Plasticity-Creep Separation Method for Viscoelastic Deformation of Lead-Free Solders

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This paper applies a constitutive model proposed previously by the authors to three lead-free solder alloys of Sn/Ag, Sn/Bi and Sn/Zn. First, the material constants in the constitutive model are determined using the so-called “plasticity-creep separation method” by simple tests such as pure tensile tests. The constitutive model is incorporated into a general purpose Finite Element Method program ANSYS using the stress integration method. The material constants for the lead-free solders could be simply determined using only the data obtained by the pure tensile tests with three strain rates. The basic mechanical deformation such as creep and cyclic deformation are simulated by the constitutive model using the material constants determined using the “plasticity-creep separation method”. Thermal deformation during a reflow process with electronic packaging is also simulated by the constitutive model.

Key Words: Lead-Free Solder Alloy, Constitutive Model, Finite Element Method, Creep, Cyclic Loading, Strain Rate Effect, Temperature Effect, Electronic Packaging, Stress Relaxation, Residual Stress

1. Introduction

Environmental pollution caused by lead liquation from solder in waste electronic machinery has become a matter of serious concern, and immediate practical application of alternative solder alloys which contain no lead is required for electronic packaging. A great variety of chemical compositions for lead-free solder alloys have been proposed, and many studies on lead-free solder alloys have been carried out[1]–[5]. Suganuma et al.[1] evaluated the wettability of Sn-Ag solder alloys for Cu and joint strength using these solders: they also investigated the effect of the Ag content on the wettability and strength. Pan et al.[2] examined the process generating intermetallic compounds during the soldering process using lead-free solder alloys.

To select the best fitting solder alloys for electronic part mounting, the thermal fatigue strength must be estimated accurately. An accurate estimation of the thermal fatigue strength requires a constitutive model which accurately describes the viscous deformation of solder alloys caused by mechanical or thermal loading, this is because the estimation is usually conducted by a structural analysis of FEM[6], in which the constitutive model is incorporated. To achieve this, studies on viscoplastic constitutive models of solder alloys have been carried out. Desai et al.[7],[8] constructed a constitutive model for thermomechanical deformation and applied the model to Sn/Pb solder alloys. Maciucescu et al.[9] described viscoplastic deformation of Pb/Sn solder alloys using a constitutive model based on overstress. Kobayashi et al.[10] applied the Ohno and Wang model together with the steady creep law to the viscoplastic deformation of Sn-3.5Ag-0.5Cu solder alloy considering the method of implementation of the model in an FEM program.

Most constitutive models proposed require the determination of a numerous material constants using empirical methods, often trial and error, and much effort must be invested in estimating material constants. To be able to adapt the constitutive models to the selection of suitable solder alloys.
lead-free solder alloys the material constants employed in the constitutive model should be simple to determine. A previous paper\textsuperscript{(11)} proposed an inelastic constitutive model for describing viscoplastic deformation of Sn-40Pb lead solder alloy, and also proposed a numerical method termed “plasticity-creep separation method” to estimate the material constants in the model. The material constants of Sn-40Pb solder alloy were successfully determined by simple tests such as pure tensile tests at just a few strain rates using the “plasticity-creep separation method”. The determined material constants can be adapted to simulate cyclic deformation, creep, and stress relaxation of the Sn-40Pb solder alloy. This study discusses the applicability of the constitutive model to lead-free solder alloys. First the “plasticity-creep separation method” is used to determine the material constants employed in the constitutive model with pure tensile tests under three strain rates at three temperatures using three lead-free solder alloys, Sn-3.5Ag-0.75Cu, Sn-7.5Zn-3.0Bi, and Sn-57Bi-1.0Ag. To incorporate the constitutive model into a general purpose FEM program “ANSYS” an implementation method is also discussed. Some simulations of basic deformation types such as creep deformation and cyclic deformation are conducted using the material constants determined by the “plasticity-creep separation method”. Finally an FEM analysis of the electronic parts mounting process is also conducted to clarify the warping behavior of electronic packaging during the reflow process.

### 2. Constitutive Model

The total strain \( \varepsilon \) is expressed by the sum of the elastic, plastic and creep strains as follows.

\[
e = \varepsilon^e + \varepsilon^p + \varepsilon^c \tag{1}
\]

where, \( \varepsilon^e \), \( \varepsilon^p \), and \( \varepsilon^c \) are the elastic, plastic, and creep strains, respectively.

