Experimental and Theoretical Evaluations of the Effect between Diameter-to-Thickness Ratio and Curvature-Rate on the Stability of Circular Tubes under Cyclic Bending*

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This paper presents the experimental and theoretical results of the effect between diameter-to-thickness ratio (D/t ratio) and curvature-rate on the response and collapse of circular tubes subjected to cyclic bending. In experimental tests, four different D/t ratios of circular tubes and three different controlled curvature-rates were used. It was observed from experimental data that if circular tubes with a certain D/t ratio were used to test by three different curvature-rates, three parallel straight lines can be seen from the relationship between the controlled curvature and the number of cycles to produce buckling in log-log scale. In addition, it was also found that the distances among three parallel straight lines for the tubes with a higher D/t ratio are wider than that with a lower D/t ratio. Finally, theoretical formulations proposed by Lee and Pan⁽¹⁾ and Lee, et al.⁽²⁾ were combined and modified so that it can be used for simulating the relationship between the controlled curvature and the number of cycles to produce buckling for circular tubes with different D/t ratios subjected to cyclic bending with different curvature-rates. The theoretical simulation was compared with the experimental data. Good agreement between the experimental and theoretical results has been achieved.

Key Words: Circular Tube, Diameter-to-Thickness Ratio, Curvature-Rate, Stability, Cyclic Bending

1. Introduction

The circular tube components in a number of practical industrial applications, such as offshore structures, nuclear reactor components, earthquake resistant structures, transporting tubes of heat exchanger, ... etc., must be designed to resist cyclic bending. The major characteristic of the circular tube under bending is the nonlinear behavior of ovalization of the tube cross-section. It is known that the magnitude of the tube ovalization increases when the bending moment increases⁽³⁾. If the bending

moment increases cyclically, the magnitude of the ovalization also increases in ratcheting manner with the number of cycles^{(4)–(6)}. Such increase in ovalization of tube cross-section causes a progressive reduction in its bending rigidity, which can ultimately result in buckling or fracture of the circular tube. Therefore, studies concerning the response and buckling of circular tubes subjected to cyclic bending are very important for many industrial applications.

It is shown from experimental investigations that engineering alloys such as 304 and 316 stainless steels and high-strength titanium alloys, exhibit rate-dependent behavior^{(7)–(9)}. Therefore, once the pipe, which is fabricated by the aforementioned metals, is manipulated under bending with different loading-rates, the response of the metal tube for each loading-rate is expected to generate differently. In 1998, Pan and Fan⁽¹⁰⁾ experimentally investigated the effect of curvature-rate at preloading stage on subsequent creep or relaxation of circular tubes under pure bending. They discovered that the higher the applied

* Received 21st April, 2003 (No. 03-5053)

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curvature-rate, the greater is the degree of hardening of the tube at the preloading stage. However, the ovalization of the tube cross-section increases when the applied curvature-rate increases. After that, Pan and Her⁽¹¹⁾ extended the research to the viscoplastic collapse of circular tubes under cyclic bending. They found that the moment-curvature loops are sensitive to the curvature-rate. The higher the applied curvature-rate, the greater is the degree of hardening exhibited in the moment. The accumulation of the ovalization with the number of cycles for higher curvature-rate increases more rapidly than that for a slow curvature-rate. In addition, a formulation among the magnitude of the applied curvature, curvature-rate and the number of cycles to produce buckling was introduced. Lee and Pan⁽¹⁾ investigated the viscoplastic response and collapse of the titanium alloy tubes subjected to cyclic bending. They modified the theoretical formulation, which was proposed by Pan and Her⁽¹¹⁾, so that it can be used for simulating the relationship between the controlled curvature and number of cycles to produce buckling for titanium alloy tubes under cyclic bending with different curvature-rates.

