Homogenized In-Plane Elastic-Viscoplastic Behavior of Long Fiber-Reinforced Laminates*

Tetsuya MATSUDA**, Nobutada OHNO***,
Hiroki TANAKA*** and Toshihiro SHIMIZU****

In this work, the homogenized elastic-viscoplastic behavior of long fiber-reinforced laminates under in-plane loading is predicted by taking into account the microscopic structure and stacking sequence of laminae. A homogenization theory of nonlinear time-dependent composites is applied to such laminates, leading to the macroscopic rate-type constitutive equation of laminates and the evolution equations of microscopic and average stresses in each lamina. The macroscopic constitutive equation is shown to have a stiffness tensor and a stress relaxation function which are evaluated explicitly in terms of the microscopic structure and stacking sequence of laminae. The established theory is then verified by performing in-plane uniaxial tensile tests of unidirectional, cross-ply, and quasi-isotropic carbon fiber/epoxy laminates. It is thus shown that the theory predicts successfully the anisotropic viscoplasticity of unidirectional and cross-ply laminates and the negligible viscoplasticity of quasi-isotropic laminates.

Key Words: Composite Material, Inelasticity, Anisotropy, Homogenization, CFRP, Laminate

1. Introduction

Fiber-reinforced laminated composites are now important engineering materials. A typical example of such composites is CFRP (carbon fiber-reinforced plastic) laminates. This type of composites consist of laminae, each of which is unidirectionally reinforced with long fibers. Macroscopic responses of laminated composites are usually predicted by attaining the overall responses of monolayers and then by averaging them in accordance with the stacking sequence of laminae. Appropriate methods such as the Mori-Tanaka⁽¹⁾ theory and the cell model of Aboudi⁽²⁾ can be employed for attaining the overall responses of

monolayers⁽³⁾⁻⁽⁶⁾. These models are based on approximate fields of microscopic stress and strain and are fairly successful, but may have errors especially when inelastic deformation occurs in constituents. It is, therefore, worthy developing a theory by which the inelastic behavior of fiber-reinforced laminates can be simulated accurately.

The present authors constructed a homogenization theory for nonlinear time-dependent composites with periodic microstructures^{(7),(8)}. This theory is based on unit cell problems, in which the so-called Y -periodicity of perturbed displacement is utilized as its boundary condition⁽⁹⁾⁻⁽¹²⁾. The theory analyzes not only the macroscopic elastic-viscoplastic behavior of composites but also the microscopic time-dependent distributions of stress and strain in unit cells. Ohno et al.⁽¹³⁾ further showed that if the microscopic distributions of stress and strain are symmetric with respect to the center of each unit cell, the field of perturbed velocity satisfies the point symmetry with respect to the cell boundary facet centers: Consequently, semiunit cells can be taken as the domain of analysis to reduce computation time. The homogenization theory was thus rebuilt using semiunit cells and

* Received 1st January 2002. Japanese original: Trans. Jpn. Soc. Mech. Eng., Vol. 68, No. 668, A (2002), pp. 543-549.

** Department of Micro System Engineering, Nagoya University, Chikusa-ku, Nagoya 464-8603, Japan

*** Department of Mechanical Engineering, Nagoya University, Chikusa-ku, Nagoya 464-8603, Japan

**** Department of Mechanical Engineering, Toyota College of Technology, 2-1 Eisei-cho, Toyota 471-8525, Japan. E-mail: ohno@mech.nagoya-u.ac.jp

applied for computing elastic-viscoplastic behavior of fiber-reinforced unidirectional composites.

The homogenization theory mentioned above does not need any approximation of microscopic stress and strain fields, if the Y -periodicity of microstructures can be assumed. We then expect that the theory accurately predicts the macroscopic elastic-viscoplastic behavior of fiber-reinforced laminates, if we can establish a framework to apply the theory to such laminates. In addition, we notice that the theory allows us to analyze the microscopic distributions of stress and strain in each lamina. These merits cannot be available if other theories are employed. It is, therefore, of significance to employ the homogenization theory for simulating the time-dependent non-linear behavior of fiber-reinforced laminates.

