The Effect of Mean Curvature on the Response and Collapse of Thin-Walled Tubes under Cyclic Bending*

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This paper presents the experimental and theoretical results of the effect of mean curvature on the response and collapse of thin-walled tubes subjected to cyclic bending. To highlight the influence of mean curvature effect, three different curvature ratios (minimum curvature/maximum curvature) were experimentally investigated in this study. It was found that the response and collapse of thin-walled tubes subjected to cyclic bending are strongly influenced by the magnitude of the mean curvature. In addition, the ordinary differential constitutive equations of endochronic theory were used to investigate the response of thin-walled tube subjected to cyclic bending. Furthermore, a theoretical formulation was proposed so that it can be used for simulating the relationship among the controlled curvature range, curvature ratio and the number of cycles to produce buckling for thin-walled tubes subjected to cyclic bending. The theoretical simulations were compared with the experimental tested data. Good agreement between the experimental and theoretical results has been achieved.

Key Words: Mean Curvature Effect, Thin-Walled Tube, Collapse, Cyclic Bending

1. Introduction

It has been known that industrial components or structures, which are made of thin-walled tubes, are constantly subjected to cyclic bending, such as structures in earthquake-prone area, various offshore pipelines and platforms in extreme weather conditions, transporting tubes of the heat exchanger for the power plant under cooling, etc. Bending of a thin-walled tube can lead to the ovalization of tube cross section. The reverse bending and subsequent repeated cyclic bending cause a gradual growth of ovalization. The growth of ovalization causes a progressive reduction of the tube's bending rigidity. Once a critical value of the ovalization is reached, the tube will buckle. Therefore, understanding the response and stability of thin-walled tubes subjected to bending is of importance in many industrial applications.

In recent years, Kyriakides and his co-workers have studied tubes subjected to monotonic or

cyclic bending with or without external pressure. Kyriakides and Shaw⁽¹⁾ investigated the response and stability of elastoplastic pipes under combined bending and external pressure. They determined the maximum moment and curvature as a function of material and geometric parameters for different pressures. Shaw and Kyriakides⁽²⁾ studied the inelastic behavior of 6061-T6 aluminum and 1018 steel tubes subjected to cyclic bending. They demonstrated that the reverse bending and subsequent repeated cyclic bending cause a gradual growth of the ovalization of the tube cross section. Kyriakides and Shaw⁽³⁾ extended the analysis of the aforementioned two metal tubes to the stability condition under cyclic bending. They discovered that under curvature-symmetric loading, the tube progressively ovalizes to a critical value at which it buckles. The ovalization's critical value was shown to be approximately equal to the value obtained just prior to buckling under monotonic bending. Corona and Kyriakides⁽⁴⁾ investigated the stability of 304 stainless steel tubes subjected to combined bending and external pressure. The curvature-pressure interaction collapse envelopes were generated for two different loading paths involving bending followed by pressure and pressure followed by bending. According to their investigation,

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the loading path strongly influences the shape of the envelopes of curvature-pressure interaction. Corona and Kyriakides⁽⁵⁾ studied the degradation and buckling of 6061-T6 aluminum and 1020 carbon steel tubes under cyclic bending and external pressure. They established the effect of the cyclic bending history and of the external pressure on the rate of accumulation of ovalization and the onset of instability. Kyriakides and Ju^{(6),(7)} studied experimentally and theoretically the bifurcation and localization instabilities on 6061-T6 aluminum tubes under pure bending. They discovered that the thinner shells develop short wavelength periodic ripples on the compressive side of the shell and buckle soon after the ripples appear. However, the thick shells exhibit a limit load instability as a direct consequence of the ovalization of the shell cross-section caused by bending. Corona and Vase⁽⁸⁾ studied the response, buckling and collapse of long, thin-walled seamless steel square tubes under bending. Rayleigh-Ritz type formulations based on the principle of virtual work were developed to predict the response and buckling of the tubes. The numerical results were in good agreement with experimental observations.