Before 2000, All of the relative investigations are dealing with the tube response or collapse with a constant D/t ratio. However, it is known that the D/t ratio is a very important factor, which has a strong influence on the behavior of a circular tube under bending. In 2001, Lee, et al.⁽²⁾ firstly studied the influence of the D/t ratio on the response and stability of circular tubes under cyclic bending. To highlight the influence of the D/t ratio, the raw tubes were slightly machined on the outside surface to obtain the desired D/t ratio. They discovered that although four groups of tested specimens had four different D/t ratios, four parallel straight lines can be seen from the relationship between the controlled curvature and the number of cycles to produce buckling in log-log scale. Furthermore, a theoretical formulation was proposed to simulate the aforementioned phenomenon. Lee and Pan⁽¹²⁾ also investigated the influence of the D/t ratio, but the loading condition for their research was the pure bending creep (the pure bending creep is to bend a tube to a desired moment and then hold that moment for a long period of time). They also proposed a formulation that can be used to simulate the relationship between the creep curvature and time for SUS 304 stainless steel tubes with different D/t ratio under pure bending creep.

Based on the material from the aforementioned references, the curvature-rate effect and the D/t ratio effect have been shown to be of importance to the response and collapse of circular tubes subjected to bending. However, those two effects were investigated separately by the aforementioned references. In this paper, the effect between D/t ratio and curvature-rate on the response and collapse of circular tubes subjected to cyclic bending

was investigated. A four-point bending machine^{(10)–(12)} was used for conducting the present tests. The circular tube material chosen for this study was SUS 304 stainless steel. The curvature-ovalization measurement apparatus, designed by Pan, et al.⁽¹³⁾, was used for conducting the curvature-controlled cyclic bending test with different curvature-rates. Finally, a theoretical formulation, which was combined and modified the formulations obtained by Lee and Pan⁽¹⁾ and Lee, et al.⁽²⁾, was proposed so that it can be used for simulating the relationship between the controlled curvature and the number of cycles to produce buckling for circular tubes with different D/t ratios subjected to cyclic bending with different curvature-rates. The theoretical simulation was compared with the experimental data. It has been shown from the comparison between the simulation and the experimental data that the proposed formulation can properly simulate the experimental result.

2. Bending Test Facilities

In this study, a bending test facility, consisting of a pure bending device and a curvature-ovalization measurement apparatus, was used for a number of experiments on circular tubes under cyclic bending. The test facility is further described below.

2.1 Pure bending device

Figure 1 shows a schematic drawing of the bending device, which was designed as a four-point bending machine. The device consists of two rotating sprockets, about 30 cm in diameter, symmetrically resting on two support beams of 1.25 m apart. The maximum length of the test specimen was 1 m. The two sprockets support two rollers, which apply point loads as a couple at each end of the test specimen. Heavy chains run around these sprockets and are connected to two hydraulic cylinders and load cells forming a closed loop. Once either the top or bottom cylinder contracts, the sprockets rotate, and pure bending of the test specimen is achieved. The contact between the tube and the rollers is free to move along axial direction during bending. The load, transfer to the test specimen, is in the form of a couple formed by concentrated loads from the two rollers. A servo-hydraulic, feedback control system was developed to prescribe cyclic loading histories. The output from either the curvature-ovalization measurement apparatus or from the load-cell can be selected as

Fig. 1 A schematic drawing of the bending device

Fig. 2 A schematic drawing of the COMA

Fig. 3 Tube deformation between two side-inclinometers under pure bending

feedback for curvature or moment control, respectively. A detailed description of such a bending device can be found in literature (Kyriakides and Shaw⁽⁴⁾; Corona and Kyriakides⁽⁵⁾; Lee and Pan⁽¹⁾; Lee et al.⁽²⁾).

