Effect of Void Shape and Its Growth on Forming Limits for Anisotropic Sheets Containing Non-Spherical Voids*

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Most failures of ductile materials in metal forming processes occurred due to material damage evolution- void nucleation, growth and coalescence of neighboring voids. Recently, Gologanu-Leblond-Devaux (J. Mech. Phys. Solids, Vol.41 (1993), pp.1723–1754) extended the classical Gurson model (ASME J. Engng. Mater. Technology, Vol.99 (1997), pp.2–15) to a ductile material containing an oblate ellipsoidal cavity. And, they proposed a new approximate yield function incorporating the initial void shape effects, which is significant especially at low stress triaxiality. In the present work, the Gologanu-Leblond-Devaux’s yield function for anisotropic sheet materials containing axisymmetric prolate ellipsoidal cavities is adopted in evaluating analytically forming limits of sheet metals under biaxial stretching by Marciniak and Kuczynski (M-K) model. The effect of a void shape and growth on the forming limits of sheet metals under biaxial tensile loading is introduced and examined within the framework of the M-K model, along with the effect of including a first-order strain gradient term in the flow stress. To confirm the validity of the proposed model, the predicted FLDs were compared with experimental results for steel sheets. The predicted forming limits for the voided sheets were found to agree well with the experimental data.

** Key Words: ** Forming Limit, Anisotropic Sheet Metal, Void Shape, Non-Spherical Voids

1. Introduction

With the increase of computational techniques, numerical predictions of forming limit diagrams (FLDs) have become more attractive and a lot of research have been conducted in order to set up a more realistic model. For example, Marciniak and Kuczynski(1) were the first to analyze the causes of necking failure and propose an analytical model, the M-K model, that theoretically predicts the forming limit based on the presence of an initially local inhomogeneity in the sheet metal. Even though the initial M-K model using a quadratic anisotropic yield criterion may be very easy to understand, there is an inconsistency between the predictions and experimental data, as the limit strains near the plane strain deformation are underestimated, whereas they are over-estimated in the equibiaxial stretching region.

Subsequently, Stören and Rice(2) incorporated the J2 deformation theory of plasticity, which is a simplified model of the corner theory, into a classical bifurcation analysis for predicting localized necking within the range of an FLD.

Much research has already been performed to set up a more realistic model using damage mechanics. Experimental researches by Jalinier(3), Parmar and Mellor(4), and Gronostajski and Zimniak(5) demonstrated that the density of a sheet material changes with straining, and there is a certain correlation between the density changes and the limit strains. These changes in the density or void volume fraction, which may govern the necking process, can result from the growth of initially existing voids and the nucleation of voids during the early stages of deformation. Commonly, microvoids nucleate as second-hard phase particles. These microvoids then grow by plastic deformation and finally coalesce to form microcracks.

To analyze the plastic flow and failure of ductile materials with such microvoids, Gurson(6) performed an upper bound analysis of simplified models containing spherical cavities and proposed an approximate yield criterion for perfectly rigid porous materials, where the matrices obey the von Mises yield criterion. His model was subse-
quent modified and developed by Tvergaard\cite{7,8}, and by Needleman and Tvergaard\cite{9}, who introduced additional parameters to model the decay of the load carrying capability. An extension of Gurson’s original model for the transformation of an isotropic matrix material into an anisotropic matrix material, such as cold-rolled sheet metals, has also been proposed in various forms, e.g. Liao et al.\cite{10} based on Hill’s quadratic or non-quadratic yield criteria, Kim et al.\cite{11} based on Hosford’s yield criterion, Kim et al.\cite{12} based on Barlat and Lian’s non-quadratic anisotropic yield criterion, and Cao et al.\cite{13} based on general anisotropic yield criterion.

Unlike a spheroidal void model, as mentioned above, voids in real materials are often non-spherical. For instance, they can be long, prolate ellipsoids if they are nucleated around segregations previously elongated by a rolling process. Conversely, they can be wide, oblate ellipsoids if they happen to grow from cleavage cracks generated in the hard phase of a dual-phase structure. As such, there is a need for models that can describe the behavior of materials containing such spherical voids. Becker et al.\cite{14} analyzed the inelastic response of periodic arrays of initially oblate spheroidal voids in a homogeneous isotropically hardening matrix for loading histories representative of axisymmetric tension tests. A modification was assumed to account for the effects of the initial void shape, namely, \( q_1 = q_1^* + m_o \) (\( m_o \) is the initial eccentricity and \( q_1^* \) is the value of \( q_1 \) for a spherical void) by Becker et al.\cite{15}, as finite element calculations to determine the growth rate of an ellipsoidal cavity are computationally expensive. Lee and Mear\cite{16} presented a study of an ellipsoidal cavity embedded in an infinite medium made of a plastic or viscoplastic material obeying the classical Noton law.