In this paper, the homogenization theory developed by the present authors is used for deriving a macroscopic constitutive equation applicable to the in-plane elastic-viscoplastic behavior of long fiber-reinforced laminates. The macroscopic equation is of the rate-type and is shown to be characterized by a stiffness tensor and a stress relaxation function which are evaluated in terms of the microscopic structure and stacking sequence of laminae. The established theory is then verified by performing in-plane uniaxial tensile tests of unidirectional, cross-ply, and quasi-isotropic carbon fiber/epoxy laminates. It is thus shown that the theory predicts successfully the anisotropic viscoplasticity of unidirectional and cross-ply laminates and the negligible viscoplasticity of quasi-isotropic laminates.

2. Theory

2.1 Basic assumptions

Let us consider a laminate in which long fiber-

reinforced laminae are stacked symmetrically as shown in Fig. 1. Let us assume that the fibers are arranged unidirectionally and periodically in each lamina, as illustrated in the figure, and also that the fibers deform elastically while the matrix exhibit elasticity and viscoplasticity. Let N and $f^{(\alpha)}$ be the number of laminae and the volume fraction of the α th lamina, respectively. It is convenient to use three kinds of Cartesian coordinates, i.e., X_i ($i=1, 2, 3$) for the laminate, $x_i^{(\alpha)}$ ($i=1, 2, 3$) for the α th lamina, and $y_i^{(\alpha)}$ ($i=1, 2, 3$) for the unit cell in the α th lamina, $Y^{(\alpha)}$, as shown in Fig. 1. The X_2 -axis is taken in the stacking direction. The $x_2^{(\alpha)}$ -axis is parallel to the X_2 -axis and thus directed perpendicularly to the lateral surface of the α th lamina. The $x_3^{(\alpha)}$ -axis is taken in the fiber direction of the α th lamina and makes an angle $\theta^{(\alpha)}$ with the X_3 -axis. The $y_i^{(\alpha)}$ -axis is parallel to the $x_i^{(\alpha)}$ -axis but is employed solely for the unit cell $Y^{(\alpha)}$.

Let us assume further that the laminate which is infinitely large in the X_1 - and X_3 -directions, is subject to in-plane loading, giving rise to no bending because of the symmetry in stacking. Then, the macroscopic stress in the laminate, Σ_{ij} , and the overall stresses in laminae, $\Sigma_{ij}^{(\alpha)}$, satisfy

$$\Sigma_{ij} = \sum_{\alpha=1}^N f^{(\alpha)} x \Sigma_{ij}^{(\alpha)}, \text{ in-plane components, } (1)$$

$$\Sigma_{22} = \Sigma_{21} = \Sigma_{23} = 0, (2)$$

$$x \Sigma_{22}^{(\alpha)} = x \Sigma_{21}^{(\alpha)} = x \Sigma_{23}^{(\alpha)} = 0, \alpha=1, 2, \dots, N, (3)$$

where $x(\)$ stands for the components with respect to the X_i coordinate system. The macroscopic strain in the laminate, E_{ij} , and the overall strains in laminae, $E_{ij}^{(\alpha)}$, can be expressed as

$$E_{11} = x E_{11}^{(\alpha)}, E_{33} = x E_{33}^{(\alpha)}, E_{13} = x E_{13}^{(\alpha)}, \alpha=1, 2, \dots, N, (4)$$

$$E_{22} = \sum_{\alpha=1}^N f^{(\alpha)} x E_{22}^{(\alpha)}, (5)$$

Fig. 1 Structure of a laminate and three kinds of Cartesian coordinates

$$E_{21}=E_{23}=0, \quad (6)$$

$${}_xE_{21}^{(a)} = {}_xE_{23}^{(a)} = 0, \alpha=1, 2, \dots, N. \quad (7)$$

Equations (3) and (7) also hold with respect to the $x_i^{(a)}$ coordinate system:

$${}_x\Sigma_{22}^{(a)} = {}_x\Sigma_{21}^{(a)} = {}_x\Sigma_{23}^{(a)} = 0, \alpha=1, 2, \dots, N, \quad (8)$$

$${}_xE_{21}^{(a)} = {}_xE_{23}^{(a)} = 0, \alpha=1, 2, \dots, N. \quad (9)$$

2.2 Homogenization in laminae

Let us denote the microscopic distributions of stress and strain in the unit cell $Y^{(a)}$ of the α th lamina as $\sigma_{ij}^{(a)}(\mathbf{y}, t)$ and $\varepsilon_{ij}^{(a)}(\mathbf{y}, t)$, where $\mathbf{y}$ and t denote y_i and time, respectively. Then, the overall stress and strain in the α th lamina, ${}_x\Sigma_{ij}^{(a)}$ and ${}_xE_{ij}^{(a)}$, are evaluated by homogenizing $\sigma_{ij}^{(a)}(\mathbf{y}, t)$ and $\varepsilon_{ij}^{(a)}(\mathbf{y}, t)$ using a volume average

