3-Dimensional Analysis of Deformation of Disk Wheels and Transverse Force of Wheel Bolts*

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Loosening of the wheel nuts, which fix the disk wheels of automobiles to the wheel hub, may be the cause of accidents where the wheel falls off while the automobile is running. When the transverse force of wheel bolts exceeds a certain proportion of the bolt shaft force, the wheel nut begins to loosen. Further, the force on the bolt shaft may also be influenced by the loads acting to the wheel through the moment caused by the offset of the wheel. This study determined the 3-dimensional deformation of the disk wheels and the transverse forces on the wheel bolt by 3-dimensional numerical analysis. The results established that the transverse force was influenced by the bolt shaft force caused by the bolt fastening and was superposed on that due to the load, and that it fluctuated greatly during the revolution of the wheel. This phenomenon may be a large factor in the loosening of wheel nuts.

Key Words: Automobile, Disk Wheel, 3-Dimensional Deformation, Wheel Bolt/Nut, Loosening, Bolt Shaft/Transverse Force, Numerical Analysis

1. Introduction

Loosening of wheel nuts, which fix the disk wheels of automobiles to the wheel hub, may be the cause of accidents where the wheel falls off while the automobile is running. When the transverse force exceeds a certain proportion of the bolt shaft force, the wheel nut begins to loosen\(^{(1)}\). The transverse force corresponds to the force shared by the load on the bolts of a wheel. However, the transverse force is not always shared equally as the disk wheel is elastic. Especially, the burdens of the bolts near the bolt where the nut has loosened are concentrated. Further, the bolt shaft force may also be influenced by the load acting on the wheel through the moment arising from the offset of the wheel.

This study determined the 3-dimensional deformation of disk wheels and the transverse force on wheel bolts by 3-dimensional numerical analysis. The designation of the assumed disk wheel is shown in Table 1 and the shape of the wheel in Fig. 1. Figure 2 suggests the inner and outer nuts. Only the outside disk wheel and the outer and inner nut pair are the objects here, because the inner nut and stud bolt pair does not loosen as long as the outer and inner nut pair remains in place.

### Table 1 Details of disk wheel and conditions of numerical analysis

| Disk wheel (rim) diameter \(D_w\) | 508 mm |
| Bolt hole pitch diameter \(D_h\) | 285 mm |
| Hub hole diameter \(D_i\) | 221 mm |
| Attaching portion diameter \(D_t\) | 340 mm |
| Number of bolt hole | 8 |
| Thickness of disk wheel | 12 mm |
| Offset \(E\) | 162 mm |
| Load \(W_w\) | 21 kN |
| Initial bolt shaft force | 40 kN |
| Bottom width of load distribution | 170 mm |
| Axial spring constant of hub bolt \(K_{ax}\) | 3.87 GN/m |
| Lateral spring constant of hub bolt \(K_{ls}\) | 0.489 GN/m |

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\(^{1}\) Received 10th May, 2005 (No. 05-4111)  
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2. Numerical Analysis

2.1 Modeling of disk wheel

The numerical analysis is carried out using the method of numerical analysis developed on the basis of the relaxation method by one of the authors(2)–(8). Figure 3 shows the disk wheel model for the numerical analysis. The disk wheel is considered circular plate that has a hub-hole center and has the outer diameter of the rim diameter. The disk wheel is divided into \( m = 21 \times n = 128 \times N_z = 5 \) elements with side length of \( \Delta r, \Delta \theta, \) and \( \Delta z \). The load, \( W \), works upwards in the lowermost part of the wheel, and the moment which originates from the offset of the wheel, \( E \), also works in this direction, shown in Fig. 3 as the pair of forces, \( F_{MO} \). The bolt shaft displacements of the wheel, \( w \), at the position of the bolt holes (8 points in the condition where there is no looseness) are fixed through a spring which has the axial spring constant of the bolt, \( K_{bol} \), determined by the thicknesses and effective lengths of the tensioned stud bolt and inner nut. The wheel in-plane displacements in these places are fixed through the spring, which has the shaft right-angled spring constant of the bolt, \( K_{bol} \), which is determined as a cantilever consisting of stud bolt and inner nut. The bolt shaft displacements of the wheel at the rear surface (right in Fig. 3), \( w_{NZ} \), inside of the attached portion diameter, \( D_s \), and excluding the parts of the bolt holes, are limited to \( w_{NZ} \leq 0 \).

