Thermal Shock Analysis of Functionally Graded Materials by Micromechanical Model*

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The transient thermoelastic behavior of the functionally graded plate due to a thermal shock with temperature dependent properties is studied in this paper. The development of a micromechanical model for the functionally graded materials is presented and its application to thermoelastic analysis is discussed for the case of the W-Cu functionally graded material for the International Thermonuclear Experimental Reactor divertor plate. The divertor plate is made of a graded layer bonded between a homogeneous substrate and a homogeneous coating, and it is subjected to a cycle of heating and cooling on the coating surface of the material. The thermal and elastic properties of the material are dependent on the temperature and the position. Numerical calculations are carried out, and the results for the transient temperature and thermal stress distributions are displayed graphically.

Key Words: Computational Mechanics, Elasticity, Thermal Shock, Functionally Graded Material, Micromechanical Model

1. Introduction

In recent years, functionally graded materials (FGMs) have gained considerable importance in extremely high temperature environments such as nuclear reactors and chemical plants. Functional gradation opens a new avenue for optimizing both material and component structures to achieve high performance and material efficiency. At the same time, it posts many challenging mechanics problems, including the prediction and measurement of the effective properties, thermal stress distributions and fracture of FGMs⁽¹⁾. Furthermore, the cracking of brittle solids, due to sudden thermal transients, has long been recognized as an important failure phenomenon. Then, it is important to develop a fundamental understanding of the thermomechanical behavior due to a thermal shock⁽²⁾⁻⁽⁶⁾.

In the fusion reactor like the International Thermonuclear Experimental Reactor (ITER), plasma facing components (PFCs) will be subjected to high heat flux. The protection of these components is a very important issue for the design of the ITER. The development of actively cooled components for the fusion reactor which can withstand the high heat flux during transient and normal operation of the reactor is one of the key issues for the development of the fusion reactor. For this purpose, the tungsten (W)-copper (Cu) graded composite has been developed for the divertor plate⁽⁷⁾.

In this paper, we consider the transient thermoelastic behavior of the functionally graded divertor plate due to a thermal shock with temperature dependent properties. The development of a micromechanical model^{(8),(9)} for the functionally graded materials is presented and its application to thermoelastic analysis is discussed for the W-Cu functionally graded divertor plate of the ITER. The divertor plate is made of a graded layer bonded between a homogeneous substrate and a homogeneous coating, and it is subjected to a cycle of heating and cooling on the coating surface of the material. Numerical calcula-

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tions are carried out, and the transient temperature distribution and the transient thermal stresses are shown graphically.

2. Statement of the Problem

The schematic divertor plate⁽⁷⁾ and the micromechanical model^{(8),(9)}, used for computations, are shown in Fig. 1. Using a micromechanical model, the thermoelastic behavior of the W and Cu phases can be analyzed individually. The divertor plate of thickness h examined in this study consists of the tungsten coating of thickness h_W , the FGM layer of thickness h_{FGM} and the copper substrate of thickness h_{Cu} . A rectangular Cartesian coordinate system (x, y, z) is introduced in the divertor plate in such a way that the z -axis is the thickness direction. ρ_W and ρ_{Cu} denote the mass densities of the tungsten and the copper, respectively. The thermal and mechanical properties of the tungsten and the copper are functions of the temperature ϕ . The tungsten has the coefficient of thermal expansion $\alpha_W(\phi)$, the thermal conductivity $\lambda_W(\phi)$, the specific heat $c_W(\phi)$, the Young's modulus $E_W(\phi)$, the shear modulus $G_W(\phi)$ and the Poisson's ratio $\nu_W(\phi) = E_W(\phi)/2G_W(\phi) - 1$. The thermal and mechanical properties of the copper are $\alpha_{Cu}(\phi)$, $\lambda_{Cu}(\phi)$, $c_{Cu}(\phi)$, $E_{Cu}(\phi)$, $G_{Cu}(\phi)$, $\nu_{Cu}(\phi) = E_{Cu}(\phi)/2G_{Cu}(\phi) - 1$. In the following, the subscripts x, y, z will be used to refer to the coordinate directions and the subscripts $\delta = W, FGM, Cu$ indicate the tungsten coating, the FGM layer and the copper substrate, respectively.

We assume that the divertor plate is suddenly heated from the initial temperature Φ_A to the boundary temperature Φ_A on the Cu surface ($z=0$) and to the boundary temperature Φ_B on the W surface ($z=h$). After the steady state is achieved in the material at the time $t=t_A$, the W surface of the material is suddenly cooled to the initial temperature Φ_A . The combination of the above temperature is one of the combination of the temperature to which the divertor plate is subjected during the sudden changing of the operations of the reactor. It is also assumed that the

Fig. 1 The schematic of the functionally graded divertor plate and the micromechanical model

resulting transient thermal stress problem is quasi-static, that is, the inertia effects are negligible.