$$\langle \# \rangle = \frac{1}{|Y^{(a)}|} \int_{Y^{(a)}} \# dY^{(a)}, \quad (10)$$

where $|Y^{(a)}|$ indicates the volume of $Y^{(a)}$. Thus,

$${}_x\Sigma_{ij}^{(a)} = \langle \sigma_{ij}^{(a)} \rangle, \quad (11)$$

$${}_xE_{ij}^{(a)} = \langle \varepsilon_{ij}^{(a)} \rangle. \quad (12)$$

We assume that the microscopic stress $\sigma_{ij}^{(a)}$ and strain $\varepsilon_{ij}^{(a)}$ are related by a constitutive relation

$$\dot{\sigma}_{ij}^{(a)} = c_{ijkl}^{(a)} [\dot{\varepsilon}_{kl}^{(a)} - \beta_{kl}^{(a)}], \quad (13)$$

where $(\dot{\cdot})$ denotes the differentiation with respect to t , and $c_{ijkl}^{(a)}$ and $\beta_{kl}^{(a)}$ indicate elastic stiffness and viscoplastic rate satisfying $c_{ijkl}^{(a)} = c_{jikl}^{(a)} = c_{ijlk}^{(a)} = c_{klij}^{(a)}$ and $\beta_{kl}^{(a)} = \beta_{lk}^{(a)}$, respectively. It is noted that $c_{ijkl}^{(a)}$ and $\beta_{kl}^{(a)}$ change from constituent to constituent in each lamina, and that $\beta_{kl}^{(a)}$ vanishes in the fibers.

Then, we can show the following relations which satisfy all fundamental equations such as the equilibrium equation of stress, the relation between displacement and strain and the constitutive relation^{(7),(8)}:

$$\dot{\sigma}_{ij}^{(a)}(\mathbf{y}, t) = a_{ijkl}^{(a)}(\mathbf{y}, t) \dot{\varepsilon}_{kl}^{(a)}(t) - r_{ij}^{(a)}(\mathbf{y}, t), \quad (14)$$

$${}_x\Sigma_{ij}^{(a)} = \langle a_{ijkl}^{(a)} \rangle {}_xE_{kl}^{(a)} - \langle r_{ij}^{(a)} \rangle, \quad (15)$$

where

$$a_{ijkl}^{(a)} = c_{ijpq}^{(a)} (\delta_{pk}\delta_{ql} + \chi_{p,q}^{kl(a)}), \quad (16)$$

$$r_{ij}^{(a)} = c_{ijkl}^{(a)} (\beta_{kl}^{(a)} + \varphi_{kl}^{(a)}). \quad (17)$$

Here, δ_{ij} denotes Kronecker's delta, $(\cdot)_{,i}$ indicates the differentiation with respect to y_i , and $\chi_{p,q}^{kl(a)}$ and $\varphi_{kl}^{(a)}$ are the functions to be determined by solving boundary value problems

$$\int_{Y^{(a)}} c_{ijpq}^{(a)} \chi_{p,q}^{kl(a)} v_{i,j}^{(a)} dY^{(a)} = - \int_{Y^{(a)}} c_{ijkl}^{(a)} v_{i,j}^{(a)} dY^{(a)}, \quad (18)$$

$$\int_{Y^{(a)}} c_{ijpq}^{(a)} \varphi_{p,q}^{(a)} v_{i,j}^{(a)} dY^{(a)} = - \int_{Y^{(a)}} c_{ijkl}^{(a)} \beta_{kl}^{(a)} v_{i,j}^{(a)} dY^{(a)}, \quad (19)$$

where $v_i^{(a)}$ signifies any Y -periodic velocity field defined in $Y^{(a)}$ at t .

