Revolution of Principal Axes of Plastic Anisotropy Developed during Deformation Process*

(Measurement of Revolution Angles by Laser Speckle Surface Strain Meter)

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The anisotropies of sheet metals originate from the texture developed by a cold rolling process. In the texture, the crystal grains assume a partial direction, and so the strength of the texture is different from that of the surrounding material. When the sheet metals are deformed with changes of the strain path, the difference in strength produces a new texture and defines a new anisotropic principal axis. Therefore, it is supposed that the principal axis of anisotropies does not behave like the material fiber printed on the sheet metals. To confirm the above assumption, a measuring method of revolution angles of principal axes is proposed by using a laser speckle surface strain meter. The revolution angle is measured by investigating the distribution of r -values with respect to the tensile directions. Finally, a dynamical revolution model of the principal axes is derived and its parameters are identified.

Key Words: Material Testing, Plasticity, Nondestructive Inspection, Strain Measurement, Laser Speckles, Strain History, Plastic Anisotropy, Anisotropic Principal Axes, Revolution of Principal Axes

1. Introduction

In the deformation process of sheet metals, plastic deformation takes place with the change of the strain path depending on the product form. Such deformation with a personal history switches the active sliding sides in a crystal, and the lattice rotation followed by these switching phenomena changes the composition of the texture.

In the deformation of an initial anisotropic steel sheet metal produced by a cold rolling process, it can

be thought that the change of strain path causes the rotation of the anisotropic principal axes. However, such rotation does not necessarily correspond with that of the material fiber printed on the sheet metals^{(1),(2)}. Inside the texture, the orientation distribution of crystal grains is concentrated in one direction, and so the structure of the texture may be considered as that of a single crystal. Therefore, sliding occurs along a specific direction, and strong resistance is experienced in directions other than that. Thus, anisotropy related to the plastic deformation appears.

Since plastic anisotropies are based on the uneven distribution of crystal grains, an anisotropic principal axis is defined as a symmetric axis with respect to the orientation distribution of crystal grains. When the strain path is changed, sliding in the texture is shifted to another specific direction and the movement of the texture becomes different from that of the surrounding material. Therefore, a new texture is formed, and

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a new anisotropic principal axis is also defined. Thus, the movement of such a new axis does not correspond to that of the material fiber printed on a sheet metal.

In this study, to investigate the movement of anisotropic principal axes, the revolution angle of a principal axis is measured by means of the identification method⁽¹⁾ of principal axes, which uses a laser speckle surface strain meter. Furthermore, a dynamical revolution model⁽²⁾ of anisotropic principal axes is proposed and its parameters are identified from experimental data.

2. Revolution of Anisotropic Principal Axes

The purpose of this study is to confirm that the movement of an anisotropic principal axis is different from that of a material fiber, when the strain path is changed. First, we consider a dynamical revolution model of an anisotropic principal axis.

We regard plastic deformation as a composite deformation of the lattice slide $\mathbf{F}^p$ and the lattice rotation $\mathbf{R}$ (see Ref. (2)), and represent the deforming gradient tensor by

$$\mathbf{F} = \mathbf{R}\mathbf{F}^p. \quad (1)$$

Furthermore, let a material fiber vector $\mathbf{s}$ be a vector selected in the sliding direction. Then, the dynamical model of the material fiber is derived by using the material time differential of the following unit vector

$$\bar{\mathbf{s}} = \frac{\mathbf{F}\mathbf{s}}{|\mathbf{F}\mathbf{s}|} = \mathbf{R}\mathbf{s}. \quad (2)$$

Consequently, we have derived the following relations in our previous study⁽²⁾.

$$\begin{aligned} \dot{\bar{\mathbf{s}}} &= (\mathbf{D} + \mathbf{W})\bar{\mathbf{s}} - \bar{\mathbf{s}}(\bar{\mathbf{s}} \cdot \mathbf{D}\bar{\mathbf{s}}) \\ &= \mathbf{W}\bar{\mathbf{s}} + \{\mathbf{D}(\bar{\mathbf{s}} \otimes \bar{\mathbf{s}}) - (\bar{\mathbf{s}} \otimes \bar{\mathbf{s}})\mathbf{D}\}\bar{\mathbf{s}} \\ &= \dot{\mathbf{R}}\mathbf{R}^{-1}\bar{\mathbf{s}} = \dot{\mathbf{R}}\mathbf{R}^t\bar{\mathbf{s}} \equiv \omega_{\bar{\mathbf{s}}}\bar{\mathbf{s}} \end{aligned} \quad (3)$$

Here, $\mathbf{D}$ and $\mathbf{W}$ are tensors composed by the symmetric and antisymmetric components of the velocity gradient tensor $\mathbf{L}$, and $\omega_{\bar{\mathbf{s}}}$ is defined as the spin tensor of $\bar{\mathbf{s}}$.

