A Crack Growth Evaluation Method for Interacting Multiple Cracks*

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When stress corrosion cracking or corrosion fatigue occurs, multiple cracks are frequently initiated in the same area. According to section XI of the ASME Boiler and Pressure Vessel Code, multiple cracks are considered as a single combined crack in crack growth analysis, if the specified conditions are satisfied. In crack growth processes, however, no prescription for the interference between multiple cracks is given in this code. The JSME Post-Construction Code, issued in May 2000, prescribes the conditions of crack coalescence in the crack growth process. This study aimed to extend this prescription to more general cases. A simulation model was applied, to simulate the crack growth process, taking into account the interference between two cracks. This model made it possible to analyze multiple crack growth behaviors for many cases (e.g. different relative position and length) that could not be studied by experiment only. Based on these analyses, a new crack growth analysis method was suggested for taking into account the interference between multiple cracks.

Key Words: Body Force Method, Crack Propagation, Life Prediction, Multiple Cracks, Maintenance Code

1. Introduction

It is well known that the interaction between multiple cracks influences the acceleration of crack growth caused by fatigue, stress corrosion cracking and so on⁽¹⁾⁻⁽⁷⁾. The author previously reported, through evaluations with a crack growth simulation considering the interaction between distributing multiple cracks by the finite element method, that the increase in the number of cracks accelerates crack propagation and shortens the use limit⁽⁸⁾. Therefore, when the lifetime of plant components is estimated by a crack growth prediction, it is necessary to consider the interaction between multiple cracks for a conservative evaluation. Section XI of the ASME Boiler and Pressure Vessel Code⁽⁹⁾ (Sec. XI) and the JSME Fitness-For-Service Code⁽¹⁰⁾ (JSME Code), which spec-

ify evaluation methods for cracks (flaws) detected at in-service inspection, provide flaw evaluation procedures in consideration of the interaction between multiple cracks.

In these codes, multiple cracks are replaced with an equivalent single crack in the stage of specifying the size of the detected cracks. In addition, in JSME Code, there is a specification in the stage of crack growth evaluation. If the initial position of cracks, which is defined by Fig. 1, satisfies the conditions of Eq. (1), it specifies that it take the interaction into consideration according to the flow of Fig. 2. According to Fig. 2, the two cracks, which satisfy the conditions of Eq. (1), are replaced with a single coalesced crack when the crack tip distance becomes $S \leq 0$, due

Fig. 1 Definition of crack location parameters

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Fig. 2 Crack growth calculation process

Fig. 3 Classification of crack growth behavior⁽¹⁰⁾

to an overlapped condition in process of crack growth evaluation, and the crack growth evaluation is carried out about the coalesced crack in future analysis.

$$\begin{aligned} S &\leq 5 \text{ mm} \quad (H \leq 10 \text{ mm}) \\ S &> 5 \text{ mm} \quad (H < 2S) \end{aligned} \quad (1)$$

The coalescence conditions of Eq. (1) specified in JSME Code were determined based on experimental results as described in Ref. (3)⁽¹⁰⁾. In that experiment, fatigue tests were conducted using plates in which semi-elliptical pre-cracks were introduced, and then the observed crack growth behaviors were catego-

Fig. 4 Crack coalesce condition prescribed in JSME maintenance code⁽¹⁰⁾

rized into classifications shown in Fig. 3. According to the relationship between initial crack positions, the boundary between the two groups was drawn as shown in Fig. 4; one group has Patterns A, B and C and the other group has Patterns D and E. It was suggested that this boundary indicates the threshold of whether two cracks coalesce during their growth process. Equation (1), prescribed in JSME Code, was determined based on the results shown in Fig. 4.

However, the intensity of the interaction is not determined by the coalescence behavior alone. It is pointed out that, when two cracks are close and grow independently, their growth behavior is almost the same as that of coalesced cracks⁽¹¹⁾. This implies that the conditions for multiple crack evaluation, such as in Fig. 4, should be evaluated for validity based on mechanistic parameters such as the stress intensity factor (K value), not only the coalescence behavior observed in experiments.