2.3 In-plane elastic-viscoplastic constitutive equation of laminates

Solving Eq. (15) for ${}_x\Sigma_{ij}^{(a)}$ and transforming the resulting equation to a matrix form gives:

$$\{ {}_xE_{ij}^{(a)} \}_{6 \times 1} = [B^{(a)}]_{6 \times 6} \{ {}_x\Sigma_{ij}^{(a)} \}_{6 \times 1} + \{ C^{(a)} \}_{6 \times 1}. \quad (20)$$

Then, Eqs. (8) and (9) allow the above equation to be reduced to

$$\{ {}_xE_{ij}^{(a)} \}_{3 \times 1} = [\bar{B}^{(a)}]_{3 \times 3} \{ {}_x\Sigma_{ij}^{(a)} \}_{3 \times 1} + \{ \bar{C}^{(a)} \}_{3 \times 1}, \quad (21)$$

$${}_xE_{22}^{(a)} = B_{21}^{(a)} {}_x\Sigma_{11}^{(a)} + B_{23}^{(a)} {}_x\Sigma_{33}^{(a)} + B_{26}^{(a)} {}_x\Sigma_{31}^{(a)} + C_{22}^{(a)}, \quad (22)$$

where $(\bar{\cdot})$ stands for the in-plane parts, i.e.,

$$\{ {}_x\Sigma_{ij}^{(a)} \} = \{ {}_x\Sigma_{11}^{(a)} \quad {}_x\Sigma_{33}^{(a)} \quad {}_x\Sigma_{31}^{(a)} \}^T, \quad (23)$$

$$\{ {}_xE_{ij}^{(a)} \} = \{ {}_xE_{11}^{(a)} \quad {}_xE_{33}^{(a)} \quad {}_xE_{31}^{(a)} \}^T, \quad (24)$$

and so on. Here $(\cdot)^T$ denotes the transpose.

Now, let us introduce further the in-plane vectors of ${}_x\Sigma_{ij}^{(a)}$ and ${}_xE_{ij}^{(a)}$, i.e.,

$$\{ {}_x\Sigma_{ij}^{(a)} \} = \{ {}_x\Sigma_{11}^{(a)} \quad {}_x\Sigma_{33}^{(a)} \quad {}_x\Sigma_{31}^{(a)} \}^T, \quad (25)$$

$$\{ {}_xE_{ij}^{(a)} \} = \{ {}_xE_{11}^{(a)} \quad {}_xE_{33}^{(a)} \quad {}_xE_{31}^{(a)} \}^T. \quad (26)$$

Then, since the $x_1^{(a)}$ - and $x_3^{(a)}$ -axes make respectively an angle $\theta^{(a)}$ with the X_1 - and X_3 -axes (see Fig. 1), we have

$$\{ {}_x\Sigma_{ij}^{(a)} \} = [P^{(a)}] \{ {}_x\Sigma_{ij}^{(a)} \}, \quad (27)$$

$$\{ {}_xE_{ij}^{(a)} \} = [Q^{(a)}] \{ {}_xE_{ij}^{(a)} \}, \quad (28)$$

where

$$[P^{(a)}] = \begin{bmatrix} \cos^2 \theta^{(a)} & \sin^2 \theta^{(a)} \\ \sin^2 \theta^{(a)} & \cos^2 \theta^{(a)} \\ -\cos \theta^{(a)} \sin \theta^{(a)} & \cos \theta^{(a)} \sin \theta^{(a)} \end{bmatrix} \quad (29)$$

$$[Q^{(a)}] = \begin{bmatrix} \cos^2 \theta^{(a)} & \sin^2 \theta^{(a)} \\ \sin^2 \theta^{(a)} & \cos^2 \theta^{(a)} \\ -2 \cos \theta^{(a)} \sin \theta^{(a)} & 2 \cos \theta^{(a)} \sin \theta^{(a)} \end{bmatrix} \quad (30)$$

Substituting Eqs. (27) and (28) into (21) yields

$$\{ {}_x\Sigma_{ij}^{(a)} \} = [P^{(a)}]^{-1} [\bar{B}^{(a)}]^{-1} [Q^{(a)}] \{ {}_xE_{ij}^{(a)} \} - [P^{(a)}]^{-1} [\bar{B}^{(a)}]^{-1} \{ \bar{C}^{(a)} \}. \quad (31)$$

Finally, substituting Eqs. (31) and (4) into Eq. (1), we obtain a macroscopic in-plane constitutive equation

$$\{ \bar{\Sigma} \} = [\bar{A}] \{ \bar{E} \} - \{ \bar{R} \}, \quad (32)$$

where

$$[\bar{A}] = \sum_{a=1}^N f^{(a)} [P^{(a)}]^{-1} [\bar{B}^{(a)}]^{-1} [Q^{(a)}], \quad (33)$$

$$\{ \bar{R} \} = \sum_{a=1}^N f^{(a)} [P^{(a)}]^{-1} [\bar{B}^{(a)}]^{-1} \{ \bar{C}^{(a)} \}. \quad (34)$$

It is noted that $[\bar{A}]$ and $\{ \bar{R} \}$ in the macroscopic constitutive relation (32) depend on $f^{(a)}$, $\theta^{(a)}$ and $[\bar{B}^{(a)}]$, which are evaluated in terms of the stacking sequence and microscopic structure of laminae.