The elastic strain \( \varepsilon^e \) in Eq. (1) is obtained by the following Hooke’s law.

\[
\varepsilon^e = D^{-1} : \sigma \tag{2}
\]

where \( D^e \) is the elastic tensor, and \( \sigma \) is the stress.

To incorporate the constitutive model into an FEM program and to apply the constitutive model to the plasticity-creep separation method, the plastic strain \( \varepsilon^p \) and creep strain \( \varepsilon^c \) in Eq. (1) are expressed in the following discrete form using the backward Euler’s method:

\[
e^p_{i+1} = \varepsilon_i^p + \Delta \varepsilon^p \tag{3}
\]

\[
e^c_{i+1} = \varepsilon_i^c + \Delta \varepsilon^c \tag{4}
\]

where the subscripts \( i \) and \( i+1 \) denote the start and end of a calculation step, \( \Delta \varepsilon^p \) and \( \Delta \varepsilon^c \) are the strain increments of the plastic and creep strains in the current calculation step.

The plastic strain increment \( \Delta \varepsilon^p \) in Eq. (3) is given by

\[
\Delta \varepsilon^p = \frac{3}{2 \bar{\sigma}^p} \frac{\Delta \bar{\sigma}^p}{H^p} (s_{i+1} - b_{i+1}) \tag{5}
\]

where \( H^p \) is the plastic tangent modulus, \( s \) is the deviatoric stress and \( \bar{\sigma}^p \) is the scalar obtained by the following equation:

\[
\bar{\sigma}^p = \sqrt{3/2 (s - b) : (s - b)} \tag{6}
\]

\( b \) in Eqs. (5) and (6) is the deviatoric back stress for the describing the cyclic deformation due to both mechanical and thermal loadings, and it is also expressed in the following discrete form.

\[
b_{i+1} = b_i + \Delta b \tag{7}
\]

where \( \Delta b \) is the deviatoric back stress increment, and is given by

\[
\Delta b = 2/3 H' \Delta \varepsilon^p \tag{8}
\]

The creep strain increment \( \Delta \varepsilon^c \) in Eq. (4) is expressed as follows.

\[
\Delta \varepsilon^c = [(1 - \theta) \dot{\varepsilon}_i^c + \theta \dot{\varepsilon}_{i+1}^c] \Delta t \tag{9}
\]

where \( \theta \) is the time integration parameter which takes values from 0 to 1; \( \Delta t \) is the time increment in the current calculation step and \( \dot{\varepsilon}_i^c \) and \( \dot{\varepsilon}_{i+1}^c \) are the creep strain rates at the start and end of a calculation step.

The creep strain rate \( \dot{\varepsilon}_i^c \) is given by

\[
\dot{\varepsilon}_i^c = (1 + c_1 \exp(-\varepsilon_i^c/(c_2))) \cdot 3/2 \cdot A \sigma_i^{n-1} s_i \tag{10}
\]

where \( c_1, c_2, A \) and \( n \) are the material constants, \( \varepsilon_i^H \) is the hardening variable expressed as follows.

\[
\dot{\varepsilon}_i^H = \dot{\varepsilon}_i^c - \min(\dot{\varepsilon}_i^N) \tag{11}
\]

where \( \min(\dot{\varepsilon}_i^N) \) is the equivalent creep strain just after the \( N \)th loading reverse occurs, while the hardening variable \( \varepsilon_i^H \) is the accumulated equivalent creep strain after the \( N \)th loading reverse and the initial value of \( \min(\dot{\varepsilon}_i^N) \) is 0.

### 3. Plasticity-Creep Separation Method

#### 3.1 Experimental procedure

The material constants required for the proposed constitutive model are estimated by the plasticity-creep separation method. In the method, the creep strains are removed from the total strain of the pure tensile tests conducted under at least three strain rates to obtain the time independent relationships between the stress and elastoplastic strain.