These results are similar to the equations derived by Asaro and Rice⁽³⁾ and Aravas and Aifantis⁽⁴⁾. Thus, we have the following fundamental equation with respect to the revolution of a material fiber.

$$\omega_{\bar{\mathbf{s}}} = \mathbf{W} + \{\mathbf{D}(\bar{\mathbf{s}} \otimes \bar{\mathbf{s}}) - (\bar{\mathbf{s}} \otimes \bar{\mathbf{s}})\mathbf{D}\} \quad (4)$$

Considering that the movement of anisotropic principal axes $\mathbf{a}_i$ ($i=1, 2, 3$) is affected by that of the material fiber, the following new dynamical model of anisotropic principal axes is proposed.

$$\omega_{\mathbf{a}_i} = \mathbf{W} + \zeta A_i \{\mathbf{D}(\mathbf{a}_i \otimes \mathbf{a}_i) - (\mathbf{a}_i \otimes \mathbf{a}_i)\mathbf{D}\} \quad (5)$$

The symbol ζ is the damping factor relating the movement of an anisotropic principal axis to that of a material fiber, and A_i is the coefficient representing the degree of anisotropies into each principal axis.

Let $\mathbf{A}$ be a second-order symmetric tensor whose principal axes correspond to the anisotropic principal

axes and whose principal values become A_i . Then, Eq. (5) becomes

$$\omega_{\mathbf{A}} = \mathbf{W} + \zeta(\mathbf{D}\mathbf{A} - \mathbf{A}\mathbf{D}). \quad (6)$$

This tensor $\mathbf{A}$ represents the degree of anisotropic properties. If the coordinate axes are selected in the direction of the principal axes of $\mathbf{A}$, then $\mathbf{A}$ is represented by the symmetric matrix

$$\mathbf{A}_0 = \begin{pmatrix} X & 0 & 0 \\ 0 & Y & 0 \\ 0 & 0 & Z \end{pmatrix}, \quad (7)$$

where X, Y, Z can be determined by r -values, yield stresses, and so on.

Now, we consider that the sheet metals are deformed with rotation tensor $\mathbf{R}$, and their principal axes retain the orthogonal properties during the deformation process. Then, we have

$$\mathbf{A} = \mathbf{R}\mathbf{A}_0\mathbf{R}^t, \quad \omega_{\mathbf{A}} = \dot{\mathbf{R}}\mathbf{R}^{-1} = \dot{\mathbf{R}}\mathbf{R}^t.$$

Thus, the dynamical model (6) becomes

$$\dot{\mathbf{R}}\mathbf{R}^t = \mathbf{W} + \zeta(\mathbf{D}\mathbf{R}\mathbf{A}_0\mathbf{R}^t - \mathbf{R}\mathbf{A}_0\mathbf{R}^t\mathbf{D}). \quad (8)$$

Let us consider the plastic deformation of an initial anisotropic sheet metal shown in Fig. 1. When the sheet metal is tensioned in the direction inclined by θ toward the direction of the principal axis $\mathbf{a}_1$ (i.e., rolling direction RD), and the revolution angle of the anisotropic principal axis is ϕ , then $\mathbf{R}$ becomes

$$\mathbf{R} = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix}, \quad (9)$$

and the following relations⁽²⁾ are given by Eq. (8).

$$W_{11} = W_{22} = 0 \quad (10)$$

$$-\dot{\phi} = W_{12} - \zeta(1 - Y)\eta \quad (11)$$

$$\eta \equiv D_{12} \cos 2\phi - \frac{D_{11} - D_{22}}{2} \sin 2\phi \quad (12)$$

Let the tensile-directional unit vector $\mathbf{n}$ shown in Fig. 1 be

$$\mathbf{n} = \cos \theta \mathbf{e}_1 + \sin \theta \mathbf{e}_2. \quad (13)$$

Because the material fiber drawn in the tensile direction does not rotate, the following relation is derived as well as Eq. (3).