In recent years, Pan and his co-workers have investigated experimentally and theoretically the response and stability of circular tubes subjected to monotonic or cyclic bending. Pan and Leu⁽⁹⁾ used the endochronic theory to investigate the collapse of thin-walled tubes subjected bending. The experimental data of 6061-T6 aluminum and 1018 steel tubes under cyclic bending tested by Kyriakides and Shaw⁽²⁾ were compared with the theoretical simulation. Pan et al.⁽¹⁰⁾ designed a new measurement apparatus, which can be placed at the mid-span of the circular tube specimen and is suitable for simultaneous experimental determination of the tube curvature and ovalization of the tube cross-section. For testing the newly designed apparatus, the tube specimen of 304 stainless steel was cyclically bent. Pan and Fan⁽¹¹⁾ investigated the effect of prior curvature-rate at preloading stage on the subsequent creep (hold constant moment for a period of time) or relaxation (hold constant curvature for a period of time) behavior. Thin-walled tubes of SUS 304 stainless steel were used in this investigation. It has been found that the curvature-rate at the preloading stage has a strong influence on the subsequent creep or relaxation deformation under pure bending. Furthermore, Pan and Her⁽¹²⁾ used the aforementioned new measurement apparatus to investigate the response and stability of SUS 304 stainless steel tubes subjected to cyclic bending with different curvature-rates. They discovered that the higher the applied curvature-rate, the greater is the degree of

hardening of metal tube. However, the ovalization of tube cross-section increases when the applied curvature-rate increases. Pan and Hsu⁽¹³⁾ experimentally and theoretically studied the viscoplastic behavior of SUS 304 stainless steel tubes subjected cyclic bending. The endochronic viscoplastic theory, which was proposed by Pan and Chern⁽¹⁴⁾, was used to investigate the viscoplastic behavior of the tubes under cyclic bending. Lee et al.⁽¹⁵⁾ studied the influence of the diameter-to-thickness ratio on the response and stability of circular tubes subjected to symmetrical cyclic bending. To highlight the influence of the diameter-to-thickness ratio on the response and stability of circular tubes under symmetrical cyclic bending, the raw tubes were slightly machined on the outside surface to obtain the desired diameter-to-thickness ratio (D/t ratio). The empirical relationship, proposed by Kyriakides and Shaw⁽³⁾, was modified so that it can be used for simulating the relationship between the controlled curvature and the number of cycles to produce buckling of circular tubes with different D/t ratios. Good agreement between the experimental result and modified empirical relationship has been achieved.

Corona and Kyriakides⁽⁵⁾ were the first to experimentally investigate the response of circular tubes under curvature-controlled cyclic bending with a mean value of curvature. It was shown from the experimental moment-curvature curve that the 1020 carbon steel tube cyclically relaxes for first few cycles. However, the moment-curvature loop gradually become stable after a few cycles. For the ovalization-curvature curve, it was found that the ovalization increases in a ratchetting manner with the number of cycles. However, the ovalization-curvature curve has a bias in the direction of the value of the mean curvature. In addition, the rate at which ovalization accumulates was found not to be significantly affected by a mean curvature in the cycles. In this study, the response of the circular tubes under curvature-controlled cyclic bending with a mean curvature was not investigated theoretically. Further, the effect of mean curvature on the collapse of tubes subjected to cyclic bending was also not investigated. Therefore, in this paper, we propose further experimental and theoretical investigation of this topic.

For the experimental aspect, a four-point bending machine (facilities reported in previous studies (Pan et al.⁽¹⁰⁾; Pan and Fan⁽¹¹⁾; Pan and Her⁽¹²⁾; Pan and Hsu⁽¹³⁾) was used to conduct the cyclic bending tests. The material of the thin-walled tube chosen for this study was SUS 304 stainless steel. The curvature-ovalization measurement apparatus (COMA), designed and reported previously by Pan et al.⁽¹⁰⁾, was

used for conducting the curvature-controlled cyclic bending test. The magnitudes of curvature and ovalization of the tube cross-section are simultaneously measured by the COMA. The magnitude of bending moment is obtained from the two load cells, mounted in the bending device. The number of cycles to produce buckling was also recorded during the test. To highlight the influence of mean curvature effect, three different curvature ratios r (r = minimum curvature/maximum curvature), $r = -1$, -0.5 and 0 , were experimentally investigated in this study.

