Accurate Displacement Measurement by Correlation of Colored Random Patterns*

Satoru YONEYAMA** and Yoshiharu MORIMOTO**

The use of colored random patterns for the calculation of the correlation coefficient is proposed to improve the accuracy of the digital image correlation technique for displacement measurements. The colored random pattern is created by spraying color paints on the surface of a sample. Then, not only the displacement distributions but the displacement gradients are determined using a set of color images before and after deformation. The effectiveness is demonstrated by applying the proposed method to the displacement measurement of rigid body rotation, and the results are compared with those obtained from the monochromatic images. The results show that the use of the color image in digital image correlation improves the measurement accuracy both of the displacements and the displacement gradients. In addition, the smaller size subset can be used for the calculation of the correlation. The proposed technique is effective in the fields of experimental mechanics and experimental stress analysis.

Key Words: Displacements, Displacement Gradients, Random Pattern, Digital Image Correlation, Image Processing, Optical Methods, Experimental Stress Analysis

1. Introduction

Experimental techniques in solid mechanics rely heavily on surface displacement field measurements. Several optical methods, such as moiré interferometry, holographic interferometry or speckle interferometry have long been used in experimental mechanics to study mechanical deformation of solids and the mechanics of fracture^{(1),(2)}. Among them, digital image correlation technique, which can obtain the deformation of a surface by comparison of digital images of the undeformed and deformed configurations, is becoming popular and widely used^{(3)–(12)}. Since this method does not need a complicated optical system, the measurement can be performed easily. In addition, unlike other methods which utilize the interference of light waves, phase analysis of the fringe pattern and subsequent phase unwrapping process are not required. Thus, applica-

tions of this method to various problems can be expected. However, in this method, it is difficult to obtain reliable results around regions near boundaries such as around a crack surface since a subset, typically about 30×30 or 40×40 pixels in an image, is used to detect the deformation. For example, at a crack surface since subsets overlap crack faces, the displacements are determined by areas on each side of the crack surface that have opposite displacements^{(5),(9)}. The difficulty is found near stress concentration regions since it is assumed that the subset is deformed uniformly. Thus, the possible smallest size subset should be used to overcome these difficulties. Lu and Cary⁽¹³⁾ proposed an accurate digital image correlation method by taking the second order displacement gradients into account. However, the difficulty is still remaining since their method cannot reduce the subset size.

In recent, on the other hand, information obtained for several wavelengths, that is, a color image is utilized for optical methods in the field of experimental mechanics^{(14)–(17)}. The information contained in color images is much greater than monochromatic images because a color image consists of three mono-

* Received 4th September, 2002 (No. 02-4156)

** Department of Opto-Mechatronics, Wakayama University, 930 Sakaedani, Wakayama 640-8510, Japan.
E-mail: yoneyama@sys.wakayama-u.ac.jp

chromatic images representing red, green and blue colors. Thus, it can be considered that a smaller size subset of color images has sufficient information for the calculation of the correlation coefficient. In addition, if color images with same subset size as monochromatic images are used, it is expected that more accurate results than the use of monochromatic images can be obtained. The colored random pattern can be created by spraying several color paints on a specimen surface. A tricolor laser may also be used to produce a colored speckle pattern.

In order to improve the accuracy of the digital image correlation and to realize the displacement measurement using small subsets, the use of colored random patterns for the calculation of the correlation coefficient is proposed in the present study. Then, the effectiveness is demonstrated by applying the proposed method to the displacement measurement of rigid body rotation, and the results are compared with those obtained from the monochromatic images. The results show that the use of color images in digital image correlation improves the measurement accuracy both of the displacements and the displacement gradients. Also, the smaller size subset can be used for the calculation of the correlation.

2. Outline of Digital Image Correlation

In the digital image correlation technique, random speckle-like black and white dot patterns on specimen surfaces are observed. These image patterns, one before and another after deformation, are digitized and stored in a computer. The digitized images are compared to match subsets from one image to another. Typically, a subset such as 30×30 or 40×40 pixels from the undeformed image is chosen and try to find its location in the deformed image. Once the location of this subset in the deformed image is found, the displacements of this subset can be determined. For the best estimate of the displacements, a normalized cross-correlation coefficient, S , defined as following equation is used^{(6),(11)}.