$$\omega_{\mathbf{n}} = \mathbf{W} + \{\mathbf{D}(\mathbf{n} \otimes \mathbf{n}) - (\mathbf{n} \otimes \mathbf{n})\mathbf{D}\}$$

Fig. 1 Revolution of the anisotropic principal axes

$$\begin{aligned}
&= \begin{pmatrix} 0 & W_{12} \\ -W_{12} & 0 \end{pmatrix} \\
&+ \begin{pmatrix} D_{11} & D_{12} \\ D_{12} & D_{22} \end{pmatrix} \begin{pmatrix} \cos^2 \theta & \cos \theta \sin \theta \\ \cos \theta \sin \theta & \sin^2 \theta \end{pmatrix} \\
&- \begin{pmatrix} \cos^2 \theta & \cos \theta \sin \theta \\ \cos \theta \sin \theta & \sin^2 \theta \end{pmatrix} \begin{pmatrix} D_{11} & D_{12} \\ D_{12} & D_{22} \end{pmatrix} \\
&= 0
\end{aligned} \quad (14)$$

Thus, we have

$$W_{12} = D_{12} \cos 2\theta - \frac{D_{11} - D_{22}}{2} \sin 2\theta. \quad (15)$$

Now, let the tensor $\mathbf{D}'$ be a tensor constructed by the coordinate transformation of $\mathbf{D}$ into the tensile direction in Fig. 1. Then, we have

$$\begin{pmatrix} D_{11}' & D_{12}' \\ D_{12}' & D_{22}' \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}^{-1} \times \begin{pmatrix} D_{11} & D_{12} \\ D_{12} & D_{22} \end{pmatrix} \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}.$$

Therefore, the following relations are given by Eqs. (15) and (12).

$$W_{12} = D_{12}' \quad (16)$$

$$\eta = D_{12}' \cos 2(\theta - \phi) + \frac{D_{11}' - D_{22}'}{2} \sin 2(\theta - \phi) \quad (17)$$

Thus, Eq.(11) with respect to the revolution of an anisotropic principal axes becomes

$$\begin{aligned}
-\dot{\phi} &= D_{12}' - \zeta(1 - Y) \\
&\times \{D_{12}' \cos 2(\theta - \phi) + \frac{D_{11}' - D_{22}'}{2} \sin 2(\theta - \phi)\}.
\end{aligned} \quad (18)$$

When the x -axis and y -axis are selected in the tensile direction and its orthogonal direction respectively, each element of $\mathbf{D}'$ represents the strain rate.

$$D_{11}' = \dot{\epsilon}_{xx}, \quad D_{12}' = \dot{\epsilon}_{xy}, \quad D_{22}' = \dot{\epsilon}_{yy} \quad (19)$$

Therefore, when the tensile-directional strain ϵ_{xx} is adopted instead of the time parameter, Eq.(18) becomes

$$\begin{aligned}
\frac{d\phi}{d\epsilon_{xx}} &= -\frac{d\epsilon_{xy}}{d\epsilon_{xx}} + \zeta(1 - Y) \\
&\times \left\{ \frac{d\epsilon_{xy}}{d\epsilon_{xx}} \cos 2(\theta - \phi) + \frac{1 - \frac{d\epsilon_{yy}}{d\epsilon_{xx}}}{2} \sin 2(\theta - \phi) \right\}.
\end{aligned} \quad (20)$$

In the above case of a plane stress, because the anisotropic principal axis is rotated, the r -value is also changed depending on the revolution angle ϕ . Therefore, Hill's anisotropic yielding function must be represented by $\bar{\theta} = \theta - \phi$ instead of θ , where the anisotropic parameters F , G , H , N are constant. Thus, we have the following relations⁽¹⁾.

$$\begin{aligned}
r_\theta(\phi) &= \frac{d\epsilon_{yy}}{-(d\epsilon_{xx} + d\epsilon_{yy})} \\
&= -\frac{(F + G - 2N)\sin^2 2\bar{\theta} - 4H \cos^2 2\bar{\theta}}{4(F \sin^2 \bar{\theta} + G \cos^2 \bar{\theta})}
\end{aligned} \quad (21)$$

$$\begin{aligned}
\Gamma_\theta(\phi) &= \frac{d\epsilon_{xy}}{d\epsilon_{xx}} = \frac{d\epsilon_{yx}}{d\epsilon_{xx}} \\
&= \frac{\sin 2\bar{\theta} \{-F \sin^2 \bar{\theta} + G \cos^2 \bar{\theta} + (2H - N) \cos 2\bar{\theta}\}}{2F \sin^4 \bar{\theta} + 2G \cos^4 \bar{\theta} + 2H \cos^2 2\bar{\theta} + N \sin^2 2\bar{\theta}}
\end{aligned} \quad (22)$$