In the present study, a crack growth simulation was developed using the body force method⁽¹²⁾ (BFM), and effects of the interaction between multiple cracks were evaluated based on mechanistic parameters. Results of the simulations under various relative position and size conditions could derive a new evaluation method based on the present procedure outlined in Fig. 2.

2. Crack Growth Simulation

2.1 Simulation procedures

Although the outline of the simulation procedure was the same as that of the present JSME Code shown in Fig. 2, the BFM was used to calculate K values. This method can derive accurate K values that take into account the interaction between multiple cracks⁽¹²⁾. Usually, cracks have three-dimensional shapes, however, in this simulation, cracks were considered as simplified two-dimensional through cracks. Although, in most cases, surface cracks can be thought of as evaluation objects, in this simulation,

they were considered simplified two-dimensional through cracks in the K value calculations. Some analytical and experimental investigations compare mechanical interference effects of through cracks and semi-elliptical surface cracks. These show that the intensity of the interaction of surface cracks depends on the aspect ratio of a crack, the ratios of wall thickness and the crack depth, etc. Also semi-elliptical surface cracks have an equal or lesser interference effect than through cracks, depending on the ratio of surface length to depth or depth to wall thickness^{(13),(14)}. In addition, the interference effects of surface cracks mainly act at surface points⁽¹³⁾. On the other hand, few effects act at the deepest point⁽¹³⁾. Therefore, from the point of view of interaction effects, through cracks can be regarded as a kind of semi-elliptical surface crack.

In the present study, the interval of the K value calculation was controlled so that each time the crack growth length did not exceed 5% of crack length. Cracks were assumed to change their growth direction according to the $\sigma_{\theta_{\max}}$ criterion⁽¹⁵⁾. According to this criterion, crack propagation direction $\theta_{\max}$, was determined by substituting K_I and K_{II} values into the following equation:

$$\theta = \mp \cos^{-1} \left(\frac{3K_{II}^2 + K_I \sqrt{8K_{II}^2 + K_I^2}}{9K_{II}^2 + K_I^2} \right) \quad (2)$$

where the sign in Eq. (2) was positive in the case of $K_{II}/K_I < 0$ and negative in other cases. Curved crack shapes were expressed by a series of short straight lines, which inclined with angle of $\theta_{\max}$. And at each step, a straight line, with length determined by crack growth length, was added to the end of the curved crack.

2.2 Target of the simulation

The target of the simulation was the growth behavior of the Ref. (3) experiments. In this experiments, type A533B steel, which is used in reactor pressure vessels, was subjected to fatigue tests in air atmosphere. Crack growth rate of that steel was formulated as:

$$\frac{dc}{dN} = 1.67 \times 10^{-12} (\Delta K)^{3.23} \quad (3)$$

where dc/dN was the crack growth rate and ΔK was the stress intensity factor range. The growth rate was given in m/cycle and ΔK in $\text{MPa}\sqrt{\text{m}}$. By substituting SIF, which was calculated using the BFM, for Eq. (3), the crack growth rate and corresponding crack growth length could be obtained. A tensile stress range of $\Delta\sigma = 529 \text{ MPa}$ was applied perpendicular to the initial crack direction.

2.3 Simulation results

Simulations were conducted for two cracks of length $2c_1 = 2c_2 = 5 \text{ mm}$. Figure 5 shows classi-

Fig. 5 Crack growth behavior obtained by the simulations (Initial crack length $2c_1 = 2c_2 = 5 \text{ mm}$)

Fig. 6 Result of the crack growth simulation (Initial conditions $S/c=8$, $H/c=2$, $2c_1 = 2c_2 = 5 \text{ mm}$)

Fig. 7 Result of the crack growth simulation (Initial conditions $S/c=8$, $H/c=8$, $2c_1 = 2c_2 = 5 \text{ mm}$)

fications, which are shown in Fig. 3, of crack growth behaviors obtained by the simulations under various initial conditions of S and H . The crack configurations obtained by the simulation conditions $(S/c, H/c) = (8, 2)$ and $(8, 8)$ (where $c = c_1 = c_2$) are shown in Figs. 6 and 7 respectively. As mentioned, the boundary indicated in Fig. 4 is the boundary of Patterns C and D from the Ref. (3) experiment. The simulation results shows almost the same tendency as those in Ref. (3); this implies that the simulation can evaluate the effect of the interaction on crack growth behavior.

