Gbhk} Q \delta Q + \bar{\gamma}^{cr} \delta Q \right] dx \quad (9)$$

with the shear modulus G and

$$\begin{aligned}N(x) &= \frac{bh}{2} \int_{-1}^1 \sigma_x(x, \zeta) d\zeta, \\ M(x) &= \frac{bh^2}{4} \int_{-1}^1 \sigma_x(x, \zeta) \zeta d\zeta, \\ \frac{1}{k} &= \frac{2}{\psi_0^2} \int_{-1}^1 \psi'^2(\zeta) d\zeta, \\ \bar{\gamma}^{cr}(x) &= \frac{1}{\psi_0} \int_{-1}^1 \gamma_{xz}^{cr}(x, \zeta) \psi'(\zeta) d\zeta\end{aligned}\quad (10)$$

After summing up all terms in Eq.(2) we obtain the following variational equation

$$\begin{aligned}\int_0^1 \left[(Q - M') \delta \varphi - (Q' + \bar{q}) \delta \bar{w} - N' \delta u_0 \right. \\ \left. + \left(\varphi + \bar{w}' - \frac{1}{Gbhk} Q - \bar{\gamma}^{cr} \right) \delta Q \right] dx = 0\end{aligned}\quad (11)$$

Assuming the variations of the functions $u_0, \varphi, \bar{w}$ and Q to be independent Eq.(11) provides the following ordinary differential equations

$$\begin{aligned}N' &= 0, \quad M' - Q = 0, \quad Q' + \bar{q} = 0, \\ Q &= Gbhk(\varphi + \bar{w}' - \bar{\gamma}^{cr})\end{aligned}\quad (12)$$

The first three equations are the classical equilibrium conditions of the linear beam theory. The last one is a constitutive equation connecting the shear force and the averaged shear strain. From this equation and with the assumed linear through-the-thickness approximation of the axial displacement we obtain

$$\begin{aligned}u(x, \zeta) &= u_0(x) - \zeta \frac{h}{2} \bar{w}'(x) \\ &+ \zeta \frac{h}{2} \frac{Q(x)}{Gbhk} + \zeta \frac{h}{2} \bar{\gamma}^{cr}(x)\end{aligned}$$

Here the second term is the rotation of the normal to the centreline (Bernoulli's hypothesis), the third term denotes the influence of the shear force in the sense of the Timoshenko theory, and the last term is the contribution of the averaged creep shear strain. The coefficient k and the average of the creep strain $\bar{\gamma}^{cr}$ are unknown while the function $\psi'(\zeta)$ is not specified. The parabolic shear stress distribution function according to the solution of the elastic Bernoulli beam obtained by

$$\psi'(\zeta) = 1 - \zeta^2 \quad (13)$$

yields the classical shear correction factor $k = 5/6$ for a homogeneous rectangular cross-section⁽²⁾. Shear correction factors for an elastic beam with various cross sections are discussed in Refs.(16), (17).

Let us consider the classical steady state creep solution of the Bernoulli beam⁽¹⁵⁾. Assuming the Norton-Bailey creep law we have

$$\dot{\varepsilon}_x \approx \dot{\varepsilon}_x^{cr} = a \sigma_x^n = -\dot{w}'' \zeta \frac{h}{2}$$

The stress σ_x can be expressed as

$$\sigma_x(x, \zeta) = \left(-\frac{\dot{w}''}{a} \right)^{1/n} |\zeta|^{(1/n)-1} \zeta \left(\frac{h}{2} \right)^{1/n}$$

$$= \frac{M(x)}{abh^2} |\zeta|^{(1/n)-1} \zeta$$

with

$$\alpha = \frac{n}{2(2n+1)}$$

After inserting this equation into the equilibrium condition

$$\sigma_{x,x} + \frac{2}{h} \tau_{xz,z} = 0 \quad (14)$$

and the integration with respect to the ζ coordinate, the distribution function can be calculated as

$$\psi'(\zeta) = 1 - \zeta^2 |\zeta|^{(1/n)-1} \quad (15)$$

Inserting this function into the first equation (10) we obtain

$$k_n = \frac{3n+2}{4n+2}$$

Setting $n=1$ this equation yields the shear correction factor of the elastic beam with rectangular cross-section. Since the value of n varies between 3 and 10 for metallic materials we can estimate, for example, if $n=3$; 10, $k_n=11/14$; 16/21, respectively. It can be seen that k_n in the case of steady state creep is influenced by the creep exponent. The value of k_n decreases with increasing creep exponent (for $n \rightarrow \infty$ we obtain $k_\infty=3/4$). Because the effect of damage is connected with the increase of the creep strain rate, the decreasing of the shear correction coefficient can be expected if damage evolution is taken into account. In addition, since the damage rate differs for tensile and compressive stresses, the thickness distribution of the transverse shear stress will be non-symmetrical. In this case the function ψ' cannot be selected a-priori. In Ref.(18) the function ψ' and the shear correction factor have been estimated for the creep-damage problem of a clamped beam based on the Ritz method. Figure 4 presents the simulation results for the uniformly loaded beam with clamped edges. For the calculations we set $l=1000$ mm, $b=50$ mm, $h=100$ mm and $q_0=50$ N/mm. The material model (1) with the introduced material constants is used. The curve 1 on the Fig.4 is the time-dependent maximum deflection calculated by use of the Bernoulli-Euler beam theory. The corresponding equations and the numerical procedure are presented in Ref.(19). The curve 2 is obtained using the beam model with the parabolic transverse shear stress according to Eq. (4) and the shear correction coefficient as 5/6. The curve 3 is the solution based on the equations discussed above with the modified transverse shear stress. The curve 4 is the ANSYS code finite element solution obtained with plane elements PLANE 42. It is obvious that the Bernoulli-Euler beam theory cannot adequately predict the deflection growth. Furthermore, the first order shear deformation theory underesti-