2.2 Curvature-Ovalization Measurement Apparatus (COMA)

Figure 2 shows a schematic drawing of the COMA. It is a lightweight instrument which can be mounted close to the tube's mid-span (Pan, et al.⁽¹³⁾). It can be used to monitor the changes in the major and minor diameters of the tube cross-section, using a magnetic detector (middle part of the COMA). Simultaneously, it can be used to measure variations in the tube curvature close to the mid-span from the signals of inclinometers. There are three inclinometers in the COMA. Two of them are fixed on two holders, (see Fig. 2). The holders are fixed on the circular tube before the test begins, and the angles of rotation detected by these two side-inclinometers are in the plane, which is the direction of the bending moment. Once the two holders are placed and fixed on the circular tube, the distance between the two side-inclinometers is fixed, and denoted as L_o . Consider that the circular tube is subjected to pure bending, the angle changes detected by the two side-inclinometers are denoted as θ_1 and θ_2 (see Fig. 3).

Fig. 4 Dimensions of four different D/t ratios of SUS 304 stainless steel tubes (in mm)

Due to the uniform bending in all sections, the circular tube, which was originally straight, is deformed into a circular arc. The distance between the center of this arc and the neutral surface is denoted as ρ (shown in Fig. 3). From Fig. 3, the value of L_o is determined as

$$L_o = \rho(\theta_1 + \theta_2). \quad (1)$$

The curvature of the tube κ is

$$\kappa = \frac{1}{\rho} = \frac{(\theta_1 + \theta_2)}{L_o} \quad (2)$$

Once the distance between the two side-inclinometers (L_o) is fixed and the angle changes (θ_1 and θ_2) of the two side-inclinometers are detected, the curvature of the tube can be determined from Eq. (2). Thus, the COMA can provide a direct measurement of the tube curvature.

3. Material and Experimental Procedures

In this section, we discuss the specimens and the test procedures of circular tubes subjected to cyclic bending with different curvature-rates. Specimens and the test procedures are given below:

3.1 Material and specimens

The material used in this study was hot-rolled SUS 304 stainless steel circular tubes with the chemical composition of Cr 18.36, Ni 8.43, Mn 1.81, Si 0.39, C 0.05, P 0.28, S 0.04, and the remainder Fe. The yield stress is 205 MPa, the tensile ultimate stress is 520 MPa and the percent elongation is 35%. The test specimens originally had a nominal outside diameter D of 38.1 mm and a wall thickness t of 1.5 mm ($D/t = 25.4$). To obtain the desired D/t ratio, the as-received tubes were slightly machined on the outside surface. The magnitudes of the D/t ratio were selected to be 60, 50, 40 and 30 in this study. Figure 4 shows the dimensions of four different D/t ratios of tested tubes.

3.2 Test procedures

The cyclic bending test was conducted by using the bending machine described in section 2.1. The test was a curvature-controlled cyclic bending test with curvature amplitudes varying from ± 0.1 to $\pm 0.7 \text{ m}^{-1}$. Tube was cyclically bent at three different curvature-rates, 0.003, 0.03 and $0.3 \text{ m}^{-1}\text{s}^{-1}$. The magnitude of the curvature was controlled and measured by the COMA, which also measured the ovalization of tube cross-section. The bending moment can be calculated from the signals detected by the two load cells, mounted on the bending device. In addition, the number of cycles to produce buckling was also being recorded.

4. Experimental Results

In this section, the experimental data of SUS 304 stainless steel tubes with different D/t ratios subjected to cyclic bending for different curvature-rates are discussed. The D/t ratios of SUS 304 stainless steel tubes are 30, 40, 50 and 60, respectively. The controlled curvature-rates are 0.003, 0.03 and $0.3 \text{ m}^{-1}\text{s}^{-1}$, respectively. The mechanical behavior and the collapse of SUS 304 stainless steel tubes under cyclic bending are discussed separately in the following.