In this study, the following idealized FGM specimen of the W-Cu system has been taken into consideration. The divertor plate is divided into N subcells (micromechanical models) in the z -direction, and to each subcell a corresponding function of concentration V_{wn} ($n=1 \sim N$) is made, where the volume concentration V_{wn} ($n=1 \sim N$) changes from one to zero. The subcell dimensions should be chosen small enough - that structure inside it might be considered as locally homogeneous. In this case the subcells become local representative volume elements (LRVE). The three sides of the cubic LRVE and the cubic tungsten phase have lengths a_0 and a_{cn} ($n=1 \sim N$), respectively. Then the LRVE and tungsten phase have the volumes a_0^3 and a_{cn}^3 ($n=1 \sim N$). From the symmetry, it is sufficient to consider the one column of the subcells only.

From the LRVE geometry it yields volume fractions of phases $V_{wn} = (a_{cn}/a_0)^3$, $V_{Cu} = 1 - (a_{cn}/a_0)^3$ ($n=1 \sim N$), and equations for Young's moduli $E_n(\phi)$, the shear moduli $G_n(\phi)$, the thermal conductivity $\lambda_n(\phi)$ and the coefficient of thermal expansion $\alpha_n(\phi)$ of the subcells are given by^{(8),(9)}

$$E_n(\phi) = E_{Cu}(\phi) \left\{ 1 + \frac{V_{wn}}{\frac{E_W(\phi)}{E_W(\phi) - E_{Cu}(\phi)} - \sqrt[3]{V_{wn}}} \right\} \quad (n=1 \sim N) \quad (1)$$

$$G_n(\phi) = G_{Cu}(\phi) \left\{ 1 + \frac{V_{wn}}{\frac{G_W(\phi)}{G_W(\phi) - G_{Cu}(\phi)} - \sqrt[3]{V_{wn}}} \right\} \quad (n=1 \sim N) \quad (2)$$

$$\lambda_n(\phi) = \lambda_{Cu}(\phi) \left\{ 1 + \frac{V_{wn}}{\frac{\lambda_W(\phi)}{\lambda_W(\phi) - \lambda_{Cu}(\phi)} - \sqrt[3]{V_{wn}}} \right\} \quad (n=1 \sim N) \quad (3)$$

$$\alpha_n(\phi) = \frac{\alpha_{Cu}(\phi) + \sqrt[3]{V_{wn}} [\Psi_n(\phi) \{ \alpha_{Cu}(\phi) - \sqrt[3]{V_{wn}} (\alpha_{Cu}(\phi) - \alpha_W(\phi)) \} - \alpha_{Cu}(\phi)]}{1 + \sqrt[3]{V_{wn}} \{ \Psi_n(\phi) - 1 \}} \quad (n=1 \sim N) \quad (4)$$

where the function $\Psi_n(\phi)$ is given by

$$\Psi_n(\phi) = \frac{1 - 2\nu_{cu}(\phi)}{\left[1 - \sqrt[3]{V_{wn}} \left\{1 - \frac{E_{cu}(\phi)}{E_w(\phi)}\right\}\right] \left[3 - \frac{E_{cu}(\phi)}{G_{cu}(\phi)} \frac{1 - \sqrt[3]{V_{wn}} \left\{1 - \frac{G_{cu}(\phi)}{G_w(\phi)}\right\}}{1 - \sqrt[3]{V_{wn}} \left\{1 - \frac{E_{cu}(\phi)}{E_w(\phi)}\right\}}\right]} \quad (n=1 \sim N) \quad (5)$$

The mass density ρ_n and the specific heat $c_n(\phi)$ of each subcell can be obtained using the rule of mixture as follows:

$$\rho_n = V_{wn}\rho_w + (1 - V_{wn})\rho_{cu} \quad (n=1 \sim N) \quad (6)$$

$$c_n(\phi) = V_{wn}c_w(\phi) + (1 - V_{wn})c_{cu}(\phi) \quad (n=1 \sim N) \quad (7)$$

The function of concentrations V_{wn} ($n=1 \sim N$) is assumed to be a power-law function of $\bar{z}_n$ ($n=1 \sim N$) given by

$$V_{wn} = \begin{cases} 0 & (0 \leq \bar{z}_n \leq h_{cu}) \\ \left(\frac{\bar{z}_n - h_{cu}}{h_{FGM}}\right)^p & (h_{cu} \leq \bar{z}_n \leq h_{cu} + h_{FGM}) \\ 1 & (h_{cu} + h_{FGM} \leq \bar{z}_n \leq h) \end{cases} \quad (n=1 \sim N) \quad (8)$$

where $\bar{z}_n$ denotes the center point of the n -th subcell and the constant p ($0 \leq p \leq \infty$) is essentially the non-homogeneity of the FGM layer. In the limiting cases $E_n(\phi)$ would approach $E_w(\phi)$ for $p \rightarrow 0$ and $E_{cu}(\phi)$ for $p \rightarrow \infty$. $p=1.0$ corresponds to the linear variation of $E_n(\phi)$ in the FGM layer, for $0 < p < 1$ the FGM layer is W-rich and for $1 < p < \infty$ it is Cu-rich.