Recently, Gologanu et al.\cite{17} extended the classical Gurson analysis of a hollow rigid ideal-plastic sphere loaded axisymmetrically to an ellipsoidal volume containing a confocal ellipsoidal cavity, in order to define approximate models for ductile metals containing non-spherical voids. Using this model and the approach of Thomason\cite{18} for the onset of void coalescence, Pardoen and Hutchinson\cite{19} then proposed a model for the axisymmetric growth and coalescence of small internal voids in elastoplastic solids to incorporate the void shape and relative void spacing on void coalescence. As discussed in their work, the effect of the void shape can be significant upon void growth and ductile failure of the material, especially in a low triaxial stress state.

In addition to the above-mentioned theoretical approaches the FEM analyses is also very effective to investigate the necking and post-necking behaviors of biaxially stretched sheets, and predict the FLDs\cite{20,25}. Narasimhan and Wagone\cite{20} and Needleman and Triantafyllidis\cite{21} have performed FEM simulation for sheet containing isolated finite defects in length and predicted the forming limits of stretched sheets. Also, Nakamachi\cite{22} and Yoshida et al.\cite{23} have adopted the Stören and Rice model to judge the onset of localized necking in the finite element simulation and predicted the FLD in part. Based on the experimental evidence of ductile fracture occurrence before the onset of localized necking in the strain paths near balanced stretching, Takuda et al.\cite{24,25} have performed the FEM simulation combined with a criterion for ductile fracture proposed by Oyane et al.\cite{26} to predict forming limit strains in biaxial stretching of sheet metals.

More recently, Brunet et al.\cite{27} used a 3D-FEM analysis of a unit cube containing an ellipsoidal void to show that a damage model that takes account of the void-shape effect and anisotropy of the matrix material could be coupled with a plastic instability criterion to produce forming limit curves that were in a good agreement with the experimental ones.

In the present work, Gologanu-Leblond-Devaux’s yield function\cite{17} is incorporated with Barlat and Lian’s non-quadratic anisotropic yield criterion\cite{28} to describe the plastic deformation of materials containing axisymmetric prolate ellipsoidal cavities with random orientations. The proposed yield criterion for voided anisotropic materials and its associated flow rules are then used to investigate the plastic behavior and predict the forming limits of various anisotropic sheet metals under biaxial tensile loading.

Moreover, a first-order strain gradient term in the constitutive equation is also included in the proposed M-K model, as suggested by Shi and Gerdeen\cite{29}. This strain gradient plasticity model can provide a more suitable description of the material behavior in sheet metal applications and account for the effect of the curvature of the punch in localized necking in anisotropic sheets.

Finally, in order to confirm the validity of the proposed model, the predicted forming limits are compared with experimental data for rimming steel and deep drawing quality (DDQ) steel in literatures.

2. Theoretical Analysis

2.1 Yield function for anisotropic materials with prolate ellipsoidal voids

A damage mechanics model requires the definition of a specific yield function for the damaging material and development of an accurate model to analyze the evolution of damage in its various stages (nucleation, growth, and coalescence). Since the first attempt by Gurson\cite{6} to develop a set of constitutive equations, including ductile damage, much research has been performed. For example, based on restricting the plastic behavior, axisymmetric loadings, and axisymmetric prolate cavities, Gologanu et al.\cite{17} extended the classical Gurson analysis of a hollow rigid ideal-plastic sphere loaded axisymmetrically to an ellipsoidal volume containing a confocal ellipsoidal cavity,
in order to define approximate models for ductile metals containing non-spherical voids. The application of these results to materials containing axisymmetric prolate ellipsoidal cavities with parallel or random orientations was also discussed in their paper. The criterion for random orientations becomes

\[
\phi = \left( \frac{\sigma}{\sigma_M^r} \right)^2 + 2q_1 C_F \cosh \left( \kappa \sigma_m \sigma_M^r \right) - (1 + q_1^2 C_F^2) = 0
\]

with \( \kappa = \left[ \frac{1}{\sqrt{3}} + (\sqrt{3} - 2) \ln \left( \frac{e_1(e_2)}{C_F} \right) \right]^{-1} \),