4.1 Mechanical behavior of SUS 304 stainless steel tubes under cyclic bending

Figure 5 (a)–(c) shows typical experimental results of cyclic moment (M)–curvature (κ) curves for SUS 304 stainless steel tubes with a D/t ratio of 30 under three different curvature-rates of 0.003, 0.03 and $0.3 \text{ m}^{-1}\text{s}^{-1}$, respectively. The curvature range is $\pm 0.3 \text{ m}^{-1}$. It can be seen that the tube was cyclically hardened and the loop becomes steady after a few cycles. In addition, the higher the applied curvature, the greater is the degree of the hardening exhibited in the moment. Similar phenomenon as shown in Fig. 5 (a)–(c) can also be found for SUS 304 stainless steel tubes with D/t ratios of 40, 50 and 60 under three different curvature-rates of 0.003, 0.03 and $0.3 \text{ m}^{-1}\text{s}^{-1}$. Figure 6 (a)–(d) presents typical experimental results of the moment (M)–curvature (κ) curves for SUS 304 stainless steel tubes with the D/t ratios of 30, 40, 50 and 60, respectively. However, the controlled curvature-rate is $0.003 \text{ m}^{-1}\text{s}^{-1}$. It is shown that due to the thinner thickness of the circular tube with a higher D/t ratio, a lower magnitude of the bending was needed to bend the tube into the desired curvature. Similar phenomenon as shown in Fig. 6 (a)–(d) can also be observed for SUS 304 stainless steel tubes with four different D/t ratios under different curvature-rates of 0.03 and $0.3 \text{ m}^{-1}\text{s}^{-1}$.

Figure 7 (a)–(c) shows the corresponding experimental results of cyclic ovalization ($\Delta D/D$)–curvature (κ) curves for Fig. 5 (a)–(c). The ovalization is defined of $\Delta D/D$, where D is the outside diameter and ΔD is the change in outside diameter. It can be noted that the oval-

Fig. 5 Typical experimental moment (M)–curvature (κ) curves under cyclic bending with $D/t = 30$ for curvature-rates of (a) 0.003, (b) 0.03, (c) $0.3 \text{ m}^{-1}\text{s}^{-1}$

Fig. 6 Typical experimental moment (M)–curvature (κ) curves under cyclic bending for curvature-rates of $0.003 \text{ m}^{-1}\text{s}^{-1}$ with D/t : (a) 30, (b) 40, (c) 50, and (d) 60

ization of the tube cross-section increases in a ratcheting manner with the number of cycles. Furthermore, the higher degree of the ovalization of the tube cross-section can be noticed under higher curvature-rate. Similar phenomenon as shown in Fig. 7 (a)–(c) can also be found for tubes with D/t ratios of 40, 50 and 60 under three different curvature-rates of 0.003 , 0.03 and $0.3 \text{ m}^{-1}\text{s}^{-1}$. Figure 8 (a)–(d) demonstrates the corresponding experimental results of cyclic ovalization ($\Delta D/D$)–curvature (κ) curves for Fig. 6 (a)–(d). It was found that the higher the D/t ratio for a circular tube, the faster the accumulation of ovalization in the tube cross-section. Persistent cycling eventually leads to buckling. Similar phenomenon as shown in Fig. 8 (a)–(d) can also be observed for tubes with four different D/t ratios under different curvature-rates of 0.03 and $0.3 \text{ m}^{-1}\text{s}^{-1}$.

4.2 Collapse of SUS 304 stainless steel tubes under cyclic bending

The SUS 304 stainless steel tubes were cyclically bent to collapse. The number of cycles to produce buckling was recorded for every tested specimen. Figures 9 (a), 9 (b), 10 (a) and 10 (b) show the experimental results of controlled cyclic curvature range (κ_c) vs. number of cycles to produce buckling (N_b) relationship with D/t ratios of 30, 40, 50 and 60, respectively. In each figure, three different curves indicate three different curvature-rates. The corresponding experimental results in Figs. 11 (a), 12 (a), 13 (a) and 14 (a) presents the κ_c vs. N_b of Figs. 9 (a), 9 (b), 10 (a) and 10 (b) in log–log scale. Three straight dot lines, determined by the least square method, denote three different curvature-rates in each figure. It can be seen that these three straight lines are almost parallel to each other. Although a factor of ten times difference for each

