Modelling of Thin Strip Cold Rolling With Friction Variation by A 3-D Finite Element Method*

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In this paper, the rolling of thin strip is simulated by a 3-D rigid plastic finite element method (FEM), which considers friction variation models. The calculated rolling pressure, spread and forward slip, taking into account a suitable friction variation model in the roll bite, are in good agreement with the experimental values. Based on the FE simulation, the calculated strip velocity at the exit of roll bite is obtained, and the effect of the friction variation on the shape and flatness of strip is presented, the characteristic of the snaking of strip is also described in this paper.

Key Words: Thin Strip, Shape and Flatness, Snaking, Friction Variation, FEM, Modelling

1. Introduction

Increasing applications of the thin strip and high quality requirement compel steel manufacturers to improve its manufacturing quality and productivity in order to maintain competitiveness in the strip rolling industry. One of the important features of the thin strip is the thickness accuracy, which affects the flatness and profile of strip and the operation of rolling process. Using the existing rolling mills to produce thinner strip, is one of the major efforts for steel manufacturers. However, the rolling of thin strip on the existing rolling mill will introduce some problems, such as shape and thickness accuracy of strip. It is a challenge for researchers to overcome these problems.

Research has shown that at the interface of the roll and strip, the tribology⁽¹⁾, that is friction, lubrication and wear, has a significant influence on the dimensional accuracy and surface quality of the strip. The traditional approach is to assume that the frictional force in the roll bite is proportional to the normal force, with the friction coefficient remaining constant in the roll bite. However, previous research has shown that the friction varies in the roll bite⁽²⁾, and it

affects the rolling pressure and the accuracy of on-line control models. Zhang⁽³⁾ and Fleck et al.⁽⁴⁾ has studied the cold rolling of foil from slab method with constant friction.

The 3-D Finite Element Method (FEM) has been used in the analysis of strip rolling⁽⁵⁾⁻⁽¹⁰⁾, shape rolling^{(11),(12)} and slab rolling⁽¹³⁾⁻⁽¹⁵⁾ in the past, and Jiang et al.⁽¹⁶⁾⁻⁽²⁰⁾ have used this finite element method to solve special shape strip rolling. There is a major handicap to producing accurate and reliable models for the cold rolling of thin strip due to the lack of well-defined friction boundary conditions⁽²¹⁾. In order to model realistically, it is necessary to employ a variable friction model in the roll bite. As far as we are aware, the modelling of thin strip rolling with friction variation has not been reported till now. In this work, the authors use the 3-D rigid plastic FEM to solve the thin strip rolling taking into account the modelling of friction variation in the deformation zone, and to compare the computed results with measured values. The characteristic of the velocity distribution of the strip along the width at the exit of the roll bite is also presented in this study. The modelling of thin strip rolling with friction variation can improve the accuracy of rolling model.

Nomenclature

Δb : Spread

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- Δb_{zk} : Spread in thickness direction at k th element
 C_H : Initial crown of strip before rolling
 C_h : Exit crown of strip after rolling
 g : Compressible factor
 K_1 : Constant for forward slip zone
 K_2 : Constant for backward slip zone
 k : Shear yield stress
 k_1, k_2, k_3 : Positive constant
 m_1 : Friction factor
 m_2 : Number of nodes at the exit of strip
 m_3 : Number of elements in the thickness direction
 S : Forward slip
 s_K : Velocity discontinuity surface within the volume V
 T_1 : Tension
 v_x, v_y, v_z : Velocity component in x, y and z directions respectively
 v_{xi} : Node velocity at exit of rolled material
 V : Volume
 V_R : Tangential velocity of roll
 V_g : Relative velocity at the interface of the strip and roll
 ΔV_f : Relative slip velocity between the rolled material and the roll
 ΔV_K : Velocity discontinuity value
 v : Velocity of the cross section with tension
 $\bar{\sigma}$: Equivalent stress
 σ_s : Yield stress
 σ_m : Hydrostatic stress
 σ'_{ij} : Deviatoric stress tensor
 $\bar{\epsilon}$: Equivalent strain rate
 $\bar{\epsilon}$: Plastic strain
 $\dot{\epsilon}_v$: Volumetric strain rate
 $\dot{\epsilon}_{ij}$: Strain rate tensor
 τ_f : Frictional shear stress
 τ_K : Shear stress
 β : Angular position of the node
 δ_{ij} : Kronecker delta
 ϕ^f : Work rate of friction
 ϕ^p : Work rate of plastic deformation
 ϕ^t : Work rate of tension
 Φ : Functional