For the theoretical investigation, the first-order ordinary differential constitutive equations of the endochronic theory, which were first employed to investigate the collapse of thin-walled tubes under cyclic bending obtained by Pan and Leu⁽⁹⁾, were used to simulate the response of thin-walled tube subjected to cyclic bending with a mean value of curvature. The response contains the relationship among bending moment, curvature and ovalization of tube cross-section. Finally, a theoretical formulation was proposed so that it can be used for simulating the relationship among the controlled curvature range, curvature ratio and the number of cycles to produce buckling for thin-walled tubes subjected to cyclic bending with mean curvature. The theoretical simulation was compared with the experimental data tested generated in this investigation. Good agreement between the experimental and theoretical results was achieved.

2. Experiments

The cyclic bending experiments on thin-walled tubes were conducted in a test facility, designed by Pan and his co-workers, consisting of a pure bending device and a curvature-ovalization measurement apparatus (COMA). The bending device was used for conducting the cyclic bending tests and the COMA was used for measuring the variation of tube curvature and the ovalization of the tube cross-section.

2.1 Bending device

A schematic drawing of the bending device is shown in Fig. 1(a). It is designed as a four-point bending machine, capable of applying bending and reverse bending (similar to the bending device reported in several references (Shaw and Kyriakides⁽²⁾; Kyriakides and Shaw⁽³⁾; Corona and Kyriakides⁽⁴⁾; Corona and Kyriakides⁽⁵⁾; Kyriakides and Ju⁽⁶⁾). The device consists of two rotating sprockets resting on two support beams. Heavy chains run around the sprockets and are connected to two hydraulic cylinders and load cells forming a closed loop. The two sprockets rest on two heavy support beams 1.25 m apart. This allows a maximum length of the test specimen to be 1 m. The bending capacity of the

Fig. 1 (a) A schematic drawing of the bending device, (b) A schematic drawing of the COMA

machine is 5300 N-m. Each tube is tested and fitted with solid rod extension. The contact between tube and the rollers is free to movement along axial direction during bending. The load transfer to the test specimen is in the form of a couple formed by concentrated loads from two of the rollers. Once either the top or bottom cylinder is contracted, the sprockets are rotated, and pure bending of the test specimen is achieved. Reverse bending can be achieved by reversing the direction of flow in the hydraulic circuit. Detail description of the bending device can be found in references (Pan et al.⁽¹⁰⁾; Pan and Fan⁽¹¹⁾; Pan and Her⁽¹²⁾; Pan and Hsu⁽¹³⁾).

2.2 Curvature-ovalization measurement apparatus (COMA)

The curvature-ovalization measurement apparatus (COMA), which was designed by Pan et al.⁽¹⁰⁾, is an instrument which can be used to measure the tube curvature and ovalization of the tube cross-section. Figure 1(b) shows a schematic drawing of the COMA. It is a lightweight instrument which can be mounted close to the tube mid-span. There are three inclinometers in the COMA. Two of them are fixed on two holders, which are denoted as side-inclinometers (see Fig. 1(b)). The holders are fixed on the circular tube before the test begins. Based on the fixed distance between the two side-inclinometers and the angle changes detected by the two side-inclinometers,

Fig. 2 Geometry of SUS 304 stainless steel tube (all dimensions in mm)

the tube curvature can be obtained by simple calculation (Pan et al.⁽¹⁰⁾). In addition, by using the magnetic detector to measure the change of the outside diameter (middle part of the COMA), the ovalization of tube cross-section can be determined. Detailed description of the COMA can be found in the work by Pan et al.⁽¹⁰⁾.

2.3 Material and experimental procedure

The material used in this study is hot-rolled SUS 304 stainless steel thin-walled tube with the chemical composition of Cr 18.36, Ni 8.43, Mn 1.81, Si 0.39, C 0.05, P 0.28, S 0.04, and the remainder Fe. The yield stress is 205 MPa, the ultimate tensile stress is 520 MPa and the percent elongation is 35%. The tested specimens had a nominal outside diameter D of 38.1 mm and a wall thickness t of 1.5 mm ($D/t=25.4$) initially. To obtain the desired D/t ratio, the raw tested specimens were slightly machined on the outside surface. The D/t ratio was selected to be 60 in this study. Therefore, the outside diameter of the raw specimens $D=38.1$ mm was machined to obtain the outside diameter of 36.31 mm. However, the inside diameters were intact and had the same magnitude for all tested specimens (inside diameter=35.1 mm). Figure 2 shows the dimension of the tested specimen.