$$S(x, y, u_x, u_y, \frac{\partial u_x}{\partial x}, \frac{\partial u_x}{\partial y}, \frac{\partial u_y}{\partial x}, \frac{\partial u_y}{\partial y}) = 1 - \frac{\sum I_u(x, y) I_d(x^*, y^*)}{\sqrt{\sum I_u(x, y)^2 \sum I_d(x^*, y^*)^2}} \quad (1)$$

where u_x and u_y are the displacement components at the center of the subset, I_u and I_d represent the gray levels of the undeformed and deformed images, respectively, and (x, y) and (x^*, y^*) are the coordinates of a point on the subset before and after deformation, respectively. The coordinate (x^*, y^*) after deformation is related to the coordinate (x, y) before deformation as

Fig. 1 Subsets before and after deformation

$$\begin{aligned} x^* &= x + u_x + \frac{\partial u_x}{\partial x} \Delta x + \frac{\partial u_x}{\partial y} \Delta y, \\ y^* &= y + u_y + \frac{\partial u_y}{\partial x} \Delta x + \frac{\partial u_y}{\partial y} \Delta y, \end{aligned} \quad (2)$$

where Δx and Δy are the x and y directional components of the distance from the center of the subset to the point (x, y) , respectively. Figure 1 shows the subsets before and after deformation. The center point P and the point Q on the undeformed subset move to the point P' and Q' on the deformed subset, respectively. Then, the moving distance from the point P to the point P' is interpreted as the displacement and the position of the point Q' is expressed as Eq. (2).

The correlation coefficient S is the function of the displacement components u_x and u_y , and the displacement gradients $\partial u_x / \partial x$, $\partial u_x / \partial y$, $\partial u_y / \partial x$ and $\partial u_y / \partial y$. Therefore, the displacements are determined by searching the best set of the displacements and the displacement gradients, which minimize the correlation coefficient S . In this process, the approximate displacements are estimated within the accuracy of one pixel with zero gradients at first. After the first estimation, the process varies to search both the displacements and the displacement gradients using Newton-Raphson method. In this step, to increase the resolution of the correlation method, an interpolation scheme is used to reconstruct a continuous intensity (gray level) instead of a discrete intensity pattern. The detail of the digital image correlation with the Newton-Raphson optimization is given in papers by Sutton and co-workers^{(4),(6),(11)}. Other approaches such as the use of Fourier transform method have also been proposed by several researchers⁽¹⁸⁾⁻⁽²⁰⁾.

3. Digital Image Correlation Using Colored Random Patterns

For the matching of images, in general, color

images are converted into monochromatic images to reduce the computational effort. In the proposed method, however, the following correlation coefficient, S_c , is proposed in order to utilize the color information.

$$S_c\left(x, y, u_x, u_y, \frac{\partial u_x}{\partial x}, \frac{\partial u_x}{\partial y}, \frac{\partial u_y}{\partial x}, \frac{\partial u_y}{\partial y}\right) = \sum_{i=r,g,b} \left\{ 1 - \frac{\sum I_{iu}(x, y) I_{id}(x^*, y^*)}{\sqrt{\sum I_{iu}(x, y)^2 \sum I_{id}(x^*, y^*)^2}} \right\}, \quad (3)$$

where i ($=r, g, b$) expresses the primary colors, that is, three monochromatic images for red, green and blue colors consisting a color image. Equation (3) means that the sum of the correlation coefficients for each color is treated as the total correlation coefficient of the color image. By using Eq.(3) in place of Eq.(1) with the Newton-Raphson optimization, the displacement components and the displacement gradients are determined. For the analysis in sub-pixel accuracy, in this study, a bilinear interpolation is used at first. Only the case that the oscillation occurs in the Newton-Raphson method, a cubic spline method is used for the interpolation.

4. Sample Preparation and Experiments

For the creation of a colored random pattern on a specimen surface, several methods, such as color spray paints or an inkjet color printer can be used. A tricolor laser may also be used to produce a colored speckle pattern. In the present study, spray paints of red, green and blue colors are employed because the spray paints can easily create a random pattern. In addition, a high contrast image of the random pattern is obtained by the spray paints. It is known that the size of the random pattern should be selected to oversample the intensity pattern by several sensors for accurate measurement⁽¹¹⁾. In this study, each random pattern is oversampled by 10 – 40 pixels.

