Fractal Dimension of Ductile-Fractured Surface of Copper-Brazed Steel Calculated from Its Dimple Size Distribution*

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The fractal dimension is related to dimple size distribution for the fractured surface of the copper-brazed steel joint, which has a macroscopically flat and microscopically dimple-covered texture. Under the assumption that the distribution density function of dimple size is the power function of the dimple size, and that a dimple's shape is in the cosine curve, the fractal dimension D of the modeled surface is deduced to be $D=(\alpha-1)/2$, where α is the exponent of the power function of the dimple size distribution. For the experimentally fractured surface of a copper-brazed steel joint, the fractal dimensions calculated using the conventional height-height correlation method and the deduced formula show agreement in the tendencies to the brazing filler thickness, and the obtained relationship is confirmed. Considering the geometrical similarity between the fractured surface of the copper-brazed steel joint and the cup-and-corn type ductile-fractured surface, the obtained formula could be adapted to the more general ductile-fractured surfaces.

Key Words: Fractography, Fractal, Ductile Fracture, Dimple Size Distribution, Brazing, Copper

1. Introduction

When a diffusion-bonded joint of austenitic stainless steel is tensile-loaded in a liquid nitrogen atmosphere, its fractured surface shows a macroscopically flat surface akin to a brittle fractured surface. Its microscopic texture, however, bears numerous dimples indicating that it is a ductile-fractured surface. A similar macroscopically flat but microscopically dimple-covered surface is also found on the fractured surface of the copper-brazed steel joint. The rigid steel constrains the large deformation and a macroscopically flat surface is made, but the numerous microscopic dimples are generated on the copper surface because of its ductile nature. Such an inconsistency in surface texture in the observing scales appears clearly in the above particular cases, but it is also found in more general cases. The cup-and-corn type fractured surface has a macroscopically flat

surface surrounded by a steep shear lip. The flat surface in the center bears numerous microscopic dimples. Therefore, the macroscopically flat and microscopically dimple-covered surface can be a typical surface of the ductile fracture.

The microscopic dimples on the ductile-fractured surface result from the micro-voids' growth process inside the material. It is necessary to characterize the shape of a dimple and the size distribution of dimples to understand the ductile-fractured surface, but the fractured surface has a texture which is difficult to analyze.

In this research, to characterize the complex surface, the fractal characteristics are investigated for the macroscopically flat but microscopically dimple-covered surface. First, we attempt to obtain analytically the relationship between the statistical size distribution of dimples and the fractal dimension. For the deduction of this relationship, a model for the surface is suggested. The model is based on two assumptions: all dimples have similar and cosine shapes, and their size distribution is the power function of their sizes. In addition, the opened edges of the dimples are assumed to be arranged in a line; the

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edge of every dimple is at the same position in height direction. Thereafter, for the experimentally obtained surface, the values of the fractal dimension calculated by the deduced formula are compared with those calculated by the conventional height-height correlation method⁽¹⁾. The surface used is a tensile-fractured surface of a copper-brazed joint, which has similar geometrical features to those of the diffusion-bonded surface, although without the effect of grain size.

In the following, the model for the ductile-fractured surface is described in section 2. The experimental procedures for obtaining the fractured surfaces and the results are given in section 3. Then the conventionally calculated fractal dimension is shown in section 4, and is compared with the fractal dimension calculated using the deduced formula in section 5.

2. Dimple Size Distribution and Fractal Dimension

The generation process of the ductile-fractured surface consists of the nucleation of voids, their growth and their coalescence. The growth of an advanced void induces the nucleation of other voids. Therefore, the number of voids varies by their sizes. The dimples on the fractured surface are the half parts of the voids. The principles of the void induction process may give some relationship between the sizes of dimples and their number. In the case of the fractured surface of the copper-brazed steel joint, it is macroscopically flat. Taking these characteristics into consideration, the following hypotheses are initially made in order to simplify the consideration of the structure of the ductile-fractured surface.

(1) The shape of all dimples are in similar geometry and their size can be represented by some reference length, e.g., the width of the opened edges or the depth of a dimple (Fig. 1(a)).

(2) The opened edges of dimples are on the same level, i.e., there is no distribution of position in the height direction (Fig. 1(b)).

(3) The density distribution function of dimple size can be defined.