Now, if the anisotropic parameter Y is given by

$$Y = \frac{r_{90^\circ}}{r_0}, \quad (23)$$

a dynamical model (20) of the anisotropic principal axes is finally derived as

$$\begin{aligned}
\frac{d\phi}{d\epsilon_{xx}} &= -\Gamma_\theta(\phi) + \zeta \left(1 - \frac{r_{90^\circ}}{r_0} \right) \\
&\times \left\{ \Gamma_\theta(\phi) \cos 2(\theta - \phi) + \frac{1 + \frac{r_\theta(\phi)}{1 + r_\theta(\phi)}}{2} \sin 2(\theta - \phi) \right\}.
\end{aligned} \quad (24)$$

3. Identification of Anisotropic Principal Axes by Laser Speckle Surface Strain Meter

In this section, the identification method of anisotropic principal axes is considered by using a laser speckle surface strain meter, and the estimation method of the revolution angle of the principal axes is proposed.

Since the anisotropic principal axis is defined as a symmetric axis of the orientation distribution of crystal grains, in our previous study⁽¹⁾, the identification method of anisotropic principal axes has been shown experimentally by investigating the symmetric property of r -values and Γ -values based on the tensile test. Therefore, the revolution angles can be estimated by developing the above identification method.

The laser speckle is an image of high-contrast optical grains arising from the application of a narrow laser beam to the surface of a specimen. These optical grains are produced by the interference of laser beams reflected from the irregular surfaces of the specimen. To measure displacement or strain, the speckle pattern can be used as the natural markings printed by the laser light⁽⁵⁾.

Figure 2 shows the construction of the laser speckle surface strain meter. This system is composed of a tensile testing machine and an image processing board with three paired image sensors of 1-4, 2-5 and 3-6. Let us consider one pair image sensors, for example, x -directional 2-5 sensors. In the direction of these sensors, the strain is given by the following equation⁽¹⁾, where $\Delta A_x = A_{x2} - A_{x1}$ is the quantity of the image movement detected by two sensors.

$$\epsilon_{xx} \approx \frac{\Delta A_x}{2L \tan \alpha} \quad (25)$$

The laser beam is applied perpendicularly to the

Fig. 2 Laser speckle surface strain meter

surface of the specimen. Here, α is the angle between a laser beam axis and each image sensor axis. The symbol L is the distance between the image sensor and the laser light spot appearing on the specimen surface. The output signals of six image sensors are input to a computer with multiplexing and A/D conversion. The computer also controls the displacement of the tensile testing machine.

Symbols A_{x2} and A_{x1} are the quantities of the image movements detected by the two sensors and determined by the following correlation method. Let us consider two different image data obtained at constant time intervals by each sensor. Then, the quantity of each image movement is determined by the distance of the dislocation that maximizes the cross-correlation function between the above old-time image data and the new-time image data shifted in the x -direction at various distances.

In the analysis of plastic anisotropies, it is important to know the two-dimensional strain tensor that includes shear strain. Generally, it is not easy to measure shear strain. Therefore, in this study, the strain tensor including shear strain is estimated from three independent strains ε_{1-4} , ε_{2-5} and ε_{3-6} , by using the following coordinate transformation formula.

$$\begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{pmatrix} = \begin{pmatrix} \cos^2 \beta_{1-4} & \sin^2 \beta_{1-4} & \frac{1}{2} \sin 2\beta_{1-4} \\ \cos^2 \beta_{2-5} & \sin^2 \beta_{2-5} & \frac{1}{2} \sin 2\beta_{2-5} \\ \cos^2 \beta_{3-6} & \sin^2 \beta_{3-6} & \frac{1}{2} \sin 2\beta_{3-6} \end{pmatrix}^{-1} \times \begin{pmatrix} \varepsilon_{1-4} \\ \varepsilon_{2-5} \\ \varepsilon_{3-6} \end{pmatrix} \quad (26)$$

The angle β is determined from the configuration of the image sensors and selected as $\beta_{1-4} = -60^\circ$, $\beta_{2-5} = 0^\circ$