2.4 Computation

Let us suppose that the history of either Σ_{ij} or E_{ij} , or a combination of them, is prescribed, and that the values of ${}_x\Sigma_{ij}^{(a)}$ and the distributions of $\sigma_{ij}^{(a)}$ and internal variables in $Y^{(a)}$ are known in all laminae at a current time t . Then, the increments in an interval

from t to $t + \Delta t$ are computed as follows :

(1) The boundary value problems (18) and (19) are solved using FEM to determine $\chi_i^{kl(a)}(\mathbf{y}, t)$ and $\varphi_i^{(a)}(\mathbf{y}, t)$, where $\alpha=1, 2, \dots, N$.

(2) Compute $\langle a_{ijkl}^{(a)} \rangle$ and $\langle r_{ij}^{(a)} \rangle$, leading to $[\bar{B}^{(a)}]$ and $\{\bar{C}^{(a)}\}$ in Eq.(21) and further to $[\bar{A}]$ and $\{\bar{R}\}$ using Eqs.(33) and (34).

(3) Determine the unknown components of $\{\bar{\Sigma}\}$ and $\{\bar{E}\}$ using Eq.(32) with the prescribed components of $\{\bar{\Sigma}\}$ and $\{\bar{E}\}$.

(4) Using Eqs.(27), (28) and (31) with (4), determine $\{x\bar{\Sigma}^{(a)}\}$ and $\{x\bar{E}^{(a)}\}$.

(5) From Eqs.(22) and (14), evaluate $x\dot{E}_{22}^{(a)}$ and $\{\dot{\sigma}^{(a)}\}$.

(6) Calculate the increments in the interval from t to $t + \Delta t$ as $\Delta \Sigma_{ij} = \dot{\Sigma}_{ij} \Delta t$, $\Delta \sigma_{ij}^{(a)} = \dot{\sigma}_{ij}^{(a)} \Delta t$, etc., and add the increments to the current values.

3. Experimental Procedure

To verify the theory described in the preceding section, uniaxial tensile tests at constant strain rates were performed at room temperature using coupon specimens cut out from TR30/#340 carbon fiber/epoxy plates of 300×300 mm, manufactured by Mitsubishi Rayon Co. Ltd. The plates were laminates of three kinds, i.e., unidirectional, cross-ply, and quasi-isotropic laminates with the lay-ups of $[0]_{12}$, $[0/90]_{4s}$ and $[0/\pm 60]_{3s}$, which had 1.5, 2.0 and 2.25 mm in thickness, respectively. The subscript s indicates symmetric stacking. The volume fraction of fibers, V_f , was 56 percent in each ply. The coupon specimens with rectangular GFRP tabs were prepared as shown in Fig. 2; Ψ indicates the angle between the longitudinal direction of specimens and the fiber direction of 0° -plies. From now on, Ψ will be referred to as the off-axis angle. The values of Ψ tested in the present work are listed in Table 1. A closed-loop servohydraulic testing machine with a load/strain computer controller was used for the tests. The longitudinal strain of each specimen, which was detected by two 5 mm long strain gauges adhered on both its sides, was controlled to increase at a constant rate.

4. Material Assumptions

The theory described in Section 2 was applied to the experiments on the basis of the material assump-

Table 1 Off-axis angle Ψ for unidirectional, cross-ply, and quasi-isotropic laminates

Laminate	Ψ (degree)
Unidirectional	0, 10, 20, 30, 45, 60, 90
Cross-ply	0, 10, 20, 30, 45
Quasi-isotropic	0, 10, 20, 30

tions stated in this section.