Fig. 4 Time dependent solutions for the maximum deflection of a clamped beam, 1—Bernoulli-Euler beam theory, 2—first order shear deformation theory with parabolic shear stress distribution, 3—first order shear deformation theory with modified shear stress, 4—solution using the ANSYS code with PLANE 42 elements

Fig. 5 Shear correction factor vs. time

mates the deflection particularly on the last stage of the creep process. By modification of the transverse shear stress distribution function a better agreement with the plane stress solution is obtained. Figure 5 presents the dependence on time of the shear correction factor. With decreasing value of k we can conclude that the influence of the shear correction terms in the discussed equations increases.

Figure 6 shows the distribution of the damage parameter at the last step of calculation. The damage is non-symmetrically distributed across the beam thickness. The damage evolution in the beam is dominantly controlled by the tensile stresses determining the maximum positive stress value σ_t . At the same time these stresses result in the maximum bending moments in the clamped edges.

Figure 7 presents the distribution of the transverse shear stress τ_{xz} obtained by ANSYS code with PLANE 42 elements. It can be observed that near the beam edges where the maximum damage occurs the

Fig. 6 Damage distribution in a beam at last time step

Fig. 7 Transverse shear stresses in a beam at last time step, solution with PLANE 42 elements

distribution is non-symmetrical across the thickness direction. The shear stress distribution along the beam length axis is antimetric. Note that the positive shear stress is obtained on the left-hand-side of the beam.

Figure 8(a) shows the solution for the shear force Q according to the derived beam equations. The transverse shear stress can be calculated as a product of the shear force and the distribution function ψ' with a constant factor. For the considered beam the shear force remains constant during the creep process and can be determined from the equilibrium condition $Q' + q_0 = 0$. Therefore the time-dependence of the transverse shear stress is determined by the time-dependence of the function ψ' . This function is computed based on the method discussed in Ref.(18) and plotted in Fig.8(b) for different time steps.

4. Analysis of Plates and Shells

The results for the beam show that the modified shear stress influences the deflection growth in the creep-damage process. On the other hand if we neglect the damage evolution, the steady state creep solution provides the shear stress distribution⁽¹⁵⁾ close to the parabolic one. From the beam equations we can conclude that the standard first order shear deformation theory with the parabolic function $\psi'(\xi)$ can be applied for the creep analysis if damage effects are negligible. Since damage evolution induces the stress state dependent material response, the transverse shear stress distribution has to be modified. With the modified function $\psi'(\xi)$ the transverse shear creep strain and the shear correction factor can be computed from Eq.(10). As we demonstrated on the example of a beam, these factors contribute to the time-dependent solution.

Let us emphasise that in the case of plates and shells the shear correction factor is additionally

Fig. 8 Transverse shear stresses in a beam: (a) shear force distribution according to the beam equations, (b) function of the transverse shear stress distribution for different time steps

responsible for the boundary layer stress distributions. The boundary layer solutions are known from the closed form solutions of the elastic plate equations⁽²⁰⁾. In the case of creep this kind of solution is important for the time dependent stress redistributions. In Eq.(21) the results of creep-damage analysis of a thin-walled pipe bend using the ANSYS code are reported. The computations were performed with the shell and the solid type finite elements available in ANSYS for creep analyses. The obtained solutions based on the shell and solid models disagree if the stress state dependent damage evolution is taken into account. Particularly, both models provide different edge zone stress redistributions. Various plots of illustrating boundary layer effects for the pipe bend in the elastic and creep solutions as well as stress and damage distributions are presented in Eq.(21). In Eq.(19) three different plate models were used for the simulation of a plate in bending. It was shown that the results based on the classical Kirchhoff theory, the Reissner-Mindlin type theory as well as the three-dimensional study agree only for the von Mises stress controlled creep and damage. The conclusions regarding the refinement of the shell with respect to the cross-section assumptions and numerical implementation have been made.

5. Conclusions

Starting from the creep responses under combined loading conditions we discussed the engineering models of beams, plates and shells with respect to the cross-section assumptions. Further we derived the first order shear deformation equations for beams considering the effects of creep deformation and damage evolution. We demonstrated on the beam equations that the shear correction factor and the averaged shear creep strain are determined by the function of the transverse shear stress distribution. This function should be modified by solving the creep-damage problems.

Further investigations should be directed to the examinations of higher order terms in through-the-thickness displacement or stress field approximations of beams, plates and shells in connection with creep-damage studies. Particularly, the higher order theories should be discussed with respect to the accuracy of the edge zone time-dependent stress redistributions for various types of boundary conditions.

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