Figure 8 shows the result of a simulation for different crack lengths, $2c_1 = 5 \text{ mm}$ and $2c_2 = 3 \text{ mm}$, with relative positions being set to $(S/c_1, H/c_1) = (8, 8)$. In this case, the growth behavior of crack 2 became

Fig. 8 Result of the crack growth simulation (Initial conditions $S/c_1=8$, $H/c_1=8$, $2c_1=5$ mm, $2c_2=3$ mm)

dormant due to the stress shielding effect⁽¹⁶⁾ caused by crack 1. This suggests that in addition to relative position, the relative crack length was also an important factor determining the interaction between multiple cracks.

In Fig. 5, the initial position S and H was normalized by the crack size, because the intensity of the interaction between multiple cracks depends on the distance normalized by crack size⁽¹⁷⁾.

2.4 Discussion

The relationship between the K value and number of cycles for the case of Figs. 6 and 7 are shown in Figs. 9 and 10. Three kinds of K values are used in these figures. The $K_{(BFM)}$ indicates the K value of outer crack tip obtained by the BFM. On the other hand, the $K_{(c)}$ represents the K value calculated using crack length c_1 ($=c_2$) according to the following equation:

$$K_{(c)} = \sigma \sqrt{\pi c_1} \quad (4)$$

The other K value, $K_{(Rx)}$, was calculated by replacing c_1 with R_x in Eq.(4). The $K_{(Rx)}$ represents the K value corresponding to a coalesced crack according to the evaluation flow shown in Fig. 2.

Distance S in Figs. 9 and 10 represents the change in the relative position of the two cracks. As two cracks approached, S became small. And after they crossed each other, S had a negative value. On the other hand, the intensity of the interaction between the two cracks could be evaluated by comparing three kinds of K values. In the initial stage of the simulation, cracks were located far enough apart that $K_{(BFM)}$ took almost the same value as $K_{(c)}$, corresponding to a case of no interaction. However, as the cracks grew, $K_{(BFM)}$ shifted to the $K_{(Rx)}$ curve, which represented the K value of the combined crack. And finally, $K_{(BFM)}$ took almost the same value as the K value of the coalesced crack $K_{(Rx)}$. Although the speed of the shift from $K_{(BFM)}$ to $K_{(Rx)}$ depended on the initial distance of H as shown in Figs. 9 and 10, the $K_{(BFM)}$ finally became as large as they coalesced and accelerated the crack growth. The interaction could also be observed in the condition of $S > 0$.

Fig. 9 Relationship between stress intensity factor and number of cycles (Initial conditions $S/c=8$, $H/c=2$, $2c_1=2c_2=5$ mm)

Fig. 10 Relationship between stress intensity factor and number of cycles (Initial conditions $S/c=8$, $H/c=8$, $2c_1=2c_2=5$ mm)

The crack configurations of the last stage of these figures corresponded to Figs. 6 and 7, respectively. Although, no coalescence can be seen in these figures, the $K_{(BFM)}$ took almost the same value as the $K_{(Rx)}$. This implies that not only the coalescence, but also the relative position and size, etc. should be taken into account in the evaluation of the interaction⁽¹¹⁾.

Figures 11 and 12 show crack growth behaviors predicted according to the evaluation flow shown in Fig. 2. The initial size and position conditions were the same as in Figs. 6 and 7. In these figures, the “ $2c$ (Analytical)” curves show changes in the single crack length $2c$ predicted by ignoring the interaction, equivalent to a coalescence-free case in the flow shown in Fig. 2. On the other hand, “ $2c \rightarrow R_x$ (Analytical)” curves in Fig. 2 also show changes in the crack size following the evaluation flow, but in this case coalescences were considered when $S \leq 0$. The “ $R_{x(BFM)}$ ” curves show the relationship between number of cycles and distance R_x obtained by the simulation shown in Figs. 6 and 7. As shown in Figs. 9 and 10, since the $K_{(BFM)}$ took almost the same value as the $K_{(Rx)}$ at the final stage of Figs. 11 and 12, the $R_{x(BFM)}$ obtained by the simulations corresponded to the equiv-