3. Heat Conduction Problem

The initial and boundary conditions are

$$\phi(z, 0) = \Phi_A \quad (0 \leq z \leq h) \quad (9)$$

$$\begin{cases} \phi(0, t) = \Phi_A \\ \phi(h, t) = \Phi_A + (\Phi_B - \Phi_A)\{H(t) - H(t - t_A)\} \end{cases} \quad (10)$$

where $H(t)$ denotes the Heaviside unit step function in time. The transient temperature distributions $\phi(z, t)$ may be determined from the nonlinear diffusion equation:

$$\frac{\partial}{\partial z} \left\{ \lambda(z, \phi) \frac{\partial \phi(z, t)}{\partial z} \right\} = \rho(z) c(z, \phi) \frac{\partial \phi(z, t)}{\partial t} \quad (0 < t) \quad (11)$$

where $\lambda(z, \phi)$, $\rho(z)$ and $c(z, \phi)$ denote the thermal conductivity, the mass density and the specific heat, respectively. The material is divided into N ($N \geq 2$) subcells and $a_0 = z_n - z_{n-1}$ ($n=1 \sim N$), where $0 = z_0 < z_1 < \dots < z_{N-1} < z_N = h$. As the time dimension is of an infinite extent, we shall deal with finite time domains $t_{k-1} \leq t < t_k$ ($k=1, 2, \dots$) and repeat the calculation for subsequent domains, where $0 = t_0 < t_1 < t_2 < \dots < t_k < t_{k+1} < \dots < t_K = t_A < t_{K+1} < \dots$. The temperature distribution $\phi_{n,k}(z, t)$ in the n -th subcell at time t ($t_{k-1} \leq t < t_k$, $k=1, 2, \dots$) is given by

$$\begin{aligned} \phi_{n,k}(z, t) &= \phi(z, t) \\ (z_{n-1} \leq z \leq z_n, n=1 \sim N, t_{k-1} < t \leq t_k, k=1, 2, \dots) \end{aligned} \quad (12)$$

The initial conditions for $\phi_{n,k}(z, t)$ are

$$\begin{aligned} \phi_{n,k}(z, t_{k-1}) &= \begin{cases} \Phi_A & (k=1) \\ \phi_{n,k-1}(z, t_{k-1}) & (k=2, 3, \dots) \end{cases} \\ (z_{n-1} \leq z \leq z_n, n=1 \sim N) \end{aligned} \quad (13)$$

We consider the problem in the time domain $t_{k-1} \leq t < t_k$ ($k=1, 2, \dots$). The thermal properties of the n -th subcell are approximated by the properties at the mean temperature $\phi_{Mn,k}$. The mean temperatures $\phi_{Mn,k}$ are given by

$$\begin{aligned} \phi_{Mn,k} &= \frac{1}{a_0} \int_{z_{n-1}}^{z_n} \phi_{n,k-1}(z, t_{k-1}) dz \\ (n=1 \sim N, k=1, 2, \dots) \end{aligned} \quad (14)$$

The temperature distributions $\phi_{n,k}(z, t)$ ($z_{n-1} \leq z \leq z_n$, $n=1 \sim N$) are obtained by solving the diffusion equations:

$$\begin{aligned} \frac{\partial^2 \phi_{n,k}(z, t)}{\partial z^2} &= \frac{1}{\chi_{n,k}} \frac{\partial \phi_{n,k}(z, t)}{\partial t} \\ (z_{n-1} \leq z \leq z_n, n=1 \sim N) \end{aligned} \quad (15)$$

where the thermal diffusivities $\chi_{n,k}$ are given by

$$\chi_{n,k} = \frac{\lambda_n(\phi_{Mn,k})}{\rho_n c_n(\phi_{Mn,k})} = \frac{\lambda_{n,k}}{\rho_n c_{n,k}} \quad (n=1 \sim N) \quad (16)$$

The boundary conditions for $\phi_{n,k}(z, t)$ are written as

$$\phi_{1,k}(0, t) = \Phi_A \quad (17)$$

$$\phi_{n,k}(z_n, t) = \phi_{n+1,k}(z_n, t) \quad (n=1 \sim N-1) \quad (18)$$

$$\begin{aligned} \lambda_{n,k} \left\{ \frac{\partial \phi_{n,k}(z, t)}{\partial z} \right\}_{z=z_n} \\ = \lambda_{n+1,k} \left\{ \frac{\partial \phi_{n+1,k}(z, t)}{\partial z} \right\}_{z=z_n} \end{aligned} \quad (n=1 \sim N-1) \quad (19)$$

$$\begin{aligned} \phi_{N,k}(h, t) &= \Phi_A + (\Phi_B - \Phi_A)\{H(t) - H(t - t_A)\} \\ &\quad (20) \end{aligned}$$

The diffusion Eq.(15), with the conditions of Eqs. (13), (17), (18) and (20), may be solved in a straightforward manner^{(10),(11)}. The relations between $\phi_{n,k}$ and $\dot{\phi}_{n,k}(z_n, t)$ ($n=1 \sim N-1$) are

$$\begin{aligned} \phi_{n,k}(z_n, t) &= \int_{t_{k-1}}^t \dot{\phi}_{n,k}(z_n, \tau) d\tau + \phi_{n,k}(z_n, t_{k-1}) \\ (n=1 \sim N-1) \end{aligned} \quad (21)$$

By using Eq.(21) and by substituting the solutions of Eq.(15) into Eq.(19), we obtain $N-1$ simultaneous Volterra integral equations of the first kind which can be solved to render $\dot{\phi}_{n,k}(z_n, t)$ ($n=1 \sim N-1$)⁽¹¹⁾.