Fig. 3 Relationship between r -value and cutting angle θ Fig. 4 Relationship between Γ -value and cutting angle θ

and $\beta_{3-6} = 60^\circ$, respectively. The r -values and Γ -values are determined by the following relations.

$$r = \frac{d\varepsilon_{yy}}{-(d\varepsilon_{xx} + d\varepsilon_{yy})}, \quad \Gamma = \frac{d\varepsilon_{xy}}{d\varepsilon_{xx}} = \frac{d\varepsilon_{yx}}{d\varepsilon_{xx}} \quad (27)$$

In the tensile test, the anisotropic principal axes are determined from the direction that brings about the extremum of r -values and eliminates the shear strain (i.e., $\Gamma = 0$). Therefore, the principal axis can be identified by investigating the direction distribution of r -values and Γ -values of specimens made from an initial anisotropic sheet metal with various cutting angles.

In the case of a steel sheet metal (SSPDX) of 0.85 mm thickness, the experimental results of r -values and Γ -values are shown in Figs. 3 and 4, respectively. The theoretical curves are determined by using Eqs. (21) and (22), and $\phi = 0$ is assumed. The anisotropic parameters F , G , H , $N^{(1)}$ are derived from the following relations (28).

Since the r -values of the above specimens are given by

$$r_{0^\circ} = 2.29, \quad r_{45^\circ} = 2.19, \quad r_{90^\circ} = 3.00,$$

the anisotropic parameters are calculated as

$$F = \frac{1}{r_{90^\circ}} = 0.333, \quad G = \frac{1}{r_{0^\circ}} = 0.437, \quad H = 1.00,$$

$$N = \left(r_{45^\circ} + \frac{1}{2} \right) \left(\frac{1}{r_{90^\circ}} + \frac{1}{r_{0^\circ}} \right) = 2.07. \quad (28)$$

In these figures, the r -value takes an extremum and the Γ -value becomes $\Gamma=0$ in the rolling direction (0°) and its orthogonal direction (90°). Therefore, we regard these directions as the anisotropic principal axes. The experimental results show that they correspond to the proper principal axes of the initial anisotropic sheet metal.

Using the above identification method of anisotropic principal axes, it is possible to measure the revolution angle of the principal axis. The revolution angle can be estimated by measuring the phase deviation derived from the direction distributions of r -values and Γ -values for two kinds of specimens with and without the pre-strain.

4. Measurement of Revolution Angles of Anisotropic Principal Axes

Let us consider the estimation of the revolution angles of the principle axes more concretely based on section 3. Using the steel sheet metal SSPDX adopted in section 3, Figs. 5 and 6 show the experimental results of r -values and Γ -values with and without the pre-strain of 10% in the direction inclined 45° toward

Fig. 5 Phase deviation of r -values between two cases with and without 10% pre-strain

Fig. 6 Phase deviation of Γ -values between two cases with and without 10% pre-strain

the rolling direction. By investigating the curves of these direction distributions, the deviation of the phase can be recognized easily. To obtain the deviation of the phase, in this study, suitable approximate theoretical curves are drawn by the least squares method. As a function of the least squares, the following equation is adopted.

$$E_{\phi_j} = \sum_{\theta_i} \{ r_{\theta_i} - r_{\theta_i}(\phi_j) \}^2 \quad (29)$$

The symbol r_{θ_i} are the experimental r -values with the pre-strain tensioned in the θ_i direction, and $r_{\theta_i}(\phi_j)$ are the theoretical values of Eq. (21) with various ϕ . The results are shown in Fig. 7.

Using $\phi = -8.2^\circ$, the minimum value is given and the revolution angle of the principal axis is determined. In Fig. 5 the theoretical curve of that case is shown by a dotted line.

In the same way, $\phi = -4.0^\circ$ minimizes the square error of Γ -values and the theoretical curve is shown in Fig. 6. Consequently, the revolution angle ϕ of Γ -values is slightly different from that of r -values. As the reason for this, it can be surmised that since the Γ -values are small the influence of measurement error becomes large. Furthermore, the theoretical values are considerably different from the experimental values in Fig. 6. Therefore, in order to identify a dynamical revolution model of the principal axes, we

Fig. 7 Fitting of the phase deviation ϕ by least squares method (r -value)

Fig. 8 Rotation of material fiber

Fig. 9 Rotation angles of principal axis with various ζ Fig. 10 Fitting of parameter ζ by least squares method

adopt only the results of r -values in the later section.