\[
e_1 = \sqrt{1 - e^{-25}}, \quad e_2 = \sqrt{C_F(1 - e_2^2)/(1 - e_1^2)} e_1
\]

where \( \sigma_m \) represents the macroscopic effective stress, \( \sigma_M^r \) is the matrix flow stress with strain hardening, \( C_F \) is the void volume fraction, \( q_1 \) is the material parameter, \( \sigma_M^r \) is the macroscopic mean stress, and the void shape parameter \( S \) is defined by \( S = \ln a_i/b_i \), defined by a logarithm of the ratio of major to minor radii of a prolate ellipsoidal void in its local frame, as shown schematically in Fig. 1. It should be noted that the yield criterion of Gologanu-Leblond-Devaux is identical to that of Gurson-Tvergaard with a value of \( q_2 \) comprised between 1 (for \( \kappa = 3/2 \), spherical cavities) and \( 2/\sqrt{3} \) (for \( \kappa = \sqrt{3} \), cylindrical cavities).

Since the above yield function involving a hyperbolic cosine term becomes unattractive in further analytical manipulations, the hyperbolic cosine term is expanded into a series. In most plastic deformation processes \( (\kappa \sigma_m/\sigma_M^r) \) is of the order of unity or less; hence it is acceptable to neglect powers higher than 2, as discussed by Ragab and Saleh(30).

In the mean time, unfortunately, even if the shape parameters of the various cavities are identical, their rates are different due to the variation in their direction; hence as suggested by Gologanu et al.(17), an average value for the \( S \)-rates of the different voids is used to describe the change in void shape.

\[
S = \left( 3 \left( \frac{1 - 3K_1}{C_F^2} + 3K_2 - 1 \right) \right) \bar{e}_m + 3 \frac{2 - t_r^2 + (t_r^2 - 1)e_1^2}{2} \langle \varepsilon_y \rangle
\]

where \( \bar{e}_m = C_F/[3(1 - C_F)] \), \( t = \sigma_m/\bar{\sigma} \),

\[
K_1 = \frac{1}{2e_1^2} - \frac{1 - e_1^2}{2e_1^2} \tanh^{-1} e_1,
\]

\[
K_2 = \frac{1}{2e_2^2} - \frac{1 - e_2^2}{2e_2^2} \tanh^{-1} e_2,
\]

\[
d\varepsilon_y = 2/3 [d\varepsilon_y - 0.5(d\varepsilon_{xy} + d\varepsilon_{yy})]
\]

with the angle brackets denote the averaging process. It is easy to check that the above equation predicts negative \( \dot{\varepsilon}_m \)-values for positive \( \dot{\varepsilon}_m \)-values; in other words, the voids tend to become spherical on average as they expand.

In the current work, GLD’s yield function, Eq. (1), is further modified with Barlat and Lian’s 1989 non-quadratic anisotropic yield criterion(20) under plane stress states:

\[
\bar{\sigma} = \left[ \frac{2 - c}{2} \left( |k_1 + k_2|M^2 + |k_1 - k_2|M^2 + \frac{c}{2 - c} |2k_2|^{2M} \right) \right]^{1/M}
\]

with \( k_1 = \frac{\sigma_{x} + h_1 \sigma_y}{2} \),

\[
k_2 = \sqrt{ \left( \frac{\sigma_x - h_1 \sigma_y}{2} \right)^2 + \frac{h_2^2 \sigma_y^2}{2} } \]

\[
c = 2 \sqrt{ \frac{R_0}{1 + R_0} \frac{R_0}{1 + R_0} } \]

\[
h_1 = \frac{R_0}{1 + R_0} \]

\[
h_2 = \frac{\sigma_y}{\tau_y} \frac{\sqrt{(1 + R_0)(1 + R_0)}}{2 \sqrt{(1 + R_0)(1 + R_0) + (2M - 2) \sqrt{R_0 R_0} }} \]

where \( R_0 \) and \( R_0 \) are defined as the ratio of the width strain to the thickness strain under uniaxial tension at 0 and 90 degrees to the rolling direction, respectively, and \( \sigma_y \) and \( \tau_y \) are the yield stress and shear stress in the rolling direction, respectively. Here, the yield criterion exponent \( M \geq 2 \) defines the shape of the yield locus, which was suggested to be 6 and 8 for BCC and FCC metals, respectively, from the fitting of upper bound crystallographic calculations.