4.1 Fiber arrangement

The arrangement of carbon fibers, which was unidirectional in the $x_3^{(a)}$ direction, was modeled to be hexagonally periodic on the $x_1^{(a)}-x_2^{(a)}$ plane in each lamina, as illustrated in Figs. 1 and 3(a). This is an idealization of the random distribution of fibers on that plane. The idealization can be justified as follows: The hexagonal periodicity of carbon fiber arrangement gives rise to the transverse quasi-isotropy not only in elasticity but also in elastoviscoplasticity^{(7),(8)}. Such isotropy matches with the transverse isotropy brought about by the random distribution of fibers on the $x_1^{(a)}-x_2^{(a)}$ plane. The hexagonal periodicity is therefore an appropriate assumption as far as the overall response of each lamina is concerned.

According to the above assumption, the unit cell $Y^{(a)}$ was chosen to be hexagonal for all laminae, as shown in Fig. 3(b); V_f was taken to be 56 percent. Here it is noted that $Y^{(a)}$ was taken to be 2D rather than 3D since each lamina was assumed to have no microscopic variation in the fiber direction^{(8),(14)}. Since the hexagonal unit cell has the point symmetry with respect to the cell center, it is sufficient to consider half of the unit cell as the domain of analysis for solving the boundary value problems (18) and (19)⁽¹³⁾. Hence, the upper half of the unit cell was considered and was divided into finite elements using four-noded isoparametric elements, as shown in Fig. 3(b).

4.2 Microscopic constitutive equations

The carbon fibers were regarded as a transversely isotropic elastic material. Consequently, the fibers were supposed to have five independent elastic con-

Fig. 2 Geometry of specimens

(a)

(b)

Fig. 3 Microstructure of laminae; (a) hexagonal arrangement of fibers, and (b) unit cell $Y^{(a)}$ and finite element mesh

stants, i.e., two Young's moduli E_{f1} and E_{f3} , two Poisson's ratios ν_{f12} and ν_{f31} , and one shear rigidity G_{f31} , where the subscripts 1, 2 and 3 signify the $y_1^{(a)}$, $y_2^{(a)}$ and $y_3^{(a)}$ directions, respectively. Table 2 shows the five constants of fibers employed in the present work; E_{f3} was provided by the manufacturer, Mitsubishi Rayon Co. Ltd., while other constants were obtained by referring to Kriz and Stinchcomb⁽¹⁵⁾.

The epoxy matrix, on the other hand, was regarded as an elastic-viscoplastic material characterized as

$$\dot{\epsilon}_{ij} = \frac{1+\nu_m}{E_m} \dot{\sigma}_{ij} - \frac{\nu_m}{E_m} \dot{\sigma}_{kk} \delta_{ij} + \frac{3}{2} \dot{\epsilon}_0^p \left[\frac{\sigma_e}{g(\bar{\epsilon}^p)} \right]^n \frac{s_{ij}}{\sigma_e}, \quad (35)$$

where E_m , ν_m and n are material constants, $g(\bar{\epsilon}^p)$ is a material function depending on accumulated viscoplastic strain $\bar{\epsilon}^p = \int [(2/3) \beta_{ij}^{(a)} \beta_{ij}^{(a)}]^{1/2} dt$, $\dot{\epsilon}_0^p$ is a reference strain rate, s_{ij} indicates the deviatoric part of σ_{ij} , and $\sigma_e = [(3/2) s_{ij} s_{ij}]^{1/2}$. The material constants and material function in Eq.(35) were determined by simulating the 45° off-axis tensile tests of the unidirectional and cross-ply laminates at $\dot{\epsilon}_{33} = 10^{-3}, 10^{-5}$ and 10^{-7} s^{-1} (see Figs. 4 and 5). This was because the effect of matrix viscoplasticity was expected to be significant in such off-axis tests. Table 2 lists the material parameters determined. It is noted that Eq.

Table 2 Material constants

Carbon fiber	$E_{f1} = 1.55 \times 10^4$	$\nu_{f12} = 0.49$
	$E_{f3} = 2.40 \times 10^5$	$\nu_{f31} = 0.28$
	$G_{f31} = 2.47 \times 10^4$	
Epoxy	$E_m = 3.5 \times 10^3$	$\nu_m = 0.35$
	$\dot{\epsilon}_0^p = 10^{-5}$	$n = 35$
	$g(\bar{\epsilon}^p) = 141.8(\bar{\epsilon}^p)^{0.165} + 10$	

MPa (stress), mm/mm (strain), s (time).

Fig. 4 Macroscopic tensile curves of unidirectional laminate with 45° off-axis angle at constant strain rates

Fig. 5 Macroscopic tensile curves of cross-ply laminate with 45° off-axis angle at constant strain rates

(35), which is based on the assumption of isotropic hardening, can be valid as far as monotonic loading is concerned.