2.2 Numerical analysis of deformation and force

Defining the \( r \), \( \theta \), and \( z \)-components of the displacement of an element \((i, j, k)\) as \( u(i, j, k), v(i, j, k), \) and \( w(i, j, k) \), respectively, the normal strain in the \( r \)-direction on the \( r \)-side closed to the center ("smaller" in the following), in the \( \theta \)-direction on the \( \theta \)-side closed to the angle position \( \theta = 0 \) ("smaller" in the following), and in the \( z \)-direction on the \( z \)-side closed to the outside surface ("smaller" in the following) of Element \((i, j, k)\) are given by:

\[
\varepsilon_r(i, j, k) = \frac{| u(i, j, k) - u(i + 1, j, k) |}{\Delta r}
\]

\[
\varepsilon_\theta(i, j, k) = \frac{| v(i, j, k) - v(i, j - 1, k) |}{r(i) \Delta \theta}
\]

\[
\varepsilon_z(i, j, k) = \frac{| w(i, j, k) - w(i, j, k - 1) |}{\Delta z}
\]

The normal stresses corresponding to Eqs. (1)–(3) are given by:

\[
\sigma_r(i, j, k) = 2G \varepsilon_r(i, j, k) + \lambda(\varepsilon_\theta(i, j, k) + \varepsilon_z(i, j, k))
\]

\[
\sigma_\theta(i, j, k) = 2G \varepsilon_\theta(i, j, k) + \lambda(\varepsilon_r(i, j, k) + \varepsilon_z(i, j, k))
\]

\[
\sigma_z(i, j, k) = 2G \varepsilon_z(i, j, k) + \lambda(\varepsilon_r(i, j, k) + \varepsilon_\theta(i, j, k))
\]

where \( G \) and \( \lambda \) are the modulus of rigidity and Lamé’s constant.

The shear stress in the \( r \)-direction on the smaller \( \theta \)-side, \( \tau_{r\theta}(i, j, k) \), and in the \( \theta \)-direction on the smaller \( r \)-side, \( \tau_{\theta r}(i, j, k) \), are distinguished and given by:

\[
\tau_{r\theta}(i, j, k) = \frac{G(\varepsilon_r(i + 1, j, k) - \varepsilon_r(i - 1, j, k))}{4\Delta r} - \frac{Gv(i, j, k) + v(i + 1, j, k)}{r(i) + r(i + 1)}
\]

\[
\tau_{\theta r}(i, j, k) = \frac{G(\varepsilon_\theta(i + 1, j, k) - \varepsilon_\theta(i - 1, j, k))}{2[2(r(i) + r(i + 1))]\Delta \theta} - \frac{Gv(i, j, k) + v(i + 1, j, k)}{r(i) + r(i + 1)}
\]

In the same manner, the other shear stresses in the \( r \)-direction on the smaller \( z \)-side, \( \tau_{rz}(i, j, k) \), and in the \( z \)-direction on the smaller \( r \)-side, \( \tau_{zr}(i, j, k) \), are also distinguished and given in the forms of Eqs. (7) and (8).

The residual forces in the \( r \), \( \theta \), and \( z \)-directions due to the above normal and shear stresses, acting upon Element \((i, j, k)\), are given by:

\[
F_r(i, j, k) = |\sigma_r(i - 1, j, k) - \sigma_r(i, j, k)|r(i)\Delta \theta \Delta z + |\tau_{r\theta}(i, j, k) - \tau_{r\theta}(i + 1, j, k)|r(i)\Delta \theta \Delta \theta + |\tau_{rz}(i, j, k) - \tau_{rz}(i, j, k + 1)|\Delta \theta \Delta z
\]

\[
F_\theta(i, j, k) = |\sigma_\theta(i, j + 1, k) - \sigma_\theta(i, j, k)|\Delta r \Delta z + |\tau_{\theta r}(i, j, k) - \tau_{\theta r}(i, j, k) + 1)|\Delta r \Delta \theta + |\tau_{zr}(i, j, k) - \tau_{zr}(i, j, k) + 1)|r(i)\Delta \theta \Delta \theta
\]

\[
F_z(i, j, k) = |\sigma_z(i, j, k) - \sigma_z(i, j, k + 1)|\Delta r \Delta \theta + |\tau_{rz}(i, j, k) - \tau_{rz}(i, j, k + 1)|r(i)\Delta \theta \Delta \theta + |\tau_{zr}(i, j, k) - \tau_{zr}(i, j, k)|r(i)\Delta \theta \Delta \theta
\]
The residual forces vanish. Those are the bolt shaft forces in the position of bolt holes that are fixed until all residual forces of elements vanish. However, the residual forces in the position of bolt holes that are fixed through the spring do not vanish. Those are the bolt shaft force and the transverse force.