In this study, the response and collapse of 304 stainless steel circular tubes subjected to cyclic bending with different mean curvatures were investigated. The test was a curvature-controlled cyclic bending test with the curvature ranges from 0.8 m^{-1} to 0.15 m^{-1} . The effect of the mean curvature was demonstrated by the value of the curvature ratio r , which is defined by $r = \text{minimum curvature}/\text{maximum curvature}$. Three different values of r , $r = -1$, -0.5 and 0 , were experimentally investigated to highlight the influence of the mean curvature in this study. The magnitude of the bending moment was measured by two load cells mounted in the bending device. The magnitudes of the curvature and the ovalization of the tube cross-section were measured by the COMA. In addition, the number of cycles to produce buckling was also recorded during the experiment.

3. Theoretical Formulation for Elastoplastic Response

In this section, the formulations of the elastoplas-

Fig. 3 Problem geometry

tic response of a thin-walled tube subjected to cyclic bending, which was employed by Pan and Leu⁽⁹⁾, are used to investigate the response of thin-walled tube subjected to cyclic bending with a mean value of curvature. The kinematics of the tube cross section, the constitutive model for elastoplastic response and the principle of virtual work are briefly discussed separately as follows:

3.1 Kinematics

A long, circular, thin-walled tube under combined bending and external pressure is considered in this study. Figure 3 shows the problem geometry, in which M is the applied moment, κ is the resultant curvature, R is the mean radius, and t is the wall thickness. Based on axial, circumferential and radial coordinates x , θ and z , the displacements of a point of the tube's mid-surface are denoted as u , v and w , respectively. The plane sections are assumed here to be perpendicular to the tube's mid-surface before and during bending deformation. In addition, the strains are assumed to remain small; however, finite rotations in both axes under bending are allowed. The circumferential strain of the deformation is expressed as (Kyriakides and Shaw⁽¹⁾; Shaw and Kyriakides⁽²⁾; Kyriakides and Shaw⁽³⁾)

$$\epsilon_{\theta} = \epsilon_{\theta}^0 + z\kappa_{\theta} \quad (1)$$

where

$$\epsilon_{\theta}^0 = \left(\frac{v' + w}{R} \right) + \frac{1}{2} \left(\frac{v' + w}{R} \right)^2 + \frac{1}{2} \left(\frac{v - w'}{R} \right)^2 \quad (2)$$

and

$$\kappa_{\theta} = \left(\frac{v' - w''}{R^2} \right) / \sqrt{1 - \left(\frac{v - w'}{R} \right)^2} \quad (3)$$

The axial strain is expressed as

$$\varepsilon_x = \varepsilon_x^0 + \xi \kappa \quad (4)$$

and

$$\xi = [(R+w)\cos\theta - v\sin\theta + z\cos\theta] \quad (5)$$

where ε_x^0 is the axial strain of the cylinder's axis. Note that the amount of ε_x^0 is generally not zero due to the shifting of the neutral axis under bending.

3.2 Endochronic constitutive equations

In 1980, Valanis⁽¹⁶⁾ reformulated the definition of intrinsic time by the plastic strain tensor, the endochronic theory has been extended widely and successfully to simulate various material responses subjected to diverse loading histories⁽¹⁷⁾⁻⁽²¹⁾. The theory is adopted for simulating the elastoplastic response of a thin-walled tube under cyclic bending. Owing to the highly nonproportional path of the stress history, the incremental form of theory should be considered in formulating the problem.