In this work, the pure tensile tests were performed under three strain rates and three temperatures as shown in Table 1. The specimens were made of the lead-free solder alloys of Sn-3.5Ag-0.75Cu, Sn-7.5Zn-3.0Bi, and Sn-57Bi-1.0Ag. Cylindrical ingots of these solder alloys were machined into specimens as shown in Fig. 1. After the machining, the specimens were annealed at 0.87T_m \textsuperscript{(12)} (\( T_m \) is the melting temperature in Kelvin).
3.2 Stress-strain relations of lead-free solder

As an example, Fig. 2 shows the stress-strain relations of the three lead-free solder alloys under three different strain rates, 0.5, 0.05, and 0.005%/s, at 303 K. Strain rate effects were observed in all the solder alloys. The areas in the large circles in Fig. 2 show the regions where the strain hardening was saturated. In these regions, the strain increased under constant stress and strain rates. This shows that the strain occurs with a mechanism similar to the steady state creep. Defining the constant stress as \( \sigma_{\text{lim}} \), the relationship between \( \sigma_{\text{lim}} \) and the strain rate can be expressed by a steady state creep law such as the following Norton’s law.

\[
\dot{\varepsilon}_t = \frac{3}{2} \cdot \frac{A}{\sigma_{\text{lim}}^n} \cdot \varepsilon_t
\]

where \( \dot{\varepsilon}_t \) is the strain rate and the subscript \( t \) denotes the uniaxial case such as in tensile tests. Then, the material constants \( A \) and \( n \) in Eq. (12) can be estimated from the pure tensile tests.

3.3 Numerical solution for plasticity-creep separation

To apply the plasticity-creep separation to the stress-strain relations of solder alloys, Eq. (10) is rewritten as,

\[
\dot{\varepsilon}_c = \dot{\varepsilon}_c^1 + \dot{\varepsilon}_c^2 \quad (13)
\]

\[
\dot{\varepsilon}_c^1 = 3/2 \cdot A \sigma_{\text{lim}}^{n-1} \varepsilon_t \quad (14)
\]

\[
\dot{\varepsilon}_c^2 = c_1 \exp(-\varepsilon_H / \varepsilon_0) \cdot 3/2 \cdot A \sigma_{\text{lim}}^{n-1} \varepsilon_t \quad (15)
\]

Eqs. (14) and (15) are the steady and transient creep parts in Eq. (10), respectively.

Assuming that the time independent elasto-plastic strain \( \varepsilon_{\text{EP}} \) is obtained by

\[
\varepsilon_{\text{EP}} = \varepsilon - (\varepsilon_c^1 + \varepsilon_c^2) \quad (16)
\]

where \( \varepsilon \) is the total strain, and \( \varepsilon_c^1 \) and \( \varepsilon_c^2 \) are the creep strains calculated by Eqs. (14) and (15), respectively.

The relationship between the stress \( \sigma \) and elasto-plastic strain \( \varepsilon_{\text{EP}} \) in the uniaxial case is also assumed to be expressed by the Ramberg-Osgood law as

\[
\varepsilon_{\text{EP}} = \frac{\sigma}{E} + \varepsilon_0 \left( \frac{\sigma}{D} \right)^m \quad (17)
\]

where \( E \) is Young’s modulus, \( D \) is the reference stress at a plastic strain \( \varepsilon_0 \) of \( 5.00 \times 10^{-4} \), and \( m \) is a hardening exponent. The subscript \( t \) denotes the uniaxial loading case.

Using Eqs. (14)–(17), plasticity-creep separation is performed on the stress-strain relations arising from the loading conditions in Table 1, and all material constants in the proposed model are estimated. The numerical solution for the plasticity-creep separation was calculated as follows:

(i) Determine the material constants \( A \) and \( n \) from the approximated relations between the constant stress \( \sigma_{\text{lim}} \) and strain rate \( \dot{\varepsilon}_t \) at each temperature.

(ii) Calculate the axial creep strain \( \varepsilon_c^1 \) using Eq. (14) with the material constants \( A \) and \( n \) determined in step (i).

(iii) Subtract \( \varepsilon_c^1 \) from each stress-strain relation, and define the obtained results as the temporary elasto-plastic strain \( \varepsilon_{\text{EP}}^{\text{t}} \).

(iv) Approximate the \( \sigma - \varepsilon_{\text{EP}}^{\text{t}} \) relation under the maximum strain rate (i.e. 0.5%/s in this work) by the Ramberg-Osgood law of Eq. (17) as a first approximation of the
relationship between the stress and elasto-plastic strain \( (\sigma - \varepsilon^p) \).