Assuming isotropic hardening, a modified yield function for planar anisotropic materials with prolate ellipsoidal voids is proposed as

\[
\bar{\sigma}_M = \frac{1}{1 - q_1^2 C_F} \sqrt{ \left( \frac{2 - c}{2} \left( |k_1 + k_2|M^2 + |k_1 - k_2|M^2 + \frac{c}{2 - c} |2k_2|^{2M} \right) \right) + q_1 \cdot C_F \cdot \kappa^2 \left( \frac{\sigma_x + \sigma_y}{3} \right)^2 }
\]
Most previous anisotropic yield functions have been based on Green or Gurson’s yield criterion, which uses spheroidal cavities, in conjunction with Hill or Hosford’s anisotropic yield criterion. However, in the current study, various constitutive models for anisotropic sheets considering non-spheroidal cavities can be evaluated and calculated from Eq. (4).

### 2.2 Flow rule and void growth characteristics

The corresponding flow rule gives the principal strain increments as

\[ \varepsilon_{p1} = d\varepsilon_{p1} = \frac{2}{3} \varepsilon_{V} \left[ \frac{2}{2} \left( 1 + \frac{1}{h_1} \right) M + \frac{c}{2 - c} \left( 1 - h_1 M \right) \right] \]

where \( \varepsilon_{V} \) is the volumetric plastic strain, \( d\varepsilon_{p1} \) is the principal strain increment, and \( h_1 \) is the stress ratio. The damage evolution by void growth from initially existing voids in the sheet materials can be calculated and updated using Eq. (7). Although most failures of ductile materials in metal forming processes occur due to material damage evolution (nucleation, growth, and coalescence)(7,21), the current paper only considers the damage evolution due to void growth from initially existing voids in the sheet materials, because void growth plays a more important role than void nucleation in damage evolution in metal forming processes, as previously discussed by Hu et al.23.

#### 2.3 M-K analysis and calculation of FLDs

A sheet metal with an internal imperfection, i.e. a heterogeneous distribution of voids in addition to an initial thickness imperfection, \( f_o = (l_0^2/f_0^2) \), is shown schematically in Fig. 1. The initial void volume fraction in zone \( "b" \), \( C_{V0} \), has a higher void volume fraction than that in zone \( "a" \), \( C_{V0} \). The matrix material is assumed to obey the power-hardening law in order to consider the effect of the first strain gradient term along the thickness direction.

\[ \bar{\sigma} = K_M \bar{\varepsilon}^{M}_{\text{av}} - g_M Q_{AM} \sigma_M \left[ (\alpha + t/2) \right]^{2n} \]

\[ \bar{\varepsilon}_{\text{AV}} = \frac{2 - c}{2} \left[ \left( 1 + \frac{1}{h_1} \right) M + \frac{c}{2 - c} \left( 1 - h_1 M \right) \right] \]

where \( K'_M \) is the modified coefficient of the material strength coefficient \( K_M \) defined in Ref. (29), \( \sigma_{AM} \) is the average stress at the section, and \( \sigma_{BM} \) is the stress resulting from bending only the matrix material.

From previous basic equations, the force equilibrium equation \( F_{1M} = F_{2M} \) and geometrical compatibility equation \( d\varepsilon_{V} = d\varepsilon_{V1} \), the following relationships are obtained:

\[ (1 - C_{V0}) Q_{AM} \cdot (\bar{\varepsilon}_{\text{AV}}^{M})^{n} - (1 - C_{V0}) f^{\prime} \cdot Q_{1M} \cdot (\bar{\varepsilon}_{\text{AV}}^{M})^{n} - \left[ (1 - C_{V0}) \cdot (t'/(\alpha + t'/2))^{2n} - (1 - C_{V0}) \cdot f \cdot (t'/(\alpha + t'/2))^{2n} \right] = 0 \]

with \( f = f_o \exp(\bar{\varepsilon}_{\text{AV}} - \bar{\varepsilon}_{\text{V}}) \)

where \( \bar{\varepsilon}_{\text{AV}} \) and \( \bar{\varepsilon}_{V} \) are the apparent thickness strains for zones ‘a’ and ‘b’, respectively. By using the previous equations, forming limit strains under biaxial stretching state are calculated numerically as discussed by Marciniak and Kucynshi(1).