According to the condition of small deformation for homogeneous and isotropic materials, the increment of deviatoric stress tensor $d\tilde{s}$ of endochronic theory is given as (Valanis⁽¹⁶⁾)

$$d\tilde{s} = 2\rho(0)d\tilde{e}^p + 2\tilde{h}(z)dz \quad (6)$$

and

$$\tilde{h}(z) = \int_0^z \frac{d\rho(z-z')}{dz} \frac{\partial \tilde{e}^p}{\partial z'} dz' \quad (7)$$

where z is the intrinsic time scale, $\rho(z)$ is termed the kernel function and $\tilde{e}^p$ is the deviatoric plastic strain tensor which is defined as

$$d\tilde{e}^p = d\tilde{e} - \frac{d\tilde{s}}{2\mu_0} \quad (8)$$

where $\tilde{e}$ denotes the deviatoric strain tensor, and μ_0 is the elastic shear modulus. The intrinsic time measure ξ is

$$d\xi = \|d\tilde{e}^p\| \text{ or } d\xi^2 = d\tilde{e}^p \cdot d\tilde{e}^p \quad (9.a, b)$$

where $\|\cdot\|$ is the Euclidean norm. The material function (or hardening function) $f(\xi)$ is

$$f(\xi) = \frac{d\xi}{dz} = 1 - Ce^{-\beta\xi}, \text{ for } C < 1 \quad (10)$$

in which C and β are material parameters. If the plastically incompressible is satisfied, the elastic hydrostatic response can be written as

$$d\sigma_{kk} = 3Kd\varepsilon_{kk} \quad (11)$$

where σ_{kk} and ε_{kk} are the trace of stress and strain tensors, respectively, and K is the elastic bulk modulus. According to the mathematical characteristic of the kernel function $\rho(z)$, which is a weakly singular function at the origin and integrable in the domain $0 \leq z \leq \infty$ (Valanis⁽¹⁶⁾), Murakami and Read⁽²²⁾ used a finite group of exponential decaying functions to form $\rho(z)$, i.e.,

$$\rho(z) = \sum_{i=1}^n C_i e^{-\alpha_i z}, \quad \rho(0) = \sum_{i=1}^n C_i \quad (12.a, b)$$

in which C_i , α_i are material parameters. Substituting Eq.(12.a) into Eqs.(6) and (7) yields

$$d\tilde{s} = \sum_{i=1}^n d\tilde{s}_i \quad (13)$$

where

$$d\tilde{s}_i = 2C_i d\tilde{e}^p - \alpha_i \tilde{s}_i dz, \quad i=1, 2, 3, \dots, n. \quad (14)$$

Substituting Eq.(8) into Eqs.(13) and (14) leads to

$$d\tilde{s} = \frac{\mu_0}{\mu_0 + \rho(0)} [2\rho(0)d\tilde{e} - \sum_{i=1}^n \alpha_i \tilde{s}_i dz]. \quad (15)$$

By using Eq.(11), Eq.(15) can be expressed in terms of the stress and strain tensors as

$$d\sigma = p_1 d\varepsilon + p_2 d\varepsilon_{kk} I + p_3 \sum_{i=1}^n \alpha_i (\sigma - \frac{\sigma_{kk}}{3} I)_i dz \quad (16)$$

where

$$p_1 = \frac{2\rho(0)}{1 + \frac{\rho(0)}{\mu_0}}, \quad p_2 = K - \frac{2\rho(0)}{3(1 + \frac{\rho(0)}{\mu_0})}, \quad (17.a, b)$$

$$p_3 = \frac{-1}{1 + \frac{\rho(0)}{\mu_0}}. \quad (17.c)$$

Detailed derivation of the constitutive equations for the case of biaxial loading can be found in Pan and Leu⁽⁹⁾.

3.3 Principle of virtual work

The principle of virtual work, which satisfies the equilibrium requirement, is given by

$$\int_V \sigma_{ij} \delta \varepsilon_{ij} dV = \delta W \quad (18)$$

where V is the volume of the material of the tube section considered, and δW is the virtual work of the external loads. For the case of thin-walled tube subjected to pure bending, the quantity of δW is equal to zero since the curvature is prescribed. Equation (18) is expressed for the incremental loading to be (Shaw and Kyriakides⁽²⁾)

$$\int_V (\sigma_{ij} + \dot{\sigma}_{ij}) \delta \dot{\varepsilon}_{ij} dV = 2R \int_0^\pi \int_{-t/2}^{t/2} [\bar{\sigma}_x \delta \dot{\varepsilon}_x + \bar{\sigma}_\theta \delta \dot{\varepsilon}_\theta] dT d\theta = 0. \quad (19)$$