Two kinds of experiments, no deformation and rotation, are performed in order to validate the proposed method. At first, two images are recorded without moving the sample (no deformation test). This test is performed to determine the minimum errors which are expected. The second experiment is the rigid body rotation of the sample. The sample is rotated from 0 deg to 1 deg at the increment of 0.1 deg by a rotation stage of the resolution of 0.01 deg. Figure 2 shows a schematic diagram of the experimental setup. A sample made of an aluminum plate with the colored random pattern on the surface is mounted on a rotation stage. The sample on the stage is illuminated by two white light sources (halogen lamps). A three-CCD type camera (640×480 pixels×24 bits) is used to record the colored random pattern. After images are acquired, they are analyzed by the

Fig. 2 Experimental Setup

Fig. 3 An example set of (a) the colored random pattern and (b)–(d) the separated monochromatic patterns for red, green and blue colors

proposed method. In order to compare the results with those obtained from monochromatic images, the analyses are also performed for monochromatic images. The monochromatic images are converted from the color images as NTSC/PAL luminance given by⁽²¹⁾

$$I = 0.2989I_r + 0.5866I_g + 0.1144I_b, \quad (4)$$

where I expresses the gray level for the monochromatic images, and I_r , I_g and I_b are the RGB levels, that is, the gray levels for red, green and blue images, respectively.

5. Results and Discussion

Figure 3 shows an example set of the colored random pattern and the separated monochromatic patterns for red, green and blue colors. In these images, the length of 1 mm corresponds to about 37.4 pixels. This value is obtained by the additional rigid body translation test. Figure 4 shows the distribu-

tions of the gray levels I_r , I_g and I_b along the A-A' line in Fig. 3(a). The differences of the random pattern due to the differences of the color are obviously seen. That is, the images for each color have different information each other. Thus, the accuracy improvement in displacement measurement by the proposed method is expected.

The results of the no deformation tests for the monochromatic images are shown in Table 1 and for the color images are in Table 2. Here, the average values and the standard deviation of 20 points on the images for the 11×11 , 21×21 , 31×31 subsets are shown. For both cases of the monochromatic and

Fig. 4 Gray level distributions along the A-A' line of the colored random pattern in Fig. 3(a)

color images, the standard deviations are lower for the large subset sizes. The average values for both cases are almost equal. On the other hand, it is seen that the standard deviations of both of the displacements and the displacement gradients for the color images are lower than those for the monochromatic images.

The displacement measurements along a horizontal line (A-A' line in Fig. 3(a)) under the rotation are examined. The distributions of the y directional displacements u_y obtained from the color images are compared with those obtained from the monochromatic images in Figs. 5 and 6. Here, only the results for the rigid body rotation of 0.3 deg are shown. In these figures, the subset sizes are set to 7×7 , 11×11 , 21×21 and 31×31 pixels. Since the applied displacement is the rigid body rotation, the displacement distribution along the line must be a straight line. It is observed that the displacement can be determined in sub-pixel accuracy for both cases of the monochromatic and color images and the accuracy is better for the larger size subsets. However, in case of the 7×7 subset of the monochromatic images in Fig. 5(a), the decorrelation is observed. Also, in case of the 11×11 subset of the monochromatic images in Fig. 5(b), the displacement distribution is a little bit scattering. On the other hand, the decorrelation does not occur in case of the color image in Fig. 6(a) whereas the subset size is too small as 7×7 pixels. In addition, the displacement distribution is almost straight and no scattering is observed for the color images in Fig. 6(b) even if the small size subset of

Table 1 Results of no deformation tests for the monochromatic images

	u_x , pixels	u_y , pixels	$\partial u_x / \partial x$	$\partial u_x / \partial y$	$\partial u_y / \partial x$	$\partial u_y / \partial y$
11 x 11 subset						
Average	0.0467	0.0364	0.001974	0.001056	0.001762	0.000308
Standard deviation	0.0109	0.0101	0.003319	0.002803	0.004329	0.002904
21 x 21 subset						
Average	0.0482	0.0375	0.000243	-0.000044	-0.000076	-0.000230
Standard dev.	0.0063	0.0047	0.000799	0.000756	0.000612	0.000825
31 x 31 subset						
Average	0.0490	0.0377	0.000103	0.000273	0.000001	-0.000073
Standard deviation	0.0065	0.0054	0.000414	0.000258	0.000493	0.000411