5. Comparison of Experiments and Predictions

Uniaxial tensile experiments were done at $\dot{\epsilon}_{33} = 10^{-5} \text{ s}^{-1}$ at room temperature using the coupon specimens of unidirectional, cross-ply and quasi-isotropic laminates with the off-axis angles Ψ listed in Table 1. The experiments were predicted by means of the present theory. The experiments and the predictions

Fig. 6 Macroscopic stress versus strain relations of unidirectional laminate at $\dot{E}_{33}=10^{-5}\text{s}^{-1}$

are compared with each other in this section.

Figure 6 deals with the unidirectional laminate $[0]_{12}$. As seen from the figure, significant dependence on the off-axis angle Ψ was observed in the experiments: When $\Psi=0^\circ$, the macroscopic tensile behavior was almost linear, suggesting that the fibers governed the macroscopic behavior. However, a little deviation of Ψ from 0° , such as 10° and 20° , caused noticeable nonlinearity due to the influence of matrix viscoplasticity, whereas the increase of Ψ beyond 45° gave negligible influence on the macroscopic behavior. This significant dependence on Ψ was predicted very well using the present theory, as depicted in Fig. 6. For all the values of Ψ examined, the predictions agreed accurately the experiments. We, therefore, can say that the present theory is successful in describing the in-plane elastic-viscoplastic anisotropy of long fiber-reinforced unidirectional composites.

Figure 7 compares the experimental results and predictions attained for the cross-ply laminate $[0/90]_{4s}$. It is seen from the figure that almost the same amount of in-plane anisotropy was observed as in the unidirectional laminate. This implies that the 0° - and 90° -plies in the cross-ply laminate interact very weakly with each other, resulting in a similarity to the macroscopic deformation behavior of the unidirectional laminate. As seen from the figure, the in-plane anisotropy of the cross-ply laminate was also predicted well using the present theory, though the prediction and experiment in the case of $\Psi=10^\circ$ had a little difference, attributable to the microscopic damages such as matrix cracking and interfacial debonding^{(6),(16)}. The comparison shown in Fig. 7 demonstrates that the present theory is applicable to cross-ply laminates as successfully as unidirectional laminates.

The experiments and predictions done for the quasi-isotropic laminate $[0/\pm 60]_{3s}$ are shown in

Fig. 7 Macroscopic stress versus strain relations of cross-ply laminate at $\dot{E}_{33}=10^{-5}\text{s}^{-1}$

Fig. 8 Macroscopic stress versus strain relations of quasi-isotropic laminate at $\dot{E}_{33}=10^{-5}\text{s}^{-1}$

Fig. 8. The quasi-isotropic laminate deformed almost linearly until macroscopic fracture occurred in the range of $\Sigma_{33}=500$ and $\Sigma_{33}=600$ MPa. The fracture stress was much higher than the viscoplastic flow stresses of the unidirectional and cross-ply laminates of $\Psi=45^\circ$, which were about 100 MPa. These results imply that matrix viscoplasticity did not give any noticeable influence on the macroscopic behavior of the quasi-isotropic laminate. It is also seen in the figure that the quasi-isotropic laminate exhibited negligible dependence on Ψ , i.e., in-plane isotropy. Let us discuss the validity of the present theory by comparing the predictions and the experiments. The predictions for $\Psi=0, 10, 20$ and 30° , which are given in Fig. 8, are very slightly nonlinear and almost completely identical to one another. It is seen from the figure that the predictions agree well with the experiments. The theory is thus verified and is successful in describing the negligible viscoplasticity and the in-plane isotropy observed for the quasi-isotropic laminate.

6. Conclusions

In this study, the homogenization theory of non-linear time-dependent composites developed by the present authors was employed for predicting the in-plane elastic-viscoplastic behavior of long fiber-reinforced laminates. We thus derived a macroscopic rate-type constitutive equation of laminates as well as evolution equations of microscopic and average stresses in each lamina. The macroscopic constitutive equation was shown to have a stiffness tensor and a stress relaxation function to be evaluated in terms of the microscopic structure and stacking sequence of laminae. To verify the present theory, in-plane tensile tests were performed at room temperature using coupon specimens of unidirectional, cross-ply, and quasi-isotropic carbon fiber/epoxy laminates with several off-axis angles. The arrangement of carbon fibers in each lamina was modeled using a hexagonal unit cell, and the material parameters of the matrix material were determined on the basis of 45° off-axis tensile experiments of the unidirectional and cross-ply laminates. The present theory then predicted very accurately the macroscopic characteristics observed such as the anisotropic viscoplasticity of unidirectional and cross-ply laminates and the negligible viscoplasticity of quasi-isotropic laminates.