where $\bar{\sigma}_x = \sigma + \dot{\sigma}_x$ and $\bar{\sigma}_\theta = \sigma + \dot{\sigma}_\theta$ and $(\dot{\cdot})$ denotes the increment of $(\cdot)$. The in-plane displacement v and w are assumed to be symmetrical and are approximated by the following expression (Shaw and Kyriakides⁽²⁾):

$$v \cong R \sum_{n=2}^N b_n \sin n\theta, \quad w \cong R \sum_{n=0}^N a_n \cos n\theta \quad (20.a, b)$$

where the number of terms N is chosen to ensure satisfactory convergence. Shaw and Kyriakides⁽²⁾ investigated the sensitivity of the moment-curvature and ovalization-curvature responses for monotonic pure bending to the number of expansion terms used in Eqs.(20.a) and (20.b). Those equations clearly indicate that $N=4$ or 6 is sufficient. By Substituting Eqs.(1) - (5), (20.a) and (20.b) into Eq.(19), a system of $2N+1$ non-linear algebraic equations in terms of $\dot{a}_0, \dot{a}_1, \dots, \dot{a}_N, \dot{b}_2, \dot{b}_3, \dots, \dot{b}_N, \varepsilon_x^0$ are determined.

This system of equations is solved using the Newton-Raphson method. The iterative scheme contains nested iterations for the constitutive relations. Shaw and Kyriakides⁽²⁾ provide a more detailed derivation of the system of equations for the loading cases of thin-walled tube under pure bending.

4. Results and Discussion

In this section, we compare the theoretical simulation of the elastoplastic response of SUS 304 stainless steel tubes with the experimental data. In our study, the kernel function of the endochronic theory was considered to be composed by three terms of exponentially decaying functions. Therefore, the material parameters of the theory can be determined according to the method proposed by Fan⁽²³⁾. The material parameters of endochronic theory for SUS 304 stainless steel are determined to be: $\mu_0 = 5.91 \times 10^4$ MPa, $K = 1.09 \times 10^5$ MPa, $C_1 = 7.0 \times 10^5$ MPa, $\alpha_1 = 9000$, $C_2 = 1.04 \times 10^4$ MPa, $\alpha_2 = 530$, $C_3 = 2.4 \times 10^3$ MPa, $\alpha_3 = 135$, $C = 0.15$ and $\beta = 20$. Finally, the collapse of SUS 304 stainless steel tubes subjected to cyclic bending with different mean curvature is also discussed. A new theoretical formulation was proposed for simulating the relationship among the controlled curvature range, curvature ratio and number of cycles to produce buckling.

4.1 Elastoplastic response of SUS 304 stainless steel tubes under cyclic bending with different values of mean curvature

To consider the effect of the mean curvature on the response of the SUS 304 stainless steel tube subjected to cyclic bending, three different curvature ratios r , $r = -1$, -0.5 and 0 , were investigated in this study. Figure 4(a) presents a typical experimental result of cyclic moment (M)-curvature (κ) curve for the SUS 304 stainless steel tube under symmetric cyclic bending. The magnitude of the curvature range ($\Delta\kappa$) is 0.3 m^{-1} and the curvature ratios is -1 . It is observed from Fig. 4(a) that the SUS 304 stainless steel tube cyclically hardens and gradually reaches steady state after a few cycles for symmetric cyclic bending. Figure 4(b) shows the simulated result obtained by the theoretical formulation described in Section 3. Figure 5(a) depicts the corresponding experimental ovalization of tube cross-section as a function of the applied curvature. The ovalization is defined by $\Delta D/D$ where D is the outside diameter and ΔD is the change in outside diameter. It can be noted that the ovalization of tube cross-section increases in a symmetric ratchetting manner with the number of cycles. As the cyclic process continues, the ovalization keeps accumulating. This phenomenon was also discovered by Kyriakides and Shaw⁽²⁾, who tested

Fig. 4 (a) A typical experimental result of moment (M)-curvature (κ) curve for $\Delta\kappa = 0.3 \text{ m}^{-1}$ and $r = -1$, (b) The theoretical simulated result

6061-T6 aluminum and 1018 steel tubes. Figure 5(b) shows the simulated $(\Delta D/D) - \kappa$ curve.