Now, let us consider the revolution angle β of a material fiber instead of the anisotropic principal axis. Let e be a unit material fiber vector with the rolling direction in Fig. 8. Supposing that the specimen is tensioned in the x -axis direction and e becomes $\bar{e}$, then the strains ε_x and ε_y are defined by this tension and the following relation is derived.

$$\beta = \theta - \arctan\left(\frac{1 + \varepsilon_y}{1 + \varepsilon_x}\right) \quad (30)$$

Furthermore, we obtain the following equation by using r_θ .

$$\beta = \theta - \arctan\left(\frac{1 - \frac{r_\theta}{1 + r_\theta} \varepsilon_x}{1 + \varepsilon_x}\right) \quad (31)$$

In our experiment we have $\theta = 45^\circ$, $\varepsilon_x = 0.1$ and $r_{45^\circ} = 2.19$. Thus, we have $\beta = 4.75^\circ$. Consequently, the revolution angle ϕ of the anisotropic principal axis is different from the same angle β of the material fiber.

5. Identification of Model Parameters

Let us consider the identification method of the parameter ζ with respect to the dynamical revolution model (24). Numerical integration of Eq.(24) gives the revolution angle depending on the amount of

Fig. 11 Relationship between r -value and cutting angle with and without 10% pre-strain Fig. 12 Relationship between Γ -value and cutting angle with and without 10% pre-strain

strain. In the case of $\theta = 45^\circ$ with various ζ , the simulation results are shown in Fig. 9. Here, the anisotropic parameters F , G , H , N are constant.

To take a suitable parameter ζ in Fig. 9 that satisfies $\phi = -8.2^\circ$ with a pre-strain of 10%, it is necessary to minimize the error between the measured revolution angles (i.e., $\phi = -8.2^\circ$) and the numerical angles derived from model (24) with various ζ and the same strain of 10%. Figure 10 shows the square error depending on ζ .

As a result, we have $\zeta = 5.46$ that minimizes the square error. The r -values and Γ -values for this ζ are shown in Figs. 11 and 12, respectively. Comparing the curves of Fig. 11 with the approximate theoretical curves of Fig. 5, both curves almost correspond to each other. Therefore, the usefulness of this model is confirmed.

6. Conclusions

The purpose of this study has been to confirm that the change of strain path induces the rotation of the principal axes of plastic anisotropies and they do not behave like the material fiber printed on the surface of a specimen.

1. A measuring method of the revolution angles of

anisotropic principal axes has been proposed by using a laser speckle surface strain meter. From the experimental results, it has become clear that the revolution angle ϕ of the anisotropic principal axes is different from the same angle β of the material fiber, when the plastic strain path is changed.

2. A dynamical revolution model of the anisotropic principal axes has been proposed, and its parameters have been identified from the experimental results.

3. Comparing the experimental results measured by the laser speckle surface strain meter with the simulation results of the dynamical revolution model (24), both curves in Figs. 5 and 11 almost correspond to each other. Therefore, the usefulness of our model is confirmed.

References

- (1) Izawa, Y., Tanaka, R. and Ito, K., Construction of a Laser Speckle Surface Strain Meter for Identifying the Principal Axes of the Plastic Anisotropy, *Trans. Jpn. Soc. Mech. Eng.*, (in Japanese), Vol. 65, No. 637, A (1999), pp. 1977-1982.
- (2) Izawa, Y., Ito, K. and Sagawa, T., On the Revolution of Plastic Anisotropic Principal Axes Developed from the Change of Strain Path, *Trans. Jpn. Soc. Mech. Eng.*, (in Japanese), Vol. 67, No. 653, A (2001), pp. 156-161.
- (3) Asaro, R.J. and Rice, J.R., Strain Localization in Ductile Single Crystals, *J. Mech. Phys. Solids*, Vol. 25 (1977), pp. 309-338.
- (4) Aravas, N. and Aifantis, E.C., On the Geometry of Slip and Spin in Finite Plastic Deformation, *Int. J. Plasticity*, Vol. 7 (1991), pp. 141-160.
- (5) Yamaguchi, I., A Laser-Speckle Strain Gauge, *J. Phys. E: Sci. Instrum.*, Vol. 14 (1981), pp. 1270-1273.
- (1) Izawa, Y., Tanaka, R. and Ito, K., Construction of