Fig. 7 Typical experimental ovalization ($\Delta D/D$)–curvature (κ) curves under cyclic bending with $D/t = 30$ for curvature-rates of (a) 0.003, (b) 0.03, (c) $0.3 \text{ m}^{-1}\text{s}^{-1}$

curvature-rate is controlled (0.003 , 0.03 and $0.3 \text{ m}^{-1}\text{s}^{-1}$), the straight line corresponding to the higher curvature-rate deviates from the straight lines corresponding to two lower curvature-rates. In addition, it was also found that the distances among three parallel straight lines for circular tubes with a higher D/t ratio are wider than that with a lower D/t ratio.

5. Discussion

In 1987, Kyriakides and Shaw⁽⁵⁾ first proposed an empirical relationship between controlled cyclic curvature range (κ_c) and number of cycles to produce buckling (N_b) for circular tubes subjected to cyclic bending to be

$$\kappa_c = A(N_b)^{-\alpha} \quad (3)$$

where A and α are the material parameters, which are related to the material properties and the D/t ratio. The material parameter A is the controlled cyclic curvature magnitude at $N_b = 1$, and α is the slope in the log–log plot. Based on the experimental data of circular tubes with $D = 31.75 \text{ mm}$ and $t = 0.889 \text{ mm}$ ($D/t = 35.7$), reported by Kyriakides and Shaw⁽⁵⁾, the magnitudes of A and α were calculated to be: $A = 0.968 \text{ m}^{-1}$ and $\alpha = 0.078$ for 1018 steel and $A = 0.988 \text{ m}^{-1}$ and $\alpha = 0.078$ or 0.12 for 6061-T6 aluminum.

In 1998, Pan and Her⁽¹¹⁾ experimentally discovered that three different parallel straight lines for κ_c vs. N_b in log–log scale can be seen to correspond three different controlled curvature-rates for circular tubes under cyclic bending. Although a factor of ten times difference for each curvature-rate was controlled (0.003 , 0.03 and $0.3 \text{ m}^{-1}\text{s}^{-1}$), the straight line corresponding to the higher curvature-rate deviated from the straight lines corresponding to two lower curvature-rates. Since three straight lines exhibit the same magnitude of slope but with different intersections on ordinate, a modified empirical formulation was proposed to be

$$\kappa_c = B(N_b)^{-\alpha} \quad (4)$$

where B is a function of curvature-rate, and was expressed to be

$$B = B_o + \beta \left[\log \frac{\dot{\kappa}}{\dot{\kappa}_o} \right]^2 \quad (5)$$

where B_o is the material parameter for lowest curvature-rate and $\dot{\kappa}_o$ is the lowest curvature-rate, $\dot{\kappa}$ is the other relative curvature-rate and β is the material parameter. Equation (5) was inspired by the investigation of viscoplastic behavior of material under multiaxial loading (Pan and Chern⁽¹⁴⁾). In their study, a formulation of the rate-sensitivity function K of endochronic theory was proposed to be

$$K = 1 + k_a \left[\log \frac{\dot{\epsilon}_{eq}^p}{(\dot{\epsilon}_{eq}^p)_o} \right] \quad (6)$$

where k_a is the material parameter, $(\dot{\epsilon}_{eq}^p)_o$ is the reference equivalent deviatoric plastic strain-rate (usually use the

Fig. 8 Typical experimental ovalization ($\Delta D/D$)–curvature (κ) curves under cyclic bending for curvature-rates of $0.003 \text{ m}^{-1} \text{ s}^{-1}$ with D/t : (a) 30, (b) 40, (c) 50, and (d) 60

lowest strain-rate) and $\dot{\epsilon}_{eq}^p$ is the relative equivalent deviatoric plastic strain-rate. For circular tubes of SUS 304 stainless steel with D/t ratio of 50 under cyclic bending with different curvature-rates, the magnitudes of B_o , α and β in Eqs. (4) and (5) were determined to be 0.357 m^{-1} , 0.118 and -0.017 , respectively.