The Laplace transform pair may be written as

$$\begin{aligned} f^*(p) &= \int_0^\infty f(t) \exp(-pt) dt \\ f(t) &= \frac{1}{2\pi i} \int_{Br} f^*(p) \exp(pt) dp \end{aligned} \quad (22)$$

and B_r denotes the Bromwich path of integration. Application of Eq.(22) to the simultaneous integral equation gives

$$\mathbf{K}_k \boldsymbol{\phi}_k = \mathbf{F}_k \quad (23)$$

where $\mathbf{K}_k$ is square matrix of order $N-1$, and $\boldsymbol{\phi}_k$ and $\mathbf{F}_k$ are column vectors of order $N-1$. The elements $K_{1,1,k}(\omega), K_{1,2,k}(\omega), \dots, K_{N-1,N-2,k}(\omega), K_{N-1,N-1,k}(\omega)$ of $\mathbf{K}_k$ and the elements $F_{1,k}(\omega), F_{2,k}(\omega), \dots, F_{N-1,k}(\omega)$ of $\mathbf{F}_k$ are given in the Appendix. The elements of $\boldsymbol{\phi}_k$ are $\exp(-\omega^2 t_{k-1}) \dot{\phi}_{n,k}^*(z_n, -\omega^2)$ ($n=1 \sim N-1$) and $-\omega^2 = p$.

Using the calculus of residues, $\dot{\phi}_{n,k}(z_n, t)$ ($n=1 \sim N-1$) can be expressed as follows:

$$\begin{aligned} \dot{\phi}_{n,k}(z_n, t) &= \frac{1}{2\pi i} \int_{B_r} \dot{\phi}_{n,k}^*(z_n, -\omega^2) \exp(pt) dp \\ &= \sum_{m=1}^{\infty} \exp\{-\omega_m^2(t-t_{k-1})\} \frac{\Delta_k^{(n)}(\omega_m)}{\left\{ \frac{d\Delta_k(\omega)}{d\omega} \right\}_{\omega \rightarrow \omega_m}} \end{aligned} \quad (n=1 \sim N-1) \quad (24)$$

where $\Delta_k(\omega)$ is the determinant of the square matrix $\mathbf{K}_k$ and $\Delta_k^{(n)}(\omega_m)$ is the determinant of the matrix obtained from $\mathbf{K}_k$ by replacing the n -th column of $\mathbf{K}_k$ by the column vector $\mathbf{F}_k$. The eigenvalues ω_m are determined from the transcendental equation:

$$\Delta_k(\omega) = 0 \quad (25)$$

By using Eqs.(21) and (24), $\phi_{n,k}(z_n, t)$ and $\dot{\phi}_{n,k}(z_n, t)$ ($n=1 \sim N-1$) may be determined, and $\phi_{n,k}(z, t)$ can be obtained. The temperature $\phi(z, t)$ of the material is determined from Eq.(12).

4. Thermal Stress Problem

Because of symmetry, the shear stress and strain components would be zero. While the stress free temperature is assumed to be Φ_A , the stresses at high temperature would be relaxed due to an inelastic deformation after a long time duration⁽¹²⁾. Then it is assumed that the stress components at the time t_A are also zero.

With the temperature distributions $\phi(z, t)$ now known, it is not difficult to determine the thermal effective stresses $\sigma_{xxn}(t), \sigma_{yy n}(t), \sigma_{zzn}(t)$ ($n=1 \sim N$) which satisfy conditions of force equilibrium through the material. The effective stress components $\sigma_{xxn}(t), \sigma_{yy n}(t), \sigma_{zzn}(t)$ in the n -th subcell are given by⁽¹³⁾

$$\left. \begin{aligned} \sigma_{xxn}(t) &= \frac{E_n(\phi_{Mn}(t))}{1-\nu_n(\phi_{Mn}(t))} \{\varepsilon_0(t) - \varepsilon_n^T(t)\} \\ \sigma_{yy n}(t) &= \frac{E_n(\phi_{Mn}(t))}{1-\nu_n(\phi_{Mn}(t))} \{\varepsilon_0(t) - \varepsilon_n^T(t)\} \\ \sigma_{zzn}(t) &= 0 \end{aligned} \right\} \quad (n=1 \sim N) \quad (26)$$

where the uniform strain $\varepsilon_0(t)$ and the thermal strain $\varepsilon_n^T(t)$ ($n=1 \sim N$) are given by

Fig. 2 Asterisk and outer regions in the micromechanical model

$$\varepsilon_0(t) = \frac{\sum_{n=1}^N \frac{E_n(\phi_{Mn}(t))}{1-\nu_n(\phi_{Mn}(t))} \varepsilon_n^T(t)}{\sum_{n=1}^N \frac{E_n(\phi_{Mn}(t))}{1-\nu_n(\phi_{Mn}(t))}} \quad (27)$$

$$\varepsilon_n^T(t) = \begin{cases} \int_{\Phi_A}^{\phi_{Mn}(t)} \alpha_n(\varphi) d\varphi & (0 \leq t \leq t_A) \\ \int_{\phi_{Mn}(t_A)}^{\phi_{Mn}(t)} \alpha_n(\varphi) d\varphi & (t_A \leq t) \end{cases} \quad (n=1 \sim N) \quad (28)$$