2. Theoretical Analysis

The slightly compressible method in 3-D rigid plastic FEM has been employed to solve many rolling problems. According to the variational principle⁽¹⁷⁾, the real velocity field must minimise the following functional:

$$\Phi = \iiint_v \bar{\sigma} \dot{\epsilon} dv + \iint_{s_f} \tau_f \Delta V_f ds + \iint_{s_K} \tau_K \Delta V_K ds \pm \iint_{s_v} T_1 v ds = \phi^p + \phi^f + \phi^j + \phi^t \quad (1)$$

where the first term on the right hand side is the work rate of plastic deformation (ϕ^p), $\bar{\sigma}$ is the equivalent stress and $\dot{\epsilon}$ is the equivalent strain rate. The second term on the right hand side is the work rate of friction (ϕ^f); ΔV_f is the relative slip velocity at the interface of strip-roll where the frictional shear stress τ_f is applied. To take into account the friction variation in the roll bite, in this study the modified frictional stress model⁽⁵⁾ can be written as follows:

$$\tau_f = K_i \frac{m_1 \sigma_s}{\sqrt{3}} \left(\frac{2}{\pi} \tan^{-1} \left\{ \frac{V_g}{k_i} \right\} \right) \quad (2)$$

where m_1 is friction factor; σ_s is yield stress; K_i is a coefficient describing the changes of frictional shear stress in the deformation zone, which has two parameters K_1 and K_2 for the forward slip zone and backward slip zone respectively, and has different value for the forward slip zone and backward slip zone; k_i is a positive constant, k_1 is for forward slip zone and k_2 backward slip zone. The distribution of the frictional shear stress can be seen in Fig. 1. The relative slip velocity, V_g , between the strip and the roll is calculated by

$$V_g = \sqrt{(v_x \sec \beta - V_R)^2 + v_y^2} \quad (3)$$

where v_x and v_y are the velocity components in the x and y directions respectively, β is the angular position of the node, V_R is the tangential velocity of the roll. ΔV_f is also the relative slip velocity between the rolled material and the roll, which can be calculated by

$$\Delta V_f = \sqrt{(v_x \sec \beta - V_R)^2 + v_y^2 + k_3^2} \quad (4)$$

where k_3 is a small positive constant to prevent a second singular point during simulation⁽⁶⁾. The third term on the right hand side of Eq. (1) is the additional shearing work rate (ϕ^j) at a surface on which there is a velocity discontinuity. ΔV_K is the velocity discontinuity across the velocity discontinuity surface s_K within the volume V , τ_K is the shear stress, $\tau_K = k$ (k is the shear yield stress). The fourth term on the right hand side of Eq. (1) is the work rate (ϕ^t) of tension. T_1 is the tension and v is the velocity of the cross

Fig. 1 Frictional shear stress model

section with tension. Here “-” indicates the front tension, and “+” the back tension. When the functional Φ is minimised by the rigid plastic FEM using the formulation for compressible materials, the equivalent stress can be written as follows⁽¹⁷⁾:

$$\bar{\sigma} = \sqrt{\frac{3}{2} \sigma'_{ij} \sigma'_{ij} + g \sigma_m^2} \quad (5)$$

where σ'_{ij} is the deviatoric stress tensor, σ_m is the hydrostatic stress and g is a small positive constant, i.e. a factor conferring a small degree of compressibility, in general, $g=0.01-0.0001$. The equivalent strain rate is written as

$$\dot{\bar{\epsilon}} = \sqrt{\frac{2}{3} \dot{\epsilon}_{ij} \dot{\epsilon}_{ij} + \frac{1}{g} \dot{\epsilon}_v^2} \quad (6)$$

where $\dot{\epsilon}_{ij}$ is the strain rate tensor and $\dot{\epsilon}_v = \dot{\epsilon}_x + \dot{\epsilon}_y + \dot{\epsilon}_z$ is the volumetric strain rate. The stress tensor can be written as follows⁽¹⁷⁾:

$$\sigma_{ij} = \frac{\bar{\sigma}}{\dot{\bar{\epsilon}}} \left(\frac{2}{3} \dot{\epsilon}_{ij} + \delta_{ij} \left(\frac{1}{g} - \frac{2}{9} \right) \dot{\epsilon}_v \right) \quad (7)$$

where δ_{ij} is the Kronecker delta.