Fig. 11 Relationship between crack length and number of cycles (Initial conditions $S/c=8$, $H/c=2$, $2c_1=2c_2=5$ mm)

Fig. 12 Relationship between crack length and number of cycles (Initial conditions $S/c=8$, $H/c=8$, $2c_1=2c_2=5$ mm)

alent crack length including the interaction effect. Therefore, the $2c$ (Analytical) was not acceptable as a conservative evaluation of crack growth prediction. On the other hand, the $2c \rightarrow R_x$ (Analytical) curve, which corresponded to the coalesced crack growth behavior of the JSME Code, was almost consistent with that of the simulation in the case of Fig. 11, although some errors were seen in $S > 0$ region. In the case of Fig. 12, however, the $2c \rightarrow R_x$ (Analytical) curve was also first compared with the $R_{x(BFM)}$ curve.

Figure 13 shows the crack growth curve for the case of Fig. 8. In this case, the growth behavior of crack 2 became dormant due to the stress shielding effect caused by crack 1. There was no acceleration by the interaction, furthermore, the $R_{x(BFM)}$ curve was slower than $2c$ (Analytical) curve, which corresponded to no interaction case. This behavior was also brought about by the stress shielding effect.

These results suggest that it is very difficult to evaluate the interaction between multiple cracks by using the simple manner described in Fig. 2, because the interaction depends on various parameters such as relative position and size. Appropriately evaluating the effect of the interaction, requires mechanical parameters in addition to observation of coalescence behavior. Furthermore, the effect should be evaluated

Fig. 13 Relationship between crack length and number of cycles (Initial conditions $S/c=8$, $H/c=8$, $2c_1=5$ mm, $2c_2=3$ mm)

by considering the relative crack size, not only the relative position.

3. Influence of Crack Shape on Interaction Effect

As shown in Figs. 6, 7 and 8, the inner tips of two approaching cracks change their direction according to Eq.(3), because K_{II} became significant by the interaction. In the simulation, a curved crack shape was expressed by a series of short straight lines, with lengths obtained using Eq.(3) and evaluated K value. The BFM was applied for the series of short straight cracks. Therefore, the number of these short lines increased as the number of steps in the simulation increased, thereby increasing calculation time.

In order to reduce the calculation time, an alternative simulation procedure was examined. In this procedure, the simulation was conducted under the assumptions that the cracks propagate without changing their progress direction. This assumption can be realized by keeping $\theta_{max}=0$ in the simulation. In this case, cracks maintained a linear shape and the relative offset distance remained constant at the initial value, H , for the whole simulation. Simulations were conducted under the same initial conditions as the case of Figs. 6 and 7, by maintaining a linear shape. The results on the relationship between number of cycles and the K value of the outer crack tip $K_{(BFM)}$ are shown in Fig. 14, and the relationship between number of cycles and distance R_x is shown in Fig. 15. These two figures suggest that the interaction between multiple cracks did not depend on the inner crack tip shape. The intensity of the interaction increased, as the cracks got closer to each other. Therefore, if the objective of the simulation is defined as evaluating the relationship between the number of cycles and parameter R_x , the assumptions relating to inner crack tip behavior, such as Eq.(2), as well as treatment of mode II, become insignificant to the simulation. The

Fig. 14 Comparison of results of crack growth simulation with flexible and linear shape cracks ($2c_1=2c_2=5$ mm) (open: flexible shape cracks, solid: linear shape cracks)

Fig. 15 Comparison of results of crack growth simulation with flexible and linear shape cracks ($2c_1=2c_2=5$ mm) (open: flexible shape cracks, solid: linear shape cracks)

curved crack shape can be approximated to a line shape in the simulation.

As for the simulation described in the latter section of this paper, the cracks were assumed to propagate with a direction of $\theta_{\max}=0$ and to maintain a linear shape and initial value of distance H . And the interval of the K value calculation, defined as the ratio of growth length to total length, was improved from 5% to 1% for precise evaluation.