Table 2 Results of no deformation tests for the color images

	u_x , pixels	u_y , pixels	$\partial u_x / \partial x$	$\partial u_x / \partial y$	$\partial u_y / \partial x$	$\partial u_y / \partial y$
11 x 11 subset						
Average	0.0411	0.0427	0.001406	0.001530	0.001173	0.000104
Standard deviation	0.0076	0.0081	0.002618	0.002677	0.002643	0.002065
21 x 21 subset						
Average	0.0445	0.0425	0.000158	-0.000042	0.000240	-0.000109
Standard dev.	0.0050	0.0041	0.000605	0.000608	0.000522	0.000555
31 x 31 subset						
Average	0.0441	0.0429	0.000042	0.000131	0.000159	-0.000026
Standard deviation	0.0051	0.0039	0.000214	0.000239	0.000297	0.000266

Fig. 5 Displacements along the A-A' line under rigid body rotation of 0.3 deg obtained from the monochromatic images using the (a) 7×7, (b) 11×11, (c) 21×21 and (d) 31×31 subsets

11×11 pixels is used. For both cases of the monochromatic and color images, the accurate results are obtained for the larger size subsets of 21×21 or 31×31 pixels. However, it is observed that the results obtained from the color images are more straight than those obtained from the monochromatic images even if the subset size is large as 21×21 or 31×31 pixels. For the subset size of 21×21 pixels in Figs. 5(c) and 6(c), the determination coefficient of the least-squares approximation of the data to a straight line is estimated as 0.999 in the case of the color image whereas the determination coefficient in case of the monochromatic image is 0.997. The coefficients are 0.998 for the monochromatic images and 0.999 for the color images for the subset size of 31×31 pixels shown in Figs. 5(d) and 6(d). That is, the accuracy of the displacement measurement can be improved by the proposed method.

The rotation angle ω is estimated from the displacement gradient terms of $\partial u_x/\partial y$ and $\partial u_y/\partial x$ using the following small angle approximation.

$$\omega = \frac{1}{2} \left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} \right). \quad (5)$$

The comparison between the actual rotation and the experimentally obtained rotation for the 21×21 subset is shown in Fig. 7. The 45 deg line in the figure indicates the perfect correspondence between the actual and measured rotations. In this figure, the results obtained from the color images are compared with those obtained from the monochromatic images again. Because the displacement gradients obtained by the digital image correlation are not so accurate and the results are usually scattering⁽⁶⁾, the results are averaged using 20 points on the images and the standard deviations are indicated by the error bars. For both cases of the color and monochromatic images, the average values of the rotation angle are estimated accurately. On the other hand, the standard deviation in case of the color image is lower than the case of the monochromatic image. That is, the displacement gradients obtained from the color images is also more accurate than those obtained from the monochromatic images.

The results show that the use of the color images improves the accuracy of not only the displacements but the displacement gradients. Moreover, the dis-

Fig. 6 Displacements along the A-A' line under rigid body rotation of 0.3 deg obtained from the color images using the (a) 7×7 , (b) 11×11 , (c) 21×21 and (d) 31×31 subsets

Fig. 7 Actual rotation versus measured rotation for the 21×21 subset obtained from (a) the monochromatic images and (b) the color images

placements can be determined accurately by the proposed method even if small subsets such as 11×11 pixels is used. On the other hand, the computation time of the proposed method increases about 2.2 times

of the case of the monochromatic images when the same subset sizes are used for both cases. The proposed method is effective when the accurate measurement or the use of smaller size subsets is required.

6. Conclusions

In the present study, the use of colored random patterns in digital image correlation for the accurate displacement measurements is proposed. The effectiveness is demonstrated by applying the proposed method to the displacement measurement of the rigid body rotation, and the results are compared with those obtained by the monochromatic images. The results show that the use of the color image in digital image correlation improve the measurement accuracy both of the displacements and the displacement gradients. In addition, the smaller size subset can be used for the calculation of the correlation.

Acknowledgments

The authors wish to express their thanks to Prof. M. Takashi of Aoyama Gakuin University for his helpful comments. Our thanks are extended to Messrs. A. Kitano and Y. Ikeda of Wakayama University for their help in the experiments.