In 2001, Lee and Pan⁽¹⁾ investigated the curvature-rate effect on the titanium alloy tubes subjected to cyclic bending. They found that three different parallel straight lines for κ_c vs. N_b in log–log scale can be seen to correspond three different controlled curvature-rates for titanium alloy tubes under cyclic bending. A factor of ten times difference for each curvature-rate was controlled (0.003 , 0.03 and $0.3 \text{ m}^{-1} \text{ s}^{-1}$). However, the straight line corresponding to the lower curvature-rate deviated from

the straight lines corresponding to two higher curvature-rates (This phenomenon is different from that of SUS 304 stainless steel tubes tested by Pan and Her⁽¹¹⁾). Therefore, Lee and Pan⁽¹⁾ modified the Eq. (5) to be

$$B = B_o + \beta \left[\log \frac{\dot{\kappa}}{\dot{\kappa}_o} \right]^n \quad (7)$$

where n is the material parameter. For circular tube of SUS 304 stainless steel, the magnitude of n is equal to 2 (which was used by Pan and Her⁽¹¹⁾). In their investigation, the magnitudes of B_o , α , β and n for titanium alloy tubes with D/t ratio of 36.3, were determined to be 1.5529 m^{-1} , 0.191 , 0.1049 and 0.9504 , respectively.

For the influence of the D/t ratio on the collapse of circular tubes under cyclic bending, Lee, et al.⁽²⁾ discovered that although four groups of tested specimens have

(a)

(b)

Fig. 9 Experimental curves of controlled cyclic curvature range (κ_c) vs. number of cycles to produce buckling (N_b) under three different curvature-rates for (a) $D/t=30$, (b) $D/t=40$

(a)

(b)

Fig. 10 Experimental curves of controlled cyclic curvature range (κ_c) vs. number of cycles to produce buckling (N_b) under three different curvature-rates for (a) $D/t=50$, (b) $D/t=60$

four different D/t ratios (30, 40, 50 and 60), four parallel straight lines can be seen from the κ_c vs. N_b curves in log-log scale. In addition, for considering Eq. (4), they found that the experimental data for the relationship between B and D/t ratio in log-log scale fell into a straight line. Therefore, an empirical formulation between B and D/t ratio was proposed

$$B = C_o(D/t)^{-\gamma} \text{ or } \log B = \log C_o - \gamma \log(D/t) \quad (8.a,b)$$

where C_o and γ are the material parameters which can be determined from the relationship between B and D/t in log-log scale. The magnitude of $\log C_o$ is the line intersection, and γ is the slope of the line. For SUS 304 stainless steel tubes with D/t ratios of 30, 40, 50 and 60, the material parameter α is the same as the magnitude of α determined by Pan and Her⁽¹¹⁾. In addition, the parameters C_o and γ were found to be 95.058 m^{-1} and 1.406.

For combining the curvature-rate effect and the D/t ratio effect, a simple theoretical formulation was proposed in this paper. The relationship between κ_c vs. N_b was also used the form in Eq. (4). However, a simple linear combination of Eqs. (7) and (8.a) was proposed to be

$$B = C_o(D/t)^{-\gamma} + \beta \left[\log \frac{\dot{\kappa}}{\dot{\kappa}_o} \right]^n \quad (9)$$

For Eq. (9), if we consider the curvature-rate effect

only. Thus, the D/t ratio is a constant. The first term of B in Eq. (9) is a constant. Therefore, Eq. (9) is equal to Eq. (7). Next, for considering D/t ratio effect only, the other relative curvature rate $\dot{\kappa}$ is a constant. Thus, the second term of B in Eq. (9) is a constant. Lee, et al.⁽²⁾ investigated the influence of the D/t ratio on the collapse of circular tubes under cyclic bending. In their experiment, the lowest curvature-rate $\dot{\kappa}_o = 0.003 \text{ m}^{-1}\text{s}^{-1}$ was used in their test. Once the other relative curvature rate $\dot{\kappa}$ is equal to lowest curvature-rate $\dot{\kappa}_o$, the second term of B in Eq. (9) is zero. Thus, Eq. (9) is equal to Eq. (8.a). In conclusion, Eq. (9) is a general form of B for the effect of D/t ratio and the curvature-rate. The functions of B in Eqs. (7) and (8.a) are the special cases of the function B in Eq. (9).