Assumption (2) reflects the macroscopically flat

Fig. 1 Schematics of our assumptions for dimple-covered ductile surface. (a) Shape of a dimple, (b) Opened edges of dimples arranged in line

texture of the generated fractured surface of a copper-brazed steel joint. Assumptions (1) and (3) also reflect the connection of dimple size and the generation process.

The shape of a dimple is axisymmetric and can be described in the cosine function form.

$$z_{\lambda}(x) = k\lambda \cos(2\pi x/\lambda). \quad (1)$$

The depth of the above dimple $k\lambda$ is proportional to its size λ (Fig. 1(1)).

The distribution density function of dimple size is assumed to be subject to the following functions.

$$p(\lambda) = \begin{cases} \beta\lambda^a & (\lambda_1 < \lambda < \lambda_2) \\ 0 & (\lambda < \lambda_1, \lambda_2 < \lambda) \end{cases} \quad (2)$$

The exponent a and the coefficient β depend on the fractured surface.

For the surface profile $z(x)$ consisting of the above dimples' shape and the size distribution function, the height-height correlation (HHC)⁽¹⁾ function $C(\tau)$ is used to calculate the fractal dimension.

$$C^2(\tau) = \{z(x+\tau) - z(x)\}^2 \\ \text{or } \frac{1}{L} \int_0^L \{z(x+\tau) - z(x)\}^2 dx \quad (3)$$

The HHC function for a single dimple $C_{\lambda}(\tau)$ is also denoted as follows:

$$C_{\lambda}^2(\tau) = \frac{1}{\lambda} \int_0^{\lambda} \{z_{\lambda}(x+\tau) - z_{\lambda}(x)\}^2 dx \\ = 2k^2\lambda^2 \sin^2(\pi\tau/\lambda) \quad (4)$$

The integration of this HHC for a single dimple with the weight of the distribution density function approximates to the HHC for the whole profile.

$$C^2(\tau) = \frac{1}{L} \int_0^L \lambda C_{\lambda}^2(\tau) p(\lambda) d\lambda \quad (5)$$

Then the substitution of Eqs. (2) and (4) into Eq. (5) leads to the following expression of the HHC.

$$C^2(\tau) = \left(\frac{2\beta k^2}{L} \right) \int_{\lambda_1}^{\lambda_2} \lambda^{3-a} \sin^2\left(\frac{\pi\tau}{\lambda}\right) d\lambda \\ \text{or } \left(\frac{2\pi^{4-a}\beta k^2}{L} \right) \int_{\xi_1}^{\xi_2} \xi^{a-5} \sin^2 \xi d\xi \tau^{4-a} \quad (6)$$

where $\xi = \pi\tau/\lambda$.

By nature, the HHC for the profile of the fractal surface is proportional to the power function of the scale τ .

$$C(\tau) = C_0 \tau^{2-D} \quad (7)$$

The comparison between Eqs. (6) and (7) gives the fractal dimension for this surface profile model.

$$D = a/2 \quad (8)$$

When the dimples on the surface have the distribution density function with the term of a' , β' ,

$$p(\lambda) = \beta' \lambda^{-a'} \quad (\lambda_1 < \lambda < \lambda_2). \quad (9)$$

The distribution density function of dimple size for the given vertical section in this surface is subject to the same form of function but its exponent is $a = a' - 1$. Then the fractal dimension in Eq. (7)

Fig. 2 Secondary electron images of the fractured surface of copper-brazed steel joint. Brazing filler thickness is 7.6 μm .

becomes

$$D = (\alpha' - 1)/2. \quad (10)$$

In the above derivation, the fractal dimension is for the scale range of $\lambda_1 < \lambda < \lambda_2$. For the microscopic scale range smaller than λ_1 , the fractal dimension is considered to be $D=1.0$. The inner surface of the dimple might have the fractal dimension of unity because of its apparent smooth texture. On the other hand, for the macroscopic scale range of $\lambda_2 > \lambda$, the fractal dimension is the value reflecting the macroscopic texture of the surface.