4. Examination of Crack Growth Evaluation Method

4.1 Case of same crack size

The previous examination showed that the simplified evaluation procedure, in which the interaction was considered by the occurrence of coalescence, could not evaluate the approximate amount of crack growth. In this study, a new procedure was examined. In this procedure, the coalesced crack size was set to $\alpha \times R_x$, instead of R_x , where R_x was equal to $2c_1+2c_2$. The case of $\alpha=0.5$ corresponded to the independent

Fig. 16 Contour plot of N_s/N_a ($\alpha=0.7$)

Fig. 17 Estimated crack coalescence factor α for approaching two crack of the same length

crack growth behavior. And the case of $\alpha=1$ was equivalent to the coalesced crack growth behavior evaluated according to the flow shown in Fig. 2. By changing α , we could control the intensity of the interaction.

As an example, the case of $\alpha=0.7$ was examined. In this case, crack size was set to $0.7 \times R_x$ when the distance S became $S \leq 0$ in the growth process. Crack growth evaluations until the distance $R_x=100$ mm with an initial crack size of $2c_1=2c_2=5$ mm were conducted, and the number of cycles was defined as N_a . The N_a under various S and H initial conditions was evaluated as shown in Fig. 5. On the other hand, the crack growth behavior was evaluated using the simulation developed in chapter 2 under the same crack size and position conditions. And the number of cycles until $R_x=100$ mm was defined as N_s . Figure 16 shows the contour plot of N_s/N_a related to S and H . The curve of $N_s/N_a=1$ shows the position condition, in which the result obtained using the new evaluation method with $\alpha=0.7$ became equivalent to that by the simulation. In the case of the position conditions belonging to the $N_s/N_a < 1$, the new evaluation method derived the larger number of cycles when α was equal to 0.7. The curve of $N_s/N_a=1$ was evaluated for above

$\alpha=0.5$ by increments of 0.1, and finally the N_s/N_a became $N_s/N_a > 1$ for all the cases of S and H when α was equal to 1.7. The curve of $N_s/N_a=1$ exhibited almost a straight-line shape, and the line of $N_s/N_a=1$ was approximated by the following equation:

(when $0.5 \leq \alpha \leq 1.0$)

$$\alpha = -0.28 \ln \left(\frac{(H/c) - 0.56(S/c) + 0.44}{5.6(S/c) + 20} \right) \quad (5)$$

(when $\alpha \geq 1.0$)

$$\alpha = \frac{(H/c) - 1.52(S/c) - 1.32}{-0.81(S/c) - 1.16} \quad (6)$$

By using the initial position S/c and H/c , α could be calculated according to Eqs. (5) and (6). Then the number of cycles until $R_x=100$ mm could be evaluated using the coalesced crack size as $\alpha \times R_x$ in the evaluation flow shown in Fig. 2.

The validity of $R_x=100$ mm used in the calculation can be explained as follows. As shown in Fig. 10, the intensity of the evaluated interaction increases, as the calculation period, which is equivalent to R_x , becomes longer. The crack depth was 50 mm when the crack was assumed to be a semi-circular shape of $R_x=100$ mm in surface length. The depth of 50 mm was sufficiently large in comparison with the thickness of pipes and reactor pressure vessels of nuclear power plants. And as shown in Fig. 10, the interaction becomes as large as that of a coalesced crack when R_x becomes $R_x=100$ mm. Therefore, $R_x=100$ mm was valid for the evaluation of Eqs. (5) and (6).

4.2 Case of dissimilar crack size

As shown in Fig. 8, the interaction between multiple cracks is influenced not only by the relative position but also the relative length of cracks. Therefore, the Eqs. (5) and (6), which evaluated for cracks of the same size, were not always available. Since cracks usually have different lengths, it is essential to evaluate the influence of the interaction between cracks of different lengths.

Figures 18 and 19 show the relationship between relative crack size and the number of cracks until $R_x=100$ mm. The simulations were conducted under initial conditions of $(S/c_1, H/c_1)=(8, 2)$ and $(8, 8)$, respectively. The number of cracks until $R_x=100$ mm, N , was normalized by $N_{(c_2=0)}$, which corresponded to the N case of $c_2=0$. The relative crack size c_2/c_1 showed the ratio obtained by changing c_2 from $c_2=0$ to 5 mm under a constant $c_1=5$ mm condition. In both cases— $(S/c_1, H/c_1)=(8, 2)$ and $(8, 8)$ — N took its minimum value at $c_2/c_1=1$ and increased as c_2 became small compared to c_1 . And, finally, N was saturated to the normalized $N=1$. This tendency of change in the parameter N against c_2/c_1 was seen under other initial position conditions, S and H , corresponding to $\alpha \geq 0.5$. In the case of $\alpha < 0.5$, the normalized N ex-