(v) Calculate the transient creep strain \( \varepsilon_t \), subtracting the \( \varepsilon_t^p \) calculated by the Ramberg-Osgood law in step (iv) from the \( \sigma_t = \varepsilon_t^p \) relations under the two strain rates different from the maximum strain rate (0.05 and 0.005%/s in this work). Also calculate the total creep strain \( \varepsilon_t \) by \( \varepsilon_t = \varepsilon_t^1 + \varepsilon_t^2 \).

(vi) Determine the axial stress \( \sigma_t \) and time \( t \) which give \( \varepsilon_t^1 \) and \( \varepsilon_t^2 \) in step (v) from the experimental data, and approximate the relationships among these four parameters by Eq. (15). Then, determine the material constants \( c_1 \) and \( c_2 \) from the approximation.

(vii) Calculate \( \varepsilon_t^2 \) by the material constants \( c_1 \) and \( c_2 \) obtained in step (vi), and subtract it from the \( \sigma_t - \varepsilon_t^p \) relation under the maximum strain rate.

(viii) Again approximate the result in step (vii) by the Ramberg-Osgood law.

(ix) Update the \( \sigma_t - \varepsilon_t^p \) relation by the obtained Ramberg-Osgood law in step (viii), and the material constants \( E \), \( D \), and \( m \) are also updated.

(x) Repeat steps (v) to (ix) until the values of the material constants \( E \), \( D \), and \( m \) converge, and consider the Ramberg-Osgood law obtained by the convergent material constants as the relationship between the stress and the elasto-plastic strain of the solder alloys.

Figure 3 shows the example of the final results of the above numerical solution applied to the stress-strain relations of the three solder alloys shown in Fig. 2. In Fig. 3, the solid, dashed, and chained lines show the original stress-strain relations and the symbols show the stress-strain relations with the creep strain subtracted. The bold lines show the relationships between the stress and the elasto-plastic strain obtained by the Ramberg-Osgood law with the convergent material constants \( E \), \( D \), and \( m \).

In Fig. 3, there are some differences between the subtracted stress-strain relations of each solder alloy due to the differences in the strain rate. However, the subtracted stress-strain relations approach the curve of the Ramberg-Osgood law for each solder alloy because most of the creep strains were removed. As a result, the plasticity-creep separation was successfully conducted.

Applying the plasticity-creep separation method to all the stress-strain relations in this work, the material constants for each solder alloy have been determined. Since they were temperature dependent, they are expressed by the following equations with the temperature \( T \) in K.

for Sn-3.5Ag-0.75Cu:

\[
A = \exp(6.02 \times 10^{-2}T - 67.9), \quad n = 12.7T^{-0.0201},
\]

\[
c_1 = 1.80 \times 10^{-4}T^{1.70},
\]

\[
c_2 = -7.52 \times 10^{-4}T + 5.26 \times 10^{-3},
\]

\[
E = 1.88 \times 10^{4}T^{-0.727} \ [\text{GPa}],
\]

\[
D = 1.86 \times 10^{-2}\exp(-5.86 \times 10^{-3}T) \ [\text{MPa}],
\]

\[
m = -3.98 \times 10^{-2}\exp(T + 5.56)
\]

for Sn-7.5Zn-3.0Bi:

\[
A = \exp(2.42 \times 10^{-2}T - 54.8), \quad n = 1.16T^{0.365},
\]

\[
c_1 = 1.25 \times 10^{6}T^{-2.33},
\]

\[
c_2 = -4.58 \times 10^{-3}T + 2.83 \times 10^{-2},
\]

\[
E = 1.39 \times 10^{6}\exp(-4.63 \times 10^{-3}T) \ [\text{GPa}],
\]

\[
D = 1.23 \times 10^{2}\exp(-3.48 \times 10^{-3}T) \ [\text{MPa}],
\]

\[
m = 3.55 \times 10^{-2}T^{-0.792}
\]

for Sn-57Bi-1.0Ag:

\[
A = \exp(0.222T - 1.07 \times 10^{5}),
\]

\[
n = 74.4\exp(-7.16 \times 10^{-3}T), \quad c_1 = -9.88\ln T + 60.7,
\]

\[
c_2 = -6.12 \times 10^{-4}T + 3.75 \times 10^{-3},
\]

\[
E = 1.82 \times 10^{5}\exp(-6.59 \times 10^{-3}T) \ [\text{GPa}],
\]

\[
D = 7.34 \times 10^{5}\exp(-8.58 \times 10^{-3}T) \ [\text{MPa}],
\]

\[
m = 68.7T^{-0.544}
\]