3. Experimental Procedures & Results

The 50 mm $\times$ ϕ 12.0 mm steel bars of SGD3MS-D and S25C steel standardized in JIS were brazed with the brazing filler metal of foils of oxygen-free 99.99 mass% pure copper. A tungsten wire spacer was also inserted to maintain a certain thickness of the brazing filler layer. Three diameters of spacer were used: ϕ 50, ϕ 100 and ϕ 500 μm . Brazing was conducted in the vacuum furnace of 6.67×10^{-2} Pa under the load of 87.0 kPa. The temperature was increased up to 1473.5 K {1200°C} in 60 sec and kept for 60 sec. Then it was allowed to stand to cool. The observation of the cross section of the obtained joint showed that the brazing filler thickness was less than the spacer wire diameter. The average brazing filler thicknesses for specimens with the spacer diameters of ϕ 50, ϕ 100 and ϕ 500 μm were 7.6, 59.2 and 385.8 μm , respectively.

For the joint specimen, the Instron-type tensile fracture test was conducted at room temperature and the fractured surfaces were generated. The tensile rate was 500 $\mu\text{m/s}$, which was sufficiently slow to regard the stress state as being static.

All fractured surfaces were obtained at the brazed joints and both side surfaces were found to be thoroughly covered by the copper. This meant the specimens fractured at the brazing filler layer. Ma-

croscopically, the surfaces were almost flat (Fig. 2 (a)). The microscopic observation revealed the typical ductile-fractured surface bearing numerous dimples (Fig. 2(b)). The minimum size of dimples was approximately 2 μm (Fig. 2(c)). The inside surface of the dimples was apparently smooth.

4. Fractal Dimension of Surface Profile

The fractal dimension for the obtained ductile-fractured surface was calculated from the vertical cross section profile. The topography scanning electron microscope (ELIONIX Inc. ERA-800) was used to record the surface profiles of the fractured surfaces. Its resolution was nominally 1 nm in the height direction. From the surface profile $z(x)$, the fractal dimension was calculated using the height-height correlation (HHC) $C(\tau)$ (Eq.(3)).

As shown in Fig. 3, the double logarithm plot of τ and $C(\tau)$ showed the line bent at the large end like the claw of a hammer. To the linear part of the $C(\tau)$ curve, the power law relation (Eq.(7)) is fitted as a regression curve and its exponent, which is the slope in Fig. 3, was obtained. The slope seems to change at around the 10 μm scale. This suggests that the profiles are the fractal geometry, and their characteristics in the lower and larger scale ranges differ. In this analysis, the value of fractal dimension calculated from the slope for around 100 μm scale is used to representative value of the surface profile.

The obtained fractal dimension is shown in Fig. 6. For a specimen, several surface profiles were measured and the values of fractal dimension are the average of those calculated for each profile. The fractal dimension decreases with increase in the brazing filler thickness.

Fig. 3 Height-height correlations for the fractured surface of copper-brazed steel joint. Brazing filler thickness is $59.2\ \mu\text{m}$. Legend indicates the magnification during recording of the surface profile.

5. Fractal Dimension and Distribution Density Function

5.1 Measurement of distribution density function

The distribution density function of the dimple size on the obtained ductile surface is obtained by manual counting of dimples from the secondary electron micrograph.

The secondary electron image (SEI) was trimmed to a square of $200\ \mu\text{m}$ on a side, which was laid at the center of the images. The dimples inside the square were counted. On the SEI, the clear white line was considered as the ridge of a dimple. A circle inscribing in the closed line of the dimple ridge was drawn and its diameter was taken as the size of the dimple (Fig. 4). For each specimen, 12 images were taken and the dimple number and size were measured to compile the histogram of dimple size distribution. Eventually, the total number of dimples compiled for the histogram became more than 2000. For the histogram, the inverse exponent function was fitted using the least-squares method (Fig. 5).

For each thickness of brazing filler layer, three specimens were used to find the dimple size distribution. The average value of their exponents was used for calculation of the fractal dimension.

To measure the dimple size, other methods could be adopted instead of the superimposition of the in-

Fig. 4 Secondary electron image of the fractured surface of the copper-brazed steel joint. Brazing filler thickness is $7.6\ \mu\text{m}$. Inscribed circles are superimposed to enable measurement of the diameter of dimples.

scribed circle. For example, the circumscribed circle could be superimposed, where the size of each dimple measured would be greater than that measured by the inscribed circle. The dimple size distribution is expected to shift to the larger scale. Though the rate of size enlargement varies from dimple to dimple, the shape of the histogram of dimple size distribution would slightly change, because the compiled number of dimples is so large. Therefore, the value of the exponent obtained by the superimposition of inscribed circles is considered not to differ from that obtained by the superimposition circumscribed circle.