2.3 Setting the bolt shaft force, $F_{bini}$

The initial bolt shaft force, $F_{bini} = 40\, \text{kN}$, is determined from the standard tightening torque of the wheel nut, 550 N·m. However, this value cannot be used as the boundary condition of the bolt shaft force, because the bolt shaft force is changed during numerical analysis by the influence of the load and moment. In this study, there is the bolt shaft displacement of the wheel in the position of the nut seat of the bolt hole $w_{b}$; and the initial value of $w_{b}$, $w_{bini}$, is set, so that the calculated bolt shaft force $F_{b}$ agrees with the initial bolt shaft force $F_{bini} = 40\, \text{kN}$. The $w_{bini}$ value shows the position of the nut seat when the bolt is not extended having the initial length. By the progress of the numerical analysis, the value of $w_{b}$ decreases by extension of the bolt. Here, trial and error was used in the process of determination of $w_{bini}$ in the condition that neither load $W_{b}$ nor moment by $E$ result in an effect, and ensure that $F_{b}$ agrees with $F_{bini} = 40\, \text{kN}$. As a result a $w_{bini} \approx 12.96\, \mu\text{m}$ was obtained, accompanied by the value of $w_{b} = 11.25\, \mu\text{m}$.

3. Results under Usual Conditions

Figure 4 shows the results of the numerical analysis in the case of usual conditions, an isochromatic line, and a vector of transverse force on the wheel bolt. As shown in Fig. 4(a), the stress in the position of the bolt hole is larger than that in the lowermost part of the wheel being affected by the load. As shown in Fig. 4(b), the transverse force is almost all upward and fluctuates greatly during one revolution of the wheel: smallest in the lowest position, nearest to the load, and largest in the top position, farthest from the load. The reason for this paradoxical fact is verified in chapter 4.

Figure 5 shows changes in the transverse force of the wheel bolt during one revolution of the wheel corresponding to Fig. 4(b). The largest transverse force arising in the top position amounts to 4.21 kN. The possibility that loosening of the wheel nut may occur by this transverse force is shown in chapter 5.

The transverse force of the wheel bolt is considered to correspond to the share of the load acting on the wheel divided by the number of bolts. However, in Fig. 5, the average of the transverse force of the whole angle position, 3.04 kN, is larger than the share of the load, 2.63 kN. The reason for this paradoxical fact is also discussed in chapter 4.

Figure 6 shows the deformation of the disk wheel under usual conditions. It is clear from Fig. 6 that the deformation of the wheel by the bolt tightening force is far

$$
F_{r}(i, j, k) = \{\sigma_{z}(i, j, k + 1) - \sigma_{z}(i, j, k)\}r(i)\Delta\theta\Delta r
+ \{\sigma_{z}(i, j, k) - \sigma_{z}(i - 1, j, k)\}r(i)\Delta\theta\Delta \theta
+ \{\tau_{z}(i, j, k) - \tau_{z}(i - 1, j, k)\}r(i)\Delta\theta\Delta z
$$

The $r$-, $\theta$-, and $z$-components of the corrected displacement of Element$(i, j, k)$ are calculated from the residual forces and the stiffness equivalent of the element so that the residual forces vanish. The process is repeated until all residual forces of elements vanish.
larger than that by the load $W_W$ and moment, and the part of the wheel in the border of the hub hole inside the bolt hole pitch diameter deforms and extends outside the disk plane.

4. Influence of the Bolt Tightening Force

To examine the influence of the bolt tightening force on the result shown in Figs. 4 (b) and 5, a numerical analysis was carried out under the condition that only bolt shaft forces affect, neither the load $W_W$ nor the moment by $E$ affects. The results are shown in Figs. 7 and 8. It is clear from Fig. 7 (b) that the transverse force due to the bolt tightening force acts in the radial direction, and the value, 1.53 kN, amounts half the share of the load of a bolt, 2.63 kN. From the above, the reason that the average of the transverse force in Fig. 5 is larger than the share of the load on a bolt, can be explained. It is clear from Fig. 8 that the part of wheel in the border region of the hub hole inside the bolt hole pitch diameter deforms and extends outside the disk plane due to the bolt tightening force.

The reason that the transverse force in Fig. 4 (b) fluctuates greatly and is smallest nearest to the load and largest farthest from the load can also be explained. The transverse forces arising from the bolt tightening force and that arising due to the load are added in the upper position of the wheel, and cancel each other out in the lower position.

5. Possibility of Loosening of Wheel Nut

The friction force of the thread surface arising due to the bolt shaft force and the friction force of the nut seat also arising due to the bolt shaft force maintain the fastening force of the bolt-nut. When the transverse force exceeds a certain proportion of the bolt shaft force, the wheel nut begins to loosen\(^\text{11}\). The nut seat has been considered a flat surface in the literature\(^\text{11}\), but the nut seat of a wheel nut is a spherical or conic surface. In this case, the nut seat is handled as a conic surface with the base angle $\beta = 51^\circ$ (for the outer nut). When the transverse force $W_L$ reaches the value of the following equation, the nut seat begins to slip.

$$W_L = \frac{\mu F_A}{\cos^2\beta}$$ (12)

Here $\mu$ is the friction coefficient, and $F_A$ is the bolt shaft force. The thread surface also begins to slip, but the pressure angle of the thread, $\alpha = 30^\circ$, is smaller than the base angle of the nut seat, $\beta = 51^\circ$, and the thread