Minimization of the functional Eq.(1) with respect to the nodal point velocities within element and sum of the element functional yield the nonlinear algebraic equations, which can be solved by the modified Newton-Raphson method⁽¹⁷⁾. In this study, the forward slip can be obtained from Eq.(8):

$$S = \frac{\sum_{i=1}^{m_2} v_{xi}}{m_2 V_R} - 1 \quad (8)$$

where v_{xi} is the node velocity at exit of rolled material; m_2 is the number of nodes at the exit of strip. The spread can be formulated by Eq.(9):

$$\Delta \bar{b} = \frac{1}{m_3} \sum_{k=1}^{m_3} \Delta b_{zk} \quad (9)$$

where m_3 is the number of element in the thickness direction, and Δb_{zk} is the spread in thickness direction at k th element.

3. Simulation Conditions

3.1 FE mesh

As the deformation is symmetric about both the y - x and z - x planes, one quarter of the deforming workpiece as shown in Fig. 2, was studied. Isoparametric hexahedral elements with eight Gauss points were used throughout the strip, including the corner at entry to the roll gap. The number of elements along the x , y and z directions can vary according to the size of studied objective. There is a thin element layer at the entry of the roll bite to consider the effect of the additional work rate causing by the velocity discontinuity in roll bite⁽²⁰⁾.

3.2 Velocity Boundary Conditions

The velocity boundary conditions are shown in Table 1. v_x , v_y and v_z are the velocity components in the directions x , y and z , respectively.

Table 1 Velocity boundary conditions (Fig. 2)

Position	Velocity
AA ₁ B ₁ B	$v_y = v_z = 0$
EHH ₁ E ₁	$v_y = v_z = 0$
CDD ₁ C ₁	$v_z/v_x = -\tan \beta$
BCC ₁ B ₁	$v_z = 0$
HH ₁ A ₁ A	$v_z = 0$
EHAB	$v_y = 0$

Fig. 2 FE mesh for analysis of thin strip rolling

3.3 Flow model of metal

A model material, low carbon steel, was rolled on an experimental rolling mill, the elastic flattening of the rolls was considered in the simulation and width of strip is 30 mm. The constitutive model⁽⁶⁾ used to describe the behavior of this low carbon steel was $\sigma = 245.98 + 702.66 \bar{\epsilon}^{0.58}$ (MPa) (10) where $\bar{\epsilon}$ is the plastic strain.

4. Results

4.1 Effect of friction conditions on simulation results

For the rolling of low carbon strip, the front and back tension was involved, and the calculated results are obtained with the assumptions of friction variation model and constant friction. In the simulation, $K_1 = K_2 = 1.0$, $k_1 = k_2 = 0.1$, $m_1 = 0.12$ and $g = 0.01$. The comparison of the calculated rolling pressure with the measured value is shown in Table 2. It is seen that the calculated rolling pressure is closer to the measured value than that with constant friction model except for the case 0.04 mm reduction.

When the thickness of strip before rolling is 0.26 mm, reduction of 0.06 mm, and $k_1 = k_2 = 0.1$, $m_1 = 0.12$, $g = 0.01$, the effect of the change of K_1 and K_2 on

Table 2 Comparison of calculated rolling pressure with measured value

Initial thickness mm	Reduction mm	Equivalent roll radius mm	Flow stress MPa	Rolling pressure, MPa		
				Friction variation	Constant friction	Measured
0.26	0.06	142.11	617.4	777.9	756.0	777.1
0.20	0.04	167.70	627.2	789.9	792.7	793.8
0.16	0.03	255.76	642.9	997.6	942.6	980.0
0.13	0.015	430.10	652.7	1125.1	1056.1	1156.4

Table 3 Effect of K_1 and K_2 on calculated rolling pressure

K_1	K_2	Calculated rolling pressure, MPa
0.5	1.4	778.0
1.0	1.0	777.9
1.2	0.8	777.9

Fig. 3 Effect of friction factor (m_1) on rolling pressure

Fig. 4 Effect of reduction on rolling pressure

rolling pressure is shown in Table 3. It is seen that the effect of constant K_1 and K_2 on calculated rolling pressure is not significant. This is because of the lower reduction value, which is in accord with the results in Ref.(22). However, the friction factor m_1 has a significant influence on the calculated rolling pressure for $K_1=K_2=1.0$, $k_1=k_2=0.1$, $g=0.01$ and thickness of strip before rolling is 0.26 mm, reduction value 0.06 mm, which is as shown in Fig. 3. It is seen that the rolling pressure increases when the friction factor increases, and the calculated value is equal to the measured one when a suitable friction factor is selected.