References

- (1) Dally, J.W. and Riley, W.F., *Experimental Stress Analysis*, 3rd Ed., (1991), McGraw-Hill.
- (2) Smith, C.W. and Kobayashi, A.S., *Experimental Fracture Mechanics, Handbook on Experimental Mechanics*, 2nd Ed., Kobayashi, A.S., Ed., (1993), pp. 905-968, VCH Publishers.
- (3) Peters, W.H. and Ranson, W.F., *Digital Imaging Technique in Experimental Stress Analysis*, Opt. Eng., Vol. 21, No. 3 (1982), pp. 427-431.
- (4) Sutton, M.A., Cheng, M., Peters, W.H., Chao, Y.J. and McNeill, S.R., *Application of an Optimized Digital Correlation Method to Planar Deformation Analysis*, Image and Vision Computing, Vol. 4, No. 3 (1986), 143-150.
- (5) McNeill, S.R., Peters, W.H. and Sutton, M.A., *Estimation of Stress Intensity Factor by Digital Image Correlation*, Eng. Fract. Mech., Vol. 28, No. 1 (1987), 101-112.
- (6) Bruck, H.A., McNeill, S.R., Sutton, M.A. and Peters, W.H., *Digital Image Correlation Using Newton-Raphson Method of Partial Differential Correction*, Exp. Mech., Vol. 29, No. 3 (1989), pp. 261-268.
- (7) Kahn-Jetter, Z.L., Jha, N.K. and Bhatia, H., *Optimal Image Correlation in Experimental Mechanics*, Opt. Eng., Vol. 33, No. 4 (1994), pp. 1099-1105.
- (8) Lyons, J.S., Liu, J. and Sutton, M.A., *High-temperature Deformation Measurements Using Digital-image Correlation*, Exp. Mech., Vol. 36, No. 1 (1996), pp. 64-70.
- (9) Chao, Y.-J., Luo, P.F. and Kalthoff, J.F., *An Experimental Study of the Deformation Fields around a Propagating Crack Tip*, Exp. Mech., Vol. 38, No. 2 (1998), pp. 79-85.
- (10) Vendroux, G. and Knauss, W.G., *Submicron Deformation Field Measurements: Part 2, Improved Digital Image Correlation*, Exp. Mech., Vol. 38, No. 2 (1998), pp. 86-92.
- (11) Sutton, M.A., McNeill, S.R., Helm, J.D. and Chao, Y.-J., *Advances in Two-Dimensional and Three-Dimensional Computer Vision*, Photomechanics, Rastogi, P.K., Ed., (2000), pp. 323-372, Springer-Verlag.
- (12) Yoneyama, S., Misawa, A. and Takashi, M., *Time Variation of Displacement Fields around a Crack Tip in a Viscoelastic Strip*, Proc. 3rd Int. Conf. Mech. Time-Dependent Materials, (2000), pp. 124-126.
- (13) Lu, H. and Cary, P.D., *Displacement Measurement by Digital Image Correlation: Implementation of a Second-order Displacement Gradient*, Exp. Mech., Vol. 40, No. 4 (2000), pp. 393-400.
- (14) Ajovalasit, A., Barone, S. and Petrucci, G., *Towards RGB Photoelasticity: Full-field Automated Photoelasticity in White Light*, Exp. Mech., Vol. 35, No. 3 (1995), pp. 193-200.
- (15) Yoneyama, S., Gotoh, J. and Takashi, M., *Tricolor Photoviscoelastic Technique and Its Application to Moving Contact*, Exp. Mech., Vol. 38, No. 3 (1998), pp. 211-217.
- (16) Gilbert, B.S. and Blatt, J.H., *Enhanced Three-dimensional Reconstruction of Surfaces Using Multicolor Gratings*, Opt. Eng., Vol. 39, No. 1 (2000), pp. 52-60.
- (17) Voloshin, A.S. and Freire, J. L. F., *Displacement Measurement Using RGB*, Proc. SEM Int. Cong. Exp. Mech., (2000), pp. 140-143.
- (18) Umezaki, E., Shimamoto, A. and Watanabe, H., *Improvement in Speed and Accuracy of Digital Image Correlation Method*, Proc. Conf. Advanced Tech. Exp. Mech., (1993), pp. 173-178.
- (19) Broggiato, G.B., Gallo, M., Newaz, G. and Salvi, J., *Non-contact Strain Measurement Using Digital Speckle Correlation*, Proc. SEM Annual Conf. Exp. Mech., (2001), pp. 358-360.
- (20) Kato, A. and Wate, H., *Deformation Measurement Using Digital Image Correlation—Interpolation Method for Sub-pixel Accuracy—*, Proc. APCFS & ATEM '01, (2001), pp. 840-845.
- (21) Kusaka, H., *Color Image Processing*, (in Japanese), (1997), Ohmsha.