The next chapter simulates the electronic parts mounting process to clarify the differences in thermal deformation caused by replacing the solder from the lead solder alloys with the lead-free solder alloys. Then, the material constants for the traditional lead solder alloy Sn-37Pb were also estimated by the “plasticity-creep separa-
tion method”, and they were expressed as the following equations, for Sn-37Pb:

\[ A = \exp(1.73 \times 10^{-1} T - 90.3), \quad n = 1.94 \times 10^{5} T^{-1.76}, \]
\[ c_1 = 7.42 \times 10^{-7} T^{2.53}, \]
\[ c_2 = \exp(-2.84 \times 10^{-2} T + 1.60), \]
\[ E = 1.47 \times 10^{4} T^{-1.13} \text{[MPa]}, \]
\[ D = -32.01 n T + 2.13 \times 10^{2} \text{[MPa]}, \]
\[ m = -2.05 \times 10^{-2} T + 10.7 \] (21)

The plastic tangent modulus \( H' \) in Eq. (5) and the flow stress \( R \) for the loading function \( F = 374 \) can be calculated from the material constants \( D \) and \( m \) in Eqs. (18)–(21). Because the second term on the right hand side in Eq. (17) expresses the relationship between the equivalent stress and equivalent plastic strain, the differentiated form of the term gives the plastic tangent modulus \( H' \) as follows.

\[ H' = \frac{(c_0 d h)}{(D \cdot (\hat{\sigma} / D)m^{-1})^{-1}} \] (22)

The flow stress \( R \) is defined as the stress at the plastic strain \( \varepsilon_p \) of 5.00\( \times \)10\(^{-5} \) in the stress-strain relation given by the Ramberg-Osgood law of Eq. (17), and then expressed as

\[ R = (\varepsilon_p / \varepsilon_0)^{1/m} \cdot D \] (23)

4. Simulations and Discussion

4.1 Implementation of constitutive model

In this study, the proposed constitutive model was incorporated into the general purpose FEM code “ANSYS” using the user subroutines of “userpl” and “usercr”. To conduct the stress integration of the constitutive model, the iterative operation with the Predictor-Corrector method was employed. The details of the operations are as follows:

(a) The total strain increment \( \Delta \varepsilon \), time increment \( \Delta t \) and temperature increment \( \Delta T \) are arbitrarily set for a calculation step.

(b) Calculating the predictor of the trial stress \( \sigma^{tr} \) by using Eq. (2) as

\[ \sigma^{tr} = D' : (\varepsilon^{e} + \Delta \varepsilon^{tr}) \] (24)

where \( \varepsilon^{e} \) is the elastic strain at the start of a calculation step, and \( \Delta \varepsilon^{tr} \) is a strain increment obtained by

\[ \Delta \varepsilon^{tr} = \Delta \varepsilon - \Delta \varepsilon^{th} \] (25)

where \( \Delta \varepsilon^{th} \) is the thermal strain increment caused by the temperature increment \( \Delta T \).

(c) Using the loading function of \( F = \sigma^{tr} - R \) with the trial stress \( \sigma^{tr} \), the yielding is checked.

(d) If \( F > 0 \) in the step (c) (where yield occurs), the trial stress \( \sigma^{tr} \) is applied to the plastic part of Eq. (5). Then, the approximation of the plastic strain \( \Delta \varepsilon^{tr} \) is calculated. When there is no yield, the operation in step (f) is carried out.

(e) Using the calculated \( \Delta \varepsilon^{tr} \), the \( \Delta \varepsilon^{tr} \) in step (b) is updated as \( \Delta \varepsilon^{tr} = \Delta \varepsilon^{tr} \) (in step (b)) - \( \Delta \varepsilon^{tr} \). Then, the trial stress \( \sigma^{tr} \) is also updated using Eq. (24) with the \( \Delta \varepsilon^{tr} \) updated in this step.

(f) If \( F > 0 \) in step (c), the approximation of the creep strain \( \Delta \varepsilon^{cr} \) is calculated by applying the \( \sigma^{tr} \) updated in step (e) to Eqs. (9)–(11). When there is no yield in step (c), \( \sigma^{tr} \) used in step (c) is applied to Eqs. (9)–(11) to calculate \( \Delta \varepsilon^{cr} \).