Fig. 5 Effect of reduction on spread

Fig. 6 Comparison of calculated spread with measured value

For the rolling of strip with thickness 1.9 mm, width 30.0 mm before rolling, the number of elements along the x , y and z directions are 10, 5 and 4 respectively, resulting in a total of 200 elements and 330 nodes and high reduction were used in the simulation. K_1 and K_2 change, $k_1=k_2=0.1$, $m_1=0.25$ and $g=0.01$ and rolling speed of experimental mill 0.316 m/s. Figure 4 shows the effect of reduction on rolling pressure. It is seen that the rolling pressure increases with reduction, and the calculated rolling pressure is significantly different for different K_1 and K_2 . When K_1 increases and K_2 decreases, the calculated rolling pressure reduces, this indicates that the friction decreases at the forward slip zone and increases at the backward slip zone, so the resistance to deformation of metal increases. As a result, the rolling pressure increases. It is also seen that the calculated rolling pressure for $K_1=0.8$, $K_2=1.2$ is close to the measured value when the reduction is higher than 43%, and for $K_1=K_2=1.0$ it is close to the measured value when the reduction is less than 43%.

Figure 5 shows the effect of the reduction on spread. It is seen that the spread of strip increases as reduction increases, and the spread increases when K_1 increases and K_2 decreases. It is also seen that the calculated result is closer to the measured value as $K_1=K_2=1.0$. A comparison of the calculated spread of

Fig. 7 Comparison of calculated forward slip with measured value

strip with results obtained from the constant friction ($\tau_f = m_1 k$, $m_1 = 0.25$)⁽⁶⁾ and friction variation ($K_1 = K_2 = 1.0$) model is conducted, which is shown in Fig. 6. It is seen that the calculated spread for friction variation model is in good agreement with the measured value.

The comparison of the calculated forward slip for friction variation and constant friction⁽⁶⁾ models with measured value is shown in Fig. 7. It is seen that the calculated forward slip with the friction variation model is higher than that with constant friction model, and it is closer to the measured result.

4.2 Effect of friction on shape and flatness of strip

For $\phi 50/200 \times 250$ 4-high rolling mill, carbon steel strip thickness 1.2 mm and width 100 mm was rolled⁽⁶⁾. In the simulation of shape, the velocity boundary condition at the exit of the roll bite was canceled. As the initial crown C_H of strip before rolling increases from zero to 40 μm and the exit crown C_h of strip after rolling remains a constant at 20 μm , the distribution of the relative velocity between the strip exit velocity and the roll surface velocity along the width of strip is shown in Fig. 8. It is seen that the distribution of the strip velocity at exit is almost uniform when the initial crown of the strip before rolling is close to the exit crown of strip after rolling, and the hill of the velocity of strip appears at the middle part of strip which causes the middle strip buckle when $C_h < C_H$. However, the hill of the relative velocity of strip appears at the sides of strip which causes the sides buckle of strip when $C_h > C_H$, the poor shape of strip is easy to result in the snaking while rolling. The calculated results are in good agreement with the experimental results. It is also found that the velocity of strip at the exit of roll bite increases when friction variation model K_1 increase and K_2 decreases. This means that the buckle becomes significant when the friction at forward slip zone increases and that at backward slip zone decreases. Because it is helpful to increase the strip deformation along the rolling direc-

Fig. 8 Distribution of calculated relative strip velocity at the exit of the roll bite

Fig. 9 Comparison of calculated spread with measured values

tion when the friction at forward slip zone increases, so the velocity of strip at exit increases. It is also seen that there is a zone where the change of the exit velocity of strip is not significant, which remains a constant for different initial crown of strip. This position is away from the side of strip about a quarter of the width. The result is useful to determine which rolling conditions produce a good shape and flatness of strip.

Figure 9 shows the comparison of the calculated spread with measured values for the rolling of strip thickness 1.2 mm, width 100 mm and material carbon steel, when the friction variation model ($K_1 = K_2 = 1.0$, $k_1 = k_2 = 0.1$) was applied in the simulation. It is seen that the calculated spreads are in good agreement with the measured values.

5. Conclusions

A model of thin strip cold rolling considering the friction variation models was produced. The results show that the friction variation model has a significant effect on the results of simulation for the rolling of thin strip when the reduction is large. If the

reduction is small, the influence is not significant, but the friction factor has a significant effect on the rolling pressure. When a suitable friction variation model is employed, the calculated results such as rolling pressure, spread and forward slip are in good agreement with the measured values. The friction variation model has also an effect on the shape and flatness of strip. The friction variation model constant K_1 and K_2 affect the strip velocity at exit, which affects the buckle of strip after rolling, and can easily cause the snaking of the strip during rolling. This is useful to determine the rolling conditions which result in a good shape and flatness of strip, in particular the rolling of thin strip. The modelling of the friction variation can produce a more accurate rolling process model that can improve the accuracy of simulation results.

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