The mean temperatures $\phi_{Mn}(t)$ ($n=1 \sim N$) are given by

$$\phi_{Mn}(t) = \frac{1}{\alpha_0} \int_{z_{n-1}}^{z_n} \phi(z, t) dz \quad (n=1 \sim N) \quad (29)$$

We have to define some properties for so-called asterisk region, where both W and Cu phases are present as shown in Fig.2. The other region in the subcell is called as outer region. Iso-strain conditions for the subcells give for the thermal and mechanical properties of the asterisk region in the subcells as follows:

$$\left. \begin{aligned} \alpha_n^*(\phi) &= \alpha_{Cu}(\phi) - \sqrt[3]{V_{Wn}} \{ \alpha_{Cu}(\phi) - \alpha_W(\phi) \} \\ E_n^*(\phi) &= \frac{E_{Cu}(\phi)}{1 - \sqrt[3]{V_{Wn}} \left\{ 1 - \frac{E_{Cu}(\phi)}{E_W(\phi)} \right\}} \\ G_n^*(\phi) &= \frac{G_{Cu}(\phi)}{1 - \sqrt[3]{V_{Wn}} \left\{ 1 - \frac{G_{Cu}(\phi)}{G_W(\phi)} \right\}} \\ \nu_n^*(\phi) &= \frac{E_n^*(\phi)}{2G_n^*(\phi)} - 1 \end{aligned} \right\} \quad (n=1 \sim N) \quad (30)$$

where the superscript * denotes the asterisk region.

The thermal-elastic stresses in the asterisk and outer regions depend only on the z -axis. The equations of equilibrium in the x, y and z -directions are given by

$$\left. \begin{aligned} \sum_{n=1}^N \left\{ \sigma_{xxn}^*(\phi_{Mn}) \sqrt[3]{V_{Wn}^2} + \sigma_{xxn}^{\text{out}}(\phi_{Mn}) \left(1 - \sqrt[3]{V_{Wn}^2}\right) \right\} &= 0 \\ \sum_{n=1}^N \left\{ \sigma_{yy n}^*(\phi_{Mn}) \sqrt[3]{V_{Wn}^2} + \sigma_{yy n}^{\text{out}}(\phi_{Mn}) \left(1 - \sqrt[3]{V_{Wn}^2}\right) \right\} &= 0 \\ \sigma_{zzn}^*(\phi_{Mn}) \sqrt[3]{V_{Wn}^2} + \sigma_{zzn}^{\text{out}}(\phi_{Mn}) \left(1 - \sqrt[3]{V_{Wn}^2}\right) &= 0 \end{aligned} \right\} \quad (n=1 \sim N) \quad (31)$$

where the superscript out denotes the outer region of the subsell.

Due to the mutual restraint between the asterisk and outer regions in each subsell, there will be internal thermal stresses set up. After straightforward analysis with Eqs. (26) and (31), the thermal stresses in the asterisk and outer regions can be obtained as follows:

$$\sigma_{xxn}^*(t) = \frac{E_n^*(\phi_{Mn}(t))}{1 - \nu_n^*(\phi_{Mn}(t))} \{ \varepsilon_n^T(t) - \varepsilon_n^{*T}(t) \} + \frac{\sigma_{xxn}(t)}{\delta_1(1 - \sqrt[3]{V_{Wn}}) + \sqrt[3]{V_{Wn}}} \quad (n=1 \sim N) \quad (32)$$

$$\sigma_{yy n}^*(t) = \sigma_{xxn}^*(t) \quad (n=1 \sim N) \quad (33)$$

$$\sigma_{zzn}^*(t) = \frac{E_n^*(\phi_{Mn}(t))}{1 - \nu_n^*(\phi_{Mn}(t))} \{ \varepsilon_n^T(t) - \varepsilon_n^{*T}(t) \} \quad (n=1 \sim N) \quad (34)$$

$$\sigma_{xxn}^{\text{out}}(t) = \frac{E_{Cu}(\phi_{Mn}(t))}{1 - \nu_{Cu}(\phi_{Mn}(t))} \{ \varepsilon_n^T(t) - \varepsilon_n^{\text{out}T}(t) \} + \frac{\sigma_{xxn}(t)}{1 - \sqrt[3]{V_{Wn}} + \sqrt[3]{V_{Wn}} \delta_2} \quad (n=1 \sim N) \quad (35)$$

$$\sigma_{yy n}^{\text{out}}(t) = \sigma_{xxn}^{\text{out}}(t) \quad (n=1 \sim N) \quad (36)$$

$$\sigma_{zzn}^{\text{out}}(t) = \frac{E_{Cu}(\phi_{Mn}(t))}{1 - \nu_{Cu}(\phi_{Mn}(t))} \{ \varepsilon_n^T(t) - \varepsilon_n^{\text{out}T}(t) \} \quad (n=1 \sim N) \quad (37)$$

where δ_1 and δ_2 are given by

$$\left. \begin{aligned} \delta_1 &= \frac{1 - \nu^*(\phi_{Mn}(t))}{1 - \nu_{Cu}(\phi_{Mn}(t))} \frac{E_{Cu}(\phi_{Mn}(t))}{E_n^*(\phi_{Mn}(t))} \\ \delta_2 &= \frac{1 - \nu_{Cu}(\phi_{Mn}(t))}{1 - \nu_n^*(\phi_{Mn}(t))} \frac{E_n^*(\phi_{Mn}(t))}{E_{Cu}(\phi_{Mn}(t))} \end{aligned} \right\} \quad (38)$$