### 3. Results and Discussions

To clarify the plastic deformation characteristics of the series form of GLD’s yield function in conjunction with Barlat and Lian’s 1989 anisotropic criterion(28), the shape of the yield locus and initial void shape parameter according to straining are investigated in Figs. 2–4. The yield loci for \( M = 6 \) and \( q_1 = 1.5 \) with the same planar anisotropy of Low-Carbon steel for different void volume fractions in the normalized principal stress plane are shown in Fig. 2. To assure its qualitative accuracy of present model, the measured yield loci from the biaxial tensile testing of a cruciform specimen.
Fig. 2 Comparison of proposed yield criteria with Kuwabara’s experiment

Fig. 3 Growth of void volume fraction $CV$ with tensile strain for different deformation modes relative to initial void shape parameter of $S_o = 0.0, 1.6,$ and $3.2$

by Kuwabara$^{(32)}$ is included for comparison. From the figure, the shrinkage of the yield locus due to damage growth was self-evident as the void volume fraction increased. Namely, the flow stress due to strain softening of the material decreased as the material damage increased. However, the initial void shape parameter exhibited no effect on the shrinkage of the yield locus, except for a very small difference at an equibiaxial tension.

For an initially different void shape parameter, the current void fraction $CV$ relative to the strain $\varepsilon_1$ for different deformation modes is illustrated in Fig. 3 in the case of $M = 6, q_1 = 1.5,$ and $CV_0 = 0.02$ with planar anisotropy. The current void volume fraction was highest at an equibiaxial tension, followed by plane strain, and then uniaxial tension. It should be noted that the deformation mode had a significant effect on void growth, as experimentally confirmed by Jalinier$^{(3)}$ and Luo et al.$^{(33)}$ Also, the growth of the void volume fraction was more pronounced when the initial void shape parameter $S_o$ was increased. However, in contrast to the case of an equibiaxial tension, the growth of the void volume fraction with a uniaxial tension was relatively uninfluenced by the initial void shape parameter.

Figure 4 represents the variation in the void shape parameter $S$ with tensile strain for three loading modes in the case of an initial void shape parameter of $S_o = 1.60$ and the same properties used in Fig. 3. As discussed in the previous section, the ellipsoidal voids tended to become spherical on average as the material experienced plastic deformation, due to Eq. (2) predicting negative $\dot{S}$-values for positive $\dot{\varepsilon}_m$-values. The tendency to become a perfect spherical void was slowest at a uniaxial tension, followed by plane strain, and then equibiaxial tension.

As shown in Figs. 5 – 7, basically the same mechanical properties of rimming steel were used in the parameter study to determine the characteristic factors affecting the shape and level of the forming limits. Thus, a parameter study for various factors, including an initial geometric imperfection, initial void volume fraction, and initial void shape parameter, was performed and discussed on the right-hand side of the FLDs. Since the effect of other mechanical properties, such as a strain hardening exponent, planar anisotropy parameter, strain rate sensitivity exponent, etc. has already been extensively studied in previous literature$^{(20),(34),(35)}$, this was not the concern of the current study.

Figure 5 shows the effect of an initial geometric imperfection on the analytical FLD. The figure illustrates that a change in the initial geometric imperfection significantly affected the FLD. As the initial geometric im-
perfection increased from $f_v = 0.99981$ to $f_v = 0.9999$, the forming limit strains increased without any change in the curve shape. The effect of damage evolution by void growth on the FLD is shown in Fig. 6. In this case, the influence of the initial void volume fraction during biaxial straining on the FLD was dramatic. When damage evolution due to void growth was included in the M-K model, the predicted forming limit strain in an equibiaxial strain state significantly decreased compared with that for the void-free sheet metal, where the FLD was generally overestimated(21). Also, when the forming limit strain was compared at each strain path, the decrease in the forming limit strain in an equibiaxial strain state was found to be larger than that in a plane strain state with an increase in the initial void volume fraction.

Figure 7 shows the effect of the initial void shape parameter on the analytical FLD. The forming limit strain in an equibiaxial strain state slightly decreased when the initial void shape parameter was increased. Also, the initial void shape parameter had a relatively minimal impact on the forming limits when the loading was directed toward a plane strain state, thereby indicating that the amount of decrease in the forming limit strain during biaxial straining depended on the increase in the void volume fraction at each strain path, as discussed in Fig. 3. It should be noted that the effect of the initial void shape parameter affecting the forming limit strain was not remarkable due to the assumption that $C^{o}_{V_o} = 0.0003$. Therefore, it would seem that the initial void shape parameter defined by $S = \ln a_1 / b_1$ can also be considered as an additional parameter for the precise prediction of an FLD.