For the same amount of curvature range ($\Delta\kappa = 0.3 \text{ m}^{-1}$) as in Fig. 4(a), the cases of $r = -0.5$ (maximum curvature $= 0.2 \text{ m}^{-1}$, minimum curvature $= -0.1 \text{ m}^{-1}$) and $r = 0$ (maximum curvature $= 0.3 \text{ m}^{-1}$, minimum curvature $= 0 \text{ m}^{-1}$) are demonstrated in Figs. 6(a) and 7(a), respectively. It is observed in Figs. 6(a) and 7(a) that the experimental moment-curvature curve cyclically relaxes for first few cycle. But after a few cycles, the moment-curvature loop becomes steady. The aforementioned phenomenon of the moment-curvature curves can also be obtained by the theoretical simulation shown in Figs. 6(b) and 7(b), respectively. Figures 8(a) and 9(a) depict the corresponding experimental ovalization $(\Delta D/D) - \kappa$ curve for $r = -0.5$ and 0 , respectively. It can be seen that the ovalization of the tube cross-section increases in a ratchetting manner with the number of cycles. However, the curve is found to be

Fig. 5 (a) The corresponding experimental result of ovalization ($\Delta D/D$)-curvature (κ) curve for $\Delta\kappa=0.3 \text{ m}^{-1}$ and $r=-1$, (b) The theoretical simulated result

Fig. 6 (a) A typical experimental result of moment (M)-curvature (κ) curve for $\Delta\kappa=0.3 \text{ m}^{-1}$ and $r=-0.5$, (b) The theoretical simulated result

Fig. 7 (a) A typical experimental result of moment (M)-curvature (κ) curve for $\Delta\kappa=0.3 \text{ m}^{-1}$ and $r=0$, (b) The theoretical simulated result

Fig. 8 (a) The corresponding experimental result of ovalization ($\Delta D/D$)-curvature (κ) curve for $\Delta\kappa=0.3 \text{ m}^{-1}$ and $r=-0.5$, (b) The theoretical simulated result

Fig. 9 (a) The corresponding experimental result of ovalization ($\Delta D/D$)-curvature (κ) curve for $\Delta\kappa=0.3 \text{ m}^{-1}$ and $r=0$, (b) The theoretical simulated result

biased in the direction of the mean. The aforementioned phenomenon of the ovalization-curvature curves can also be obtained by the theoretical simulation shown in Figs. 8(b) and 9(b) for $r=-0.5$ and 0, respectively.

4.2 Collapse of SUS 304 stainless steel tubes under cyclic bending with different values of mean curvature

Figure 10 demonstrates the experimental result of the curvature range ($\Delta\kappa$) versus the number of cycles to produce buckling (N_b) curves with three different curvature ratios r of -1 , -0.5 and 0 , respectively. In these tests, the magnitudes of $\Delta\kappa$ vary from 0.15 to 0.8 m^{-1} . For each curvature ratio r , five specimens were carefully conducted for constructing the $\Delta\kappa$ - N_b curve. It is found that the collapse of circular tubes subjected to cyclic bending is strongly influenced by the value of mean curvature. Although SUS 304 stainless steel tubes were conducted with same magnitude of the curvature range, the higher

curvature ratio leads to shorter number of cycles to produce buckling. The same result of Fig. 10 plotted on log-log scale is shown in Fig. 11(a). Three straight dashed lines in this figure, determined by least-square method, denote three different amounts of curvature ratio.