(g) If \( F > 0 \) in step (c), the \( \Delta \varepsilon^{tr} \) calculated in step (e) is updated by using the calculated \( \Delta \varepsilon^{cr} \) as \( \Delta \varepsilon^{tr} = \Delta \varepsilon^{tr} \) (in step (e)) - \( \Delta \varepsilon^{cr} \). When there is no yield in step (c), \( \Delta \varepsilon^{tr} \) is used as \( \Delta \varepsilon^{tr} = \Delta \varepsilon^{tr} \) (in step (b)) - \( \Delta \varepsilon^{cr} \) to update \( \Delta \varepsilon^{tr} \) of step (b). Then, the trial stress \( \sigma^{tr} \) is also updated using Eq. (24) with the updated \( \Delta \varepsilon^{cr} \) in this step.

(h) Repeat steps (c) to (g) until the value of the trial stress \( \sigma^{tr} \) converges. When it converges, the calculated \( \sigma^{tr} \) is regarded as the stress at the end of the current calculation step. Also \( \Delta \varepsilon^{tr} \) and \( \Delta \varepsilon^{cr} \) are regarded as the plastic and creep strain increments in the current calculation step, and then the plastic and creep strains are updated by applying these strain increments to Eqs. (3) and (4).

4.2 Comparison of simulations with experiments

The simulations of the creep and cyclic tension-compression loading were carried out using “ANSYS”, into which the constitutive model is incorporated.

Figure 4 shows the creep curves of experiments together with those of the simulations. Figure 4(a)–(c) show the creep curves of Sn-3.5Ag-0.75Cu, Sn-7.5Zn-3.0Bi and Sn-57Bi-1.0Ag at 303 K. The loading conditions were pure tension under a stress rate of 20 MPa/s until the stress reached the required stress levels. All the simulations in Fig. 4 were calculated as creep curves subsequent to the pure tension. Namely, the simulations were performed considering the pure tension until the stress reached the required stress level for the creep.

The experiments show that the Sn-3.5Ag-0.75Cu solder alloy had the largest creep strain, and Sn-7.5Zn-3.0Bi the smallest. Comparing the simulation with the experiments in Fig. 4, there is a little difference between the simulations and the experiments. In Fig. 4(a), for example, the experiment with Sn-3.5Ag-0.75Cu at the stress level of 30 MPa (c) shows a larger transient creep region than that of the simulation. The difference arises from the deviation of the experiments from the approximations in Fig. 3. However, the simulations approximate the experimental results well. Especially, the differences of the creep strains due to the differences of the solder alloys are described well by the simulations in Fig. 4.

Figure 5 shows the experimental stress-strain relations caused by the cyclic tension-compression loading under the three strain rates at the three temperatures and with the three strain amplitudes. Only the stress-strain relations at the stabilized cycle are shown in Fig. 5. The
effects of strain rate, temperature, and strain amplitude are shown in Fig. 5 (a), (b), and (c).

Figure 6 shows the simulation using the material constants of Eqs. (18) – (20), which corresponds to Fig. 5. Comparing Fig. 5 with Fig. 6, there are some differences between the experiments and simulations. However, the terminal stresses of the hysteresis loops are well predicted by the simulations. Moreover, the differences in the stress levels of the hysteresis loop caused by the differences in both loading conditions and solder alloys are well described by the simulations.

As shown above, the constitutive model can simulate the creep and cyclic loading of the lead-free solder alloys with the material constants determined by “the plasticity-creep separation method” using only the data of the pure tensile tests. This also shows that the constitutive model was suitably incorporated into ANSYS by the stress integration method shown in the previous section.
4.3 Simulation of electronic parts mounting process

Thermal deformations of electronic packages during the electronic parts mounting process were simulated using the material constants of Eqs. (18)–(21). Figure 7 shows the 3D FEM model of the electronic package for the analysis. The solder alloys were cooled from the melting temperature to the room temperature of 298 K in 300 sec., and the temperature was kept at 298 K for 1 hour. The deformations of the electronic package components except for the solder were assumed to be elastic deformation due to the thermal loading, and the material constants are shown in Table 2. The melting temperature \( T_m \) and thermal expansion coefficient \( \alpha \) of each solder alloy are shown in Fig. 8 together with a schematic figure of the thermal loading for the simulation.