Acknowledgements

The authors are grateful to Prof. K. Tohgo of Shizuoka University for his helpful comments in performing the present work.

References

- (1) Mori, T. and Tanaka, K., Average Stress in Matrix and Average Elastic Energy of Materials with Misfitting Inclusions, *Acta Metall.*, Vol. 21 (1973), pp. 571-574.
- (2) Aboudi, J., *Mechanics of Composite Materials*, (1991), Elsevier Science Publishers, Amsterdam, The Netherlands.
- (3) Pindera, M.J. and Lin, M.W., Micromechanical Analysis of the Elastoplastic Response of Metal Matrix Composites, *ASME J. Press. Vessel Technol.*, Vol. 111 (1989), pp. 183-190.
- (4) Tszeng, T.C., Micromechanics Characterization of Unidirectional Composites during Multiaxial Plastic Deformation, *J. Compos. Mater.*, Vol. 28 (1994), pp. 800-820.
- (5) Kawai, M., Masuko, Y., Kawase, Y. and Negishi, R., Micromechanical Analysis of the Off-Axis Rate-Dependent Inelastic Behavior of Unidirectional AS4/PEEK at High Temperature, *Int. J. Mech. Sci.*, Vol. 43 (2001), pp. 2069-2090.
- (6) Tohgo, K., Kawahara, K. and Sugiyama, Y., Off-Axis Tensile Properties of CFRP Laminates and Non-linear Lamination Theory Based on Micromechanics Approach, *Trans. Jpn. Soc. Mech. Eng.*, (in Japanese), Vol. 67, No. 661, A(2001), pp. 1493-1500.
- (7) Wu, X. and Ohno, N., A Homogenization Theory for Time-Dependent Nonlinear Composites with Periodic Internal Structures, *Int. J. Solids Struct.*, Vol. 36 (1999), pp. 4991-5012.
- (8) Ohno, N., Wu, X. and Matsuda, T., Homogenized Properties of Elastic-Viscoplastic Composites with Periodic Internal Structures, *Int. J. Mech. Sci.*, Vol. 42 (2000), pp. 1519-1536.
- (9) Bensoussan, A., Lions, J.-L. and Papanicolaou, G., *Asymptotic Analysis for Periodic Structures*, (1978), North-Holland Publishing Company, Amsterdam, The Netherlands.
- (10) Sanchez-Palencia, E., *Non-Homogeneous Media and Vibration Theory*, Lecture Notes in Physics 127, (1980), Springer-Verlag, Berlin, Germany.
- (11) Bakhvalov, N. and Panasenko, G., *Homogenisation: Averaging Processes in Periodic Media*, (1984), Kluwer Academic Publishers, Dordrecht, The Netherlands.
- (12) Guedes, J.M. and Kikuchi, N., Preprocessing and Postprocessing for Materials Based on the Homogenization Method with Adaptive Finite Element Methods, *Comput. Methods Appl. Mech. Eng.*, Vol. 83 (1990), pp. 143-198.
- (13) Ohno, N., Matsuda, T. and Wu, X., A Homogenization Theory for Elastic-Viscoplastic Composites with Point Symmetry of Internal Distributions, *Int. J. Solids Struct.*, Vol. 38 (2001), pp. 2867-2878.
- (14) Noguchi, H. and Shimizu, E., Study on Fracture Mode Transition of Unidirectional CFRP by Using Homogenization Method (1st Report, Formulation), *Trans. Jpn. Soc. Mech. Eng.*, (in Japanese), Vol. 65, No. 630, A(1999), pp. 225-231.
- (15) Kriz, R.D. and Stinchcomb, W.W., Elastic Moduli of Transversely Isotropic Graphite Fibers and Their Composites, *Exper. Mech.*, Vol. 19 (1979), pp. 41-49.
- (16) Tohgo, K., Akizuki, K. and Sugiyama, Y., Ply Cracking Damage Theory and Damage Behavior in CFRP Cross-ply Laminates, *Trans. Jpn. Soc. Mech. Eng.*, (in Japanese), Vol. 64, No. 621, A (1998), pp. 1160-1167.