To confirm the accuracy of the proposed damage model for anisotropic sheets with prolate ellipsoidal cavities, the predicted forming limits for two kinds of sheet materials were compared with experimental data, as shown in Figs. 8 and 9. In the current study, it was assumed that the initial void shape parameter was 1.6, because ellipsoidal inclusions in a material are only, on average, approximately 4.5 times longer than the smallest principal axis length, as examined by Spitzig(36). Since Marciniak and Kuczynski(1) assumed an unrealistic value of initial inhomogeneity, many researchers have also used and adopted it in the M-K model. Azrin and Backofen(37) concluded that an unrealistically large thickness inhomogeneity would be needed for the predictions based on the M-K model to be meaningful. Commonly, a value of initial geometric imperfection has been chosen to make the best fit between experimental data for sheet metals and theoretical predictions, as discussed by Kim et al.(38) However, in the current study, it was assumed that the sheet metal was free from any thickness imperfection, i.e. $f_v = 0.9999 (\approx 1)$. Thus, the only existing inhomogeneity

![Fig. 5](image1.png)  
**Fig. 5** Effect of initial geometric imperfection on analytical FLD

![Fig. 6](image2.png)  
**Fig. 6** Effect of initial void volume fraction on analytical FLD

![Fig. 7](image3.png)  
**Fig. 7** Effect of initial void shape parameter on analytical FLD
was that the initial void volume fraction in zone ‘b’, \( C_{V_0}^{b} \), had a higher void volume fraction than that in zone ‘a’, \( C_{V_0}^{a} \), i.e., \( C_{V_0}^{b} > C_{V_0}^{a} \) as discussed by Ragab and Saleh (39).

The results in Fig. 8 show a comparison between the experimental FLD obtained by Tadros and Mellof (40) and the predicted one using \( M = 6, q_1 = 1.5, C_{V_0} = 0.0003, S_0 = 1.60 \) for rimming steel. The values for \( C_{V_0} \) and \( q_1 \) were also chosen as representative values for steel from previous literatures (7), (39). In Fig. 8, the rectangular symbols (□, △) represent the results measured from an out-of-plane test in a plane strain mode, while the circular symbols (○, •) represent the results measured from an in-plane test in a biaxial tension mode. A clear distinction was exhibited in the plane strain mode between the maximum uniform strain measured away from the fracture and the nearest-circle strain adjacent to the fracture, because a severe strain gradient existed at the onset of tensile instability in the plane strain mode. As also noted by Tadros and Mellof (40), these results were from an out-of-plane test where the strain gradient is of greater importance. Predicting the FLD at the lowest point on the FLD requires the effect of the strain gradient resulting from deforming a flat sheet into a curved sheet. As such, the proposed M-K model method including a first-order strain gradient term in the constitutive equation was used to calculate a more suitable material behavior for the formability of sheet metals. The agreement between the experimental FLD and the predicted one with the same punch radius \( (a = 50.8 \text{ mm}) \) was quite reasonable. Also, for in-plane biaxial stretching \( (a = \infty) \) the predicted FLD agreed well with the experimental data.

Figure 9 shows a comparison of the experimental FLD obtained from an out-of-plane test with a punch radius \( (a = 50.8 \text{ mm}) \) with the predicted one using \( M = 8, q_1 = 1.5, C_{V_0} = 0.0008, S_0 = 1.6 \) for DDQ steel (41). The predicted FLD exhibited a good correlation with the experimental one for DDQ steel. Therefore, the results in Figs. 8 and 9 confirm that the proposed damage model for anisotropic voided sheets provided accurate FLD predictions for rimming and DDQ steel.

In the present model, we assumed that prolate ellipsoidal voids exists in the sheet before the forming, and the ductile failure evaluated by forming limits is only affected by the growth of the voids for simplicity. However, the voids are also generated during the plastic deformation as discussed in introduction part of this study, and thus the failure is also affected by the generated voids very much. In this sense the present model mainly focused on the effect of void growth may over-estimate a little the level of FLDs. Therefore, to predict the forming limit more rigorously, we should consider the both effects of the nucleation and its growth of voids.

4. Conclusions

To describe the plastic deformation of materials containing axisymmetric prolate ellipsoidal cavities with random orientations, the current paper adopted a series form of Gologanu-Leblond-Devaux’s yield function. As such, based on the consideration of ellipsoidal void growth during biaxial straining and the introduction of a first strain gradient term to the M-K model, the effects of damage evolution on the forming limit strain of sheet metal were discussed. In addition, the predicted FLDs were compared with experimental data for rimming and DDQ steel. Accordingly, it was concluded that the introduction of damage evolution into the M-K model can lead to quite reasonable and accurate predictions for limit strains.

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