![Figure 7: Electronic package model for FEM analysis](image)

![Table 2: Material constants of electronic package components in Fig. 7 other than the solder](image)

Figure 9 shows the \( z \) directional displacements of the circled edge in the FEM model of Fig. 7. The displacements occur in the negative \( z \) direction during the cooling process irrespective of the compositions of the solder alloys. The negative displacement in \( z \) direction leads to a warping of the electronic packaging due to the cooling process. The maximum value of the warp occurs just after the cooling, i.e. at 300 sec. After 300 sec. the deformation recovers while the temperature is maintained at room temperature. The values of deformation increase in the order, Sn-37Pb, Sn-57Bi-1.0Ag, Sn-3.5Ag-0.75Cu, and Sn-7.5Zn-3.0Bi. Namely, the warp of the electronic packaging using the lead-free solder alloys is larger than that using the lead solder alloys. Especially, the warp of the electronic package using Sn-7.5Zn-3.0Bi is about twice that using Sn-37Pb.

![Figure 8: Schematic figure of the thermal loading for the simulation](image)

![Figure 9: The \( z \) directional displacement of an edge of the electronic package](image)

Figure 10 shows an example of the equivalent stress contour in the solder joint of Sn-3.5Ag-0.75Cu together with the deformation figures. Figure 10 (a) and (b) correspond to the contour at 300 sec (the end of the cooling process) and that at 3900 sec (after 1 hour of temperature maintenance), respectively. The maximum equivalent stress in Fig. 10 (a) is 44.0 MPa, while that in Fig. 11 (b) is...
28.0 MPa. This means that the stress was relaxed by about 16 MPa by the temperature maintaining process.

Figure 11 shows the relationships between the equivalent stress in the solder joint and time during the thermal loading. The relations were obtained at the node which recorded the maximum equivalent stress among the solder joints. The positions of the node were the same irrespective of the compositions of the solder alloys, and they were located in the circled solder joint in Fig. 10.

In Fig. 11, the equivalent stress increases in the cooling process and the maximum equivalent stresses occur just after completion of the cooling process, i.e. at 300 sec. After 300 sec, there is stress relaxation during the temperature maintaining process, and it occurs simultaneously with the deformation recovery of the electronic package shown in Fig. 9. Namely, the recovery is caused by the stress relaxation in the solder joint, and it suggests that a part of the electronic package warp generated during cooling in electronic part mounting could be recovered by the temperature maintaining process which induces stress relaxation in the solder joints.

Though stress relaxation occurred in all solder alloys in Fig. 11, the values of the residual stress at 3,900 sec. are different. It is obvious that the largest residual stress occurs in the case of Sn-7.5Zn-3.0Bi. This is why the largest warp was observed in the case of Sn-7.5Zn-3.0Bi as shown in Fig. 9.

5. Conclusions

This paper applied the constitutive model previously proposed by the authors, to the lead-free solder alloys of Sn-3.5Ag-0.75Cu, Sn-7.5Zn-3.0Bi, and Sn-57Bi-1.0Ag. Also the implementation method for the constitutive model was discussed to incorporate the model into a general purpose FEM program “ANSYS” and the warp of the electronic packaging using both lead-free and lead solder alloys was simulated. The following conclusions were obtained:

(1) The plasticity-creep separation method could be successfully applied to the lead-free solder alloys Sn-3.5Ag-0.75Cu, Sn-7.5Zn-3.0Bi, and Sn-57Bi-1.0Ag. Then, the material constants of the lead-free solder alloys for the constitutive model could be determined by using the data of only the pure tensile tests.

(2) The creep and cyclic tension-compression loading of the lead-free solder alloys could be simulated by the FEM program “ANSYS”, into which the constitutive model was incorporated.

(3) A structural analysis of the electronic parts mounting process using the lead-free solders of Sn-3.5Ag-0.75Cu, Sn-57Bi-1.0Ag, and Sn-7.5Zn-3.0Bi and the lead solder of Sn-37Pb were conducted. As a result, the warp of the electronic package using the lead free solder alloys becomes larger than that using the lead solder alloy, and the largest warp was observed when the Sn-7.5Zn-3.0Bi was used.

(4) There was stress relaxation in the solder joints after the cooling of the electronic parts mounting irrespective of the composition of solder alloys. The largest residual stress was observed when the Sn-7.5Zn-3.0Bi solder alloy was used for the packaging.

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