The thermal strains $\varepsilon_n^{*T}(t)$, $\varepsilon_n^{\text{out}T}(t)$ ($n=1 \sim N$) are given by

$$\varepsilon_n^{*T}(t) = \begin{cases} \int_{\phi_A}^{\phi_{Mn}(t)} \alpha_n^*(\phi) d\phi & (0 \leq t \leq t_A) \\ \int_{\phi_{Mn}(t_A)}^{\phi_{Mn}(t)} \alpha_n^*(\phi) d\phi & (t_A \leq t) \end{cases} \quad (n=1 \sim N) \quad (39)$$

$$\varepsilon_n^{\text{out}T}(t) = \begin{cases} \int_{\phi_A}^{\phi_{Mn}(t)} \alpha_{Cu}(\phi) d\phi & (0 \leq t \leq t_A) \\ \int_{\phi_{Mn}(t_A)}^{\phi_{Mn}(t)} \alpha_{Cu}(\phi) d\phi & (t_A \leq t) \end{cases} \quad (n=1 \sim N) \quad (40)$$

5. Numerical Results and Discussion

Numerical calculations have been carried out at the temperatures of $\Phi_A = 293$ [K] (room temperature), $\Phi_B = 1300$ [K] and for the geometric parameters of $h_W = 2$ [mm], $h_{FGM} = 8.5$ [mm], $h_{Cu} = 4$ [mm]⁽⁷⁾. The

Fig. 3 The temperature distribution $\phi(z, t)$ for $p=1.0$

parameter p is varied from 0.2 to 5.0. The number of the subsells $N=290$ and the time interval $\kappa_{Cu}(\Phi_A)(t_k - t_{k-1})/h^2 = 0.001$ ($k=1, 2, \dots$) are chosen from the requirements of accuracy of the analysis. The mass densities of the tungsten and the copper are $\rho_W = 1930.0$ [kg/m³] and $\rho_{Cu} = 8930.0$ [kg/m³]. Using the experimental results^{(14)–(16)}, the thermal and mechanical properties of the tungsten and the copper are approximated by the following functions of the temperature ϕ :

$$\alpha_W(\phi) = 4.6 [10^{-6}/\text{K}] \quad (41)$$

$$\alpha_{Cu}(\phi) = 16.8 + 0.005\phi - 4 \times 10^{-6}\phi^2 [10^{-6}/\text{K}] \quad (42)$$

$$\lambda_W(\phi) = 162.3 + 0.07\phi + 2.0 \times 10^{-5}\phi^2 [\text{W/mK}] \quad (43)$$

$$\lambda_{Cu}(\phi) = 401.4 - 0.0625\phi [\text{W/mK}] \quad (44)$$

$$c_W(\phi) = 10.011\sqrt{\phi} - 0.1362\phi [\text{J/kgK}] \quad (45)$$

$$c_{Cu}(\phi) = 15.37 + 7.18 \times 10^{-3}\phi - 4.0 \times 10^{-6}\phi^2 [\text{J/kgK}] \quad (46)$$

$$E_W(\phi) = 411.4 - 0.044\phi [\text{GPa}] \quad (47)$$

$$E_{Cu}(\phi) = 128.0 - 0.029\phi [\text{GPa}] \quad (48)$$

$$G_W(\phi) = 159.5 - 0.018\phi [\text{GPa}] \quad (49)$$

$$G_{Cu}(\phi) = 47.0 [\text{GPa}] \quad (50)$$

In the first series of calculations, we consider the transient temperature and stress distributions for $p=1.0$. Figure 3 shows the transient temperature distributions $\phi(z, t)$ for various values of the Fourier number $F = \kappa_{Cu}(\Phi_A)t/h^2$, where $\kappa_{Cu}(\Phi_A) = \lambda_{Cu}(\Phi_A)/\rho_{Cu}c_{Cu}(\Phi)$ is the thermal diffusivity of the copper at the initial temperature Φ_A and $F_A = \kappa_{Cu}(\Phi_A)t_A/h^2$ denotes the Fourier number at the time $t=t_A$. In the figure, the solid and dashed lines indicate the temperature in the heating and cooling processes, respectively. Since the Fourier number F in the cooling process is $F_A \leq F$, we represent the Fourier number in the cooling process as $F - F_A$. When $F \rightarrow F_A$ in the heating process and $F - F_A \rightarrow \infty$ in the cooling process, the temperature distribution approaches the steady state, and the extreme temperature gradient occurs at the top surface ($z/h=1.0$).