In this paper, the parameters $\alpha = 0.118$, $C_o = 95.058 \text{ m}^{-1}$ and $\gamma = 1.406$ found by Lee, et al.⁽²⁾, and parameters $\beta = -0.017$ and $n = 2$ found by Pan and Her⁽¹¹⁾ were used. Based on Eqs. (4) and (9), the correlated results for D/t ratios of 30, 40, 50 and 60 are shown in Figs. 11(b), 12(b), 13(b) and 14(b). Good agreement between the theoretical and experimental results has been achieved when compared with the experimental results in Figs. 11(a), 12(a), 13(a) and 14(a).

Fig. 11 Controlled cyclic curvature range (κ_c) vs. number of cycles to produce buckling (N_b) under three different curvature-rates for $D/t = 30$ (log-log scale). (a) Approximation by least-square method, (b) simulation by present formulation

6. Conclusions

The effect between the diameter-to-thickness ratio (D/t ratio) and the curvature-rate on the response and collapse of circular tubes subjected to cyclic bending is investigated in this study. A tube bending machine and a curvature-ovalization measurement apparatus were used for conducting the cyclic bending tests. The following important conclusions can be drawn from this investigation:

(1) For certain amount of D/t ratio under three different curvature-rates, the higher the applied curvature, the greater is the degree of the hardening and the higher degree of the ovalization of the tube cross-section. For a certain amount of controlled cyclic curvature range, the number of cycles to produce buckling is found to be reduced for higher curvature-rate.

(2) For certain amount of curvature-rate, tubes with smaller outside diameters (higher D/t ratios) need a lower magnitude of bending moment to bend the tube into a desired curvature. Furthermore, the higher the D/t ratio for a circular tube, the fast the accumulation of ovalization in the tube cross-section. For a certain amount of controlled cyclic curvature range, tubes with smaller D/t ratios re-

Fig. 12 Controlled cyclic curvature range (κ_c) vs. number of cycles to produce buckling (N_b) under three different curvature-rates for $D/t = 40$ (log-log scale). (a) Approximation by least-square method, (b) simulation by present formulation

quire fewer number of cycles to produce buckling than those with larger D/t ratios.

(3) For each D/t ratio, three almost parallel straight lines for κ_c vs. N_b relationship in log-log scale can be seen when SUS 304 stainless steel tubes were conducted under cyclic bending with three different curvature-rates. Besides, it was also found that the distances among three parallel straight lines with a higher D/t ratio are wider than that with a lower D/t ratio.

(4) A formulation, which included the influence of the curvature-rate and D/t ratio, between the κ_c and N_b was proposed in this paper. The formulation was used to simulate the experimental results of SUS 304 stainless steel tubes with four different D/t ratios under cyclic bending with three different controlled curvature-rates. It can be found from Figs. 11 (b), 12 (b), 13 (b) and 14 (b) that the formulation can properly simulated the experimental results.

Acknowledgements

The work presented was carried out with the support of National Science Council under grant NSC 91-2212-E-

Fig. 13 Controlled cyclic curvature range (κ_c) vs. number of cycles to produce buckling (N_b) under three different curvature-rates for $D/t = 50$ (log-log scale). (a) Approximation by least-square method, (b) simulation by present formulation

Fig. 14 Controlled cyclic curvature range (κ_c) vs. number of cycles to produce buckling (N_b) under three different curvature-rates for $D/t = 60$ (log-log scale). (a) Approximation by least-square method, (b) simulation by present formulation

006-093. Its support is gratefully acknowledged.

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