5.2 Fractal dimension

The fractal dimension calculated from the exponent of the size distribution is shown in Fig. 6. When the value of the fractal dimension is larger than 2 or smaller than unity, the surface profile cannot be regarded as fractal. In this paper, the fractal characteristics of surface profiles have already been confirmed by height-height correlation method as described in section 4. Therefore, when the values of fractal dimension are not in the suitable range of $1 < D < 2$, the validity of this model and its derivation is rejected. The results shown in Fig. 6 confirm that this model and derivation yield the suitable fractal dimension and they are acceptable as the model of ductile-fractured surfaces.

Although the values of these derived fractal dimensions are not equal to those obtained from the height-height correlation, but the tendency of its value of fractal dimension to the brazing filler thick-

Fig. 5 Distribution density function of the dimple size

ness is in agreement with those obtained from the height-height correlation.

6. Discussion

The equation deduced above eventually adopts a very simple form and this is due to the assumptions and simplification made during the deduction.

The assumption of the cosine-type dimple shape is made on the basis of the dimple generation process. The microvoids, which finally grow to form the dimples, are expected to have a 3-dimensional ellipsoidal shape⁽²⁾. Their coalescence begins with the necking of the remained pillars between the ellipsoids and the edges of dimples are formed⁽³⁾. Then the shape of a dimple might be hat-like with a round bottom and narrow edge. In Fig. 2(c), the center void has a round bottom and its left side edge runs to the next void but not in the direction extended from the bottom curve. To approximate the round hat dimple shape, the cosine function is used.

The similarity of dimples is introduced just for the first approximation. For the simplicity of the mathematical dealing, a dimple's shape is thought to be better described by a single parameter. If the depth of the dimple is not proportional to its width, the form of the fractal dimension will change. The amount of the change is left for the future study.

The dimple size distribution of the power function form is based on the empirical knowledge and Ishikawa's work⁽⁴⁾. As shown in Fig. 5, the distribution density of the dimple number tends to increase geometrically with the decrease of dimple size, and then suddenly drops at the minimum dimple size. The Poisson's distribution function or Fisher's distribution

Fig. 6 Comparison of fractal dimensions calculated from dimple size distribution and surface profile using height-height correlation. Both values are obtained for fractured surfaces of copper-brazed steel joints.

function⁽⁵⁾ may fit the tendency better. However, because the drop at the minimum size is so sharp, the difference between the power function and Poisson's or Fisher's function will be so small and the complexity introduced in the deduction by using those sophisticated functions may not be rewarded.

The lined edges of dimples reflect the flatness of the macroscopic texture, and this lined-edges assumption also simplifies the mathematical calculations. The ductile fracture of the tensile-loaded bar usually generates cup-and-corn type fractured surfaces. The lips of the fractured surfaces incline sharply but the fractured surface at the center is macroscopically flat. The proposed model can be applied to this kind of surface. However, the actual fractal dimension for these fractured surfaces is expected to be higher than the value calculated from the deduced formula. On these fractured surfaces, the dimples' edges differ in the height direction. The difference introduces greater complexity in the deduction of the relationship of the dimple size distribution and the fractal dimension, and may increase the value of the fractal dimension. For example, in the Fig. 6, the fractal dimension values calculated from the surface profile are always higher than those calculated from the model.

As mentioned above, the quantitative assessments are still intact, and the proposed model does not exactly reflect the ductile-fractured surface. But it is

revealed that the power-function-type distribution function may yield the fractal characteristics of the geometry, and if the ductile-fractured surface is essentially the fractal geometry, the dimple size distribution of the ductile-fractured surface may be subjected to the power function. The investigation of the process of making the dimple size distribution of the power-function-type is expected to lead to a clearer understanding of the ductile fracture process.

7. Summary

The fractal characteristics of the ductile-fractured surface observed in copper-brazed steel are considered to be originated from the size distribution of dimples. Under the assumption that the dimple shape is given by the axisymmetric cosine curve and that the density distribution function of the dimple size has the form of the inverse power function, the fractal dimension of the ductile-fractured surface was analytically calculated. The obtained relationship between the

exponent of the density distribution function of the dimple size and the surface fractal dimension is verified experimentally.

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