Figures 4, 5, 6 and 7 show the transient thermal stress distributions $\sigma_{xxn}^*(t)$, $\sigma_{xxn}^{\text{out}}(t)$ and $\sigma_{zzn}^*(t)$, $\sigma_{zzn}^{\text{out}}(t)$

Fig. 4 The thermal stress distribution $\sigma_{xxn}^*(t)$ for $p=1.0$ Fig. 5 The thermal stress distribution $\sigma_{xxn}^{out}(t)$ for $p=1.0$ Fig. 6 The thermal stress distribution $\sigma_{zzn}^*(t)$ for $p=1.0$ Fig. 7 The thermal stress distribution $\sigma_{zzn}^{out}(t)$ for $p=1.0$ Fig. 8 The effect of p on the thermal stress distribution $\sigma_{xxn}^*(t)$ Fig. 9 The effect of p on the thermal stress distribution $\sigma_{xxn}^{out}(t)$

for various values of F , respectively. It should be noted that the asterisk region does not exist in the Cu substrate ($0 \leq z/h \leq 0.276$) and the outer region does not exist in the W coating ($0.893 \leq z/h \leq 1.0$). The stress $\sigma_{xxn}^*(t)$ near the top surface ($z/h=1.0$) shown in Fig. 4 is large compressive at $F=0.001$ in the heating process and is large tensile at $F-F_A=0.001$ in the cooling process. The stress in the FGM layer is tensile in the heating process, and the opposite is true in the cooling process. As shown in Fig. 5, the maximum value of the stress $\sigma_{xxn}^{out}(t)$ occurs at the interface between the FGM layer and W coating in both the heating and cooling processes. The stress near the interface is compressive in the heating process, and the tensile stress in the cooling process increases with increasing F . Figure 6 shows the transient thermal stress distribution $\sigma_{zzn}^*(t)$. The stress $\sigma_{zzn}^*(t)$ in the W coating and the Cu substrate is zero, and the stress in the FGM layer is relatively small. As F increases, the absolute value of the stress in the FGM layer increases. The transient thermal stress $\sigma_{zzn}^{out}(t)$ shown in Fig. 7 occurs also in the FGM layer and the same tendency as in Fig. 5 is observed.

In the next set of calculations, we study the effect of the parameter p on the thermal stress distributions. Figures 8, 9, 10 and 11 indicate the stresses $\sigma_{xxn}^*(t)$,

Fig. 10 The effect of p on the thermal stress distribution $\sigma_{zzn}^*(t)$

Fig. 11 The effect of p on the thermal stress distribution $\sigma_{zzn}^{out}(t)$

$\sigma_{xxn}^{out}(t)$ and $\sigma_{zzn}^*(t)$, $\sigma_{zzn}^{out}(t)$ for $p=0.2, 1.0$ and 5.0 . The normalized times F in the heating process and $F-F_A$ in the cooling process, at which the maximum values of stresses occur, are chosen from the results shown in Figs. 4 to 7 for $p=1.0$. Figure 8 demonstrates the thermal stress distribution $\sigma_{xxn}^*(t)$ at $F=0.001$ in the heating process and at $F-F_A=0.001$ in the cooling process. No significant stress difference among the stress profiles calculated with the parameter $p=0.2, 1.0$ and 5.0 is seen in the results. Figure 9 shows the value of $\sigma_{xxn}^{out}(t)$ at $F \rightarrow F_A$ in the heating process and at $F-F_A \rightarrow \infty$ in the heating cooling process. While the absolute value of the stress near the bottom surface ($z/h=0.0$) increases with increasing p , the very large tensile stress at the interface between the FGM layer and the W coating decreases with increasing p in the cooling process. It may be useful for reducing the stress to increase p . The thermal stress distributions $\sigma_{zzn}^*(t)$ and $\sigma_{zzn}^{out}(t)$ at $F \rightarrow F_A$ in the cooling process and at $F-F_A \rightarrow \infty$ in the cooling process are plotted in Figs. 10 and 11. The effect of p on the stress distributions is large. While the absolute value of the stress $\sigma_{zzn}^{out}(t)$ decreases with increasing p , the opposite is true for the stress $\sigma_{zzn}^*(t)$.

6. Conclusion

Using the micromechanical model, the theoretical analysis of the thermal shock behavior of the functionally graded divertor plate with temperature dependent properties has been shown in this study. From the numerical results, the following conclusions have been reached.

1. The large stress $\sigma_{xxn}^*(t)$ occurs on the W surface in the early stage of the heating and cooling processes, and the stress rapidly decreases with time. The effect of the parameter p on the stress $\sigma_{xxn}^*(t)$ is not so large.
2. The absolute value of the stress $\sigma_{xxn}^{out}(t)$ tends to decrease with increasing z in the heating process, and the opposite is true in the cooling process. The absolute values of the stress $\sigma_{xxn}^{out}(t)$ at the bottom surface increase with increasing p .
3. The stress $\sigma_{zzn}^*(t)$ is entirely dependent on the parameter p and the Fourier number F , and the absolute values of the stress increase with increasing p and F .
4. While the distribution of the stress $\sigma_{zzn}^*(t)$ is dependent on p and F , p has little effect on the absolute maximum values of the stress.