Fig. 18 Relationship between normalized number of cycles to $R_x=100$ mm and crack length ratio c_2/c_1 ($S/c_1=8$, $H/c_1=2$)

Fig. 19 Relationship between normalized number of cycles to $R_x=100$ mm and crack length ratio c_2/c_1 ($S/c_1=8$, $H/c_1=8$)

ceeded 1 for all c_2/c_1 conditions. If the initial length c_2 was relatively small, the growth rate of crack 2 became relatively small compared to that of crack 1, due to the difference in crack length. This difference in growth rate tended to increase the difference in crack length. Finally, crack 2 was covered by crack 1, as shown in Fig. 8, and the growth of crack 2 stagnates due to the stress shielding effect of crack 1. Therefore, maximum crack growth acceleration occurred for interactions where cracks were of the same length. This means that the crack growth behavior can be evaluated conservatively by using the criteria shown in Eqs. (5) and (6).

From the results shown in Figs. 18 and 19, it was revealed that if the difference in the crack size was significantly small, the interaction became sufficiently small, allowing the effect of the shorter crack on the growth behavior of the main crack to be ignored. Then the threshold values were evaluated. The threshold values, c_2/c_1 , at which normalized N becomes 0.99, were evaluated by decreasing c_2/c_1 from 1.0 for all S , H cases. And the contour plot of threshold c_2/c_1 related to S and H is shown in Fig. 20. In this figure, the curve of threshold c_2/c_1 was evaluat-

Fig. 20 Estimated threshold of c_2/c_1 for ignoring the interference effect of approaching two cracks

ed by increments of 0.1. The approximated lines of these threshold curves were estimated conservatively and they can be expressed by following equation.

$$H/c_1 > (1.45\beta^2 + 0.26\beta - 0.11)(S/c_1) + 2.2\beta \quad (7)$$

where, $\beta = c_2/c_1$ ($c_2 \leq c_1$). If the relative position and size satisfy Eq. (7), the interaction can be ignored in the crack growth evaluation. In Fig. 20, the lines obtained by Eq. (7) are also indicated.

4.3 Crack growth evaluation procedure

The effect of the interaction in crack growth evaluation can be considered conservatively by taking into account the criteria evaluated in this study and the conventional procedure prescribed in JSME Code shown in Fig. 2. First, evaluate the relative position, S/c_1 and H/c_1 , and relative size, c_2/c_1 , at the initial condition. Second, calculate the parameter α according to Eqs. (5) and (6). If α becomes $\alpha \geq 0.5$ and Eq. (7) is not satisfied, the interaction should be considered in the evaluation. The crack growth evaluation is conducted according to the evaluation flow shown in Fig. 2, but the coalesced crack size $\alpha \times (2c_1 + 2c_2)$ is used instead of $(2c_1 + 2c_2)$.

5. Conclusions

In this study, the influence of the interaction between two cracks on the crack growth behavior was evaluated by using the BFM. First, the crack growth simulation was developed using the BFM. Then the validity of the simulation was confirmed by comparing the crack configurations with the experimental results as described in Ref. (3). By using the simulation, the following conclusions could be made.

(1) When the cracks are close and overlapped, from the point of view of a mechanical parameter such as K value, the intensity of the interaction between these cracks is almost equivalent to that of a single coalesced crack, even if they are independent cracks.

(2) Even if crack shape is approximated by a line in the simulation, there is little influence on the crack growth behavior, which is represented by the relationship between the number of cycles and K value of crack tip or the distance of outer crack tips, because the interaction does not depend on the change in progress direction of the inner crack tips.

(3) Interaction is influenced not only by the relative position but also by the relative length of cracks. If the crack length difference is greater than a certain level, the interaction is sufficiently small to allow the influence of the interaction on crack growth evaluation to be ignored.

Based on these conclusions, a new crack growth evaluation method, in which the effect of interaction is considered, was proposed.

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