Kyriakides and Shaw⁽³⁾ have proposed a formulation of the relationship between cyclic curvature κ_c (maximum curvature for symmetric cyclic bending) and number of cycles to buckling N_b to be

$$\frac{\kappa_c}{\kappa_m} \propto (N_b)^{-\alpha} \text{ or } \kappa_c = A(N_b)^{-\alpha} \quad (21.a, b)$$

where κ_m is the smaller amount of the curvature at limit moment or at bifurcation, and A and α are material parameters, which are related to the material properties and the ratio D/t . The constant A is the magnitude of cyclic curvature at $N_b=1$, and α is the slope on the log-log plot. By using the least-square method, the magnitudes of A and α for tubular specimens with $D=31.75 \text{ mm}$ and $t=0.889 \text{ mm}$

Fig. 10 The experimental result of curvature range ($\Delta\kappa$) versus the number of cycles to produce buckling (N_b) curves for different amounts of r

($D/t=35.7$), reported by Kyriakides and Shaw⁽³⁾, were calculated to be: $A=0.968\text{ m}^{-1}$ and $\alpha=0.078$ for 1018 steel tube and $A=0.988\text{ m}^{-1}$ and $\alpha=0.078$ or 0.12 for 6061-T6 aluminum tube.

Based on the formulation of the relationship between κ_c and N_b introduced by Kyriakides and Shaw⁽³⁾, a new formulation of the relationship among the controlled curvature range $\Delta\kappa$, curvature ratio and number of cycles to produce buckling N_b is proposed to be

$$\Delta\kappa = C(N_b)^{-\alpha} \quad (22)$$

where C and α are material parameters. Taking log on both sides of Eq.(22), one obtains

$$\log \Delta\kappa = \log C - \alpha \log N_b \quad (23)$$

In Fig. 11(a), three almost parallel straight dashed lines can be seen to correspond to three different values of r . Since the magnitude of α is the slope of the dashed line, the value of α can be treated as a constant for three different values of r . In our study, the magnitude of α is determined by using the case of $r=-1$. According to the trend of $\Delta\kappa$ - N_b curves on log-log scale, a simple formulation of C for considering the effect of mean curvature is proposed to be

$$C = ar + b \quad (24)$$

where a and b are material parameters. Based on the thin-walled tube of SUS 304 stainless steel with $D/t=60$ tested by us, the magnitudes of a , b and α are determined to be -0.358 m^{-1} , 0.436 m^{-1} and 0.1629 . The correlated results are shown in Fig. 11(b) in solid line.

5. Conclusions

The effect of mean curvature on the response and collapse of thin-walled tubes subjected to cyclic bending was investigated in this study. A bending machine and a curvature-ovalization measurement apparatus (COMA) were used for conducting the cyclic curvature-controlled bending tests. Based on the

Fig. 11 (a) The experimental result of curvature range ($\Delta\kappa$) versus the number of cycles to produce buckling (N_b) curves for different amounts of r , (b) The theoretical simulated result

experimental and theoretical results, the following important conclusions can be drawn from this investigation:

(1) The SUS 304 stainless steel tube cyclically hardens and gradually reaches steady state after a few cycles under symmetric cyclic bending ($r=-1$). However, the experimental moment-curvature curve cyclically relaxes for first few cycles and becomes steady after a few cycles under unsymmetric cyclic bending ($r=-0.5$ or 0). The ovalization-curvature curve varies in a resembling symmetric ratchetting manner with the number of cycles for the case of $r=-1$. However, the curves are found to be biased in the direction of the mean for the cases of $r=-0.5$ and 0 .

(2) The ordinary differential constitutive equations of endochronic theory and the principle of virtual work were used to investigate the elastoplastic response of SUS 304 stainless steel tube subjected to cyclic bending with different amounts of mean curvature. It can be seen that the theoretical simulations correlate well with the experimental results

(3) It is found from $\Delta\kappa$ vs. N_b curves with three different curvature ratios that the higher curvature

ratio leads to shorter number of cycles to produce buckling, if the thin-walled tubes were conducted with same magnitudes of the curvature range. In addition, three almost parallel straight lines can be seen if the $\Delta\kappa$ vs. N_b curves plot on log-log scale.

(4) A new formulation of the relationship among the controlled curvature range, curvature ratio and number of cycles to produce buckling for thin-walled tubes subjected to cyclic bending is proposed in this paper (Eqs. (22) and (24)). For thin-walled tube of SUS 304 stainless steel with $D/t=60$ tested by us, the magnitudes of a , b and α are determined to be -0.358 m^{-1} , 0.436 m^{-1} and 0.1629 . The simulated results are in good agreement with experimental observations.

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