Appendix

The nonzero elements $K_{1,1,k}(\omega)$, $K_{1,2,k}(\omega)$, ..., $K_{N-1,N-2,k}(\omega)$, $K_{N-1,N-1,k}(\omega)$ of $\mathbf{K}_k$ in Eq. (23) are given by

$$K_{n,n-1,k}(\omega) = \frac{\lambda_{n,k}}{\sqrt{\chi_{n,k}} \sin(a_0\omega/\sqrt{\chi_{n,k}})} \quad (n=2 \sim N-1, k=1, 2, \dots) \quad (51)$$

$$K_{n,n,k}(\omega) = -\frac{\lambda_{n,k} \cos(a_0\omega/\sqrt{\chi_{n,k}})}{\sqrt{\chi_{n,k}} \sin(a_0\omega/\sqrt{\chi_{n,k}})} - \frac{\lambda_{n+1,k} \cos(a_0\omega/\sqrt{\chi_{n+1,k}})}{\sqrt{\chi_{n+1,k}} \sin(a_0\omega/\sqrt{\chi_{n+1,k}})} \quad (n=1 \sim N-1, k=1, 2, \dots) \quad (52)$$

$$K_{n,n+1,k}(\omega) = \frac{\lambda_{n+1,k}}{\sqrt{\chi_{n+1,k}} \sin(a_0\omega/\sqrt{\chi_{n+1,k}})} \quad (n=1 \sim N-2, k=1, 2, \dots) \quad (53)$$

The elements $F_{1,k}(\omega)$, $F_{2,k}(\omega)$, ..., $F_{N-1,k}(\omega)$ of $\mathbf{F}_k$ in Eq. (23) are given by

$$F_{n,k}(\omega) = -\frac{\lambda_{n,k} \phi_{n-1,k}(z_{n-1}, t_{k-1})}{\sqrt{\chi_{n,k}} \sin(a_0\omega/\sqrt{\chi_{n,k}})} - \frac{\lambda_{n+1,k} \phi_{n+1,k}(z_{n+1}, t_{k-1})}{\sqrt{\chi_{n+1,k}} \sin(a_0\omega/\sqrt{\chi_{n+1,k}})} + \left\{ \frac{\lambda_{n,k} \cos(a_0\omega/\sqrt{\chi_{n,k}})}{\sqrt{\chi_{n,k}} \sin(a_0\omega/\sqrt{\chi_{n,k}})} + \frac{\lambda_{n+1,k} \cos(a_0\omega/\sqrt{\chi_{n+1,k}})}{\sqrt{\chi_{n+1,k}} \sin(a_0\omega/\sqrt{\chi_{n+1,k}})} \right\} \phi_{n,k}(x_n, t_{k-1}) + \frac{\lambda_{n,k} \omega}{\chi_{n,k}} \int_{z_{n-1}}^{z_n} \frac{\sin\{(\xi - z_{n-1})\omega/\sqrt{\chi_{n,k}}\}}{\sin(a_0\omega/\sqrt{\chi_{n,k}})} \times \phi_{n,k}(\xi, t_{k-1}) d\xi + \frac{\lambda_{n+1,k} \omega}{\chi_{n+1,k}} \int_{z_n}^{z_{n+1}} \frac{\sin\{(z_{n+1} - \xi)\omega/\sqrt{\chi_{n+1,k}}\}}{\sin(a_0\omega/\sqrt{\chi_{n+1,k}})} \times \phi_{n+1,k}(\xi, t_{k-1}) d\xi$$

$$\begin{aligned}
& \times \phi_{n+1,k}(\xi, t_{k-1}) d\xi \quad (n=1 \sim N-2) \quad (54) \\
F_{N-1,k}(\omega) = & -\frac{\lambda_{N-1,k} \phi_{N-2,k}(x_{N-2}, t_{k-1})}{\sqrt{\kappa_{N-1,k}} \sin(a_0 \omega / \sqrt{\kappa_{N-1,k}})} \\
& -\frac{\lambda_{N,k} \Phi(t)}{\sqrt{\kappa_{N,k}} \sin(a_0 \omega / \sqrt{\kappa_{N,k}})} \\
& + \left\{ \frac{\lambda_{N-1,k} \cos(a_0 \omega / \sqrt{\kappa_{N-1,k}})}{\sqrt{\kappa_{N-1,k}} \sin(a_0 \omega / \sqrt{\kappa_{N-1,k}})} \right. \\
& + \frac{\lambda_{N,k} \cos(a_0 \omega / \sqrt{\kappa_{N,k}})}{\sqrt{\kappa_{N,k}} \sin(a_0 \omega / \sqrt{\kappa_{N,k}})} \left. \right\} \phi_{N-1,k}(z_{N-1}, t_{k-1}) \\
& + \frac{\lambda_{N-1,k} \omega \int_{z_{N-2}}^{z_{N-1}} \frac{\sin\{(\xi - z_{N-2})\omega / \sqrt{\kappa_{N-1,k}}\}}{\sin(a_0 \omega / \sqrt{\kappa_{N-1,k}})} \\
& \times \phi_{N-1,k}(\xi, t_{k-1}) d\xi \\
& + \frac{\lambda_{N,k} \omega \int_{z_{N-1}}^{z_N} \frac{\sin\{(z_N - \xi)\omega / \sqrt{\kappa_{N,k}}\}}{\sin(a_0 \omega / \sqrt{\kappa_{N,k}})} \\
& \times \phi_{N,k}(\xi, t_{k-1}) d\xi \quad (55)
\end{aligned}$$

when $\Phi(t) = \Phi_A + (\Phi_B - \Phi_A)\{H(t) - H(t - t_A)\}$ and $\phi_{0,k}(z_0, t_{k-1}) = \Phi_A$ ($k=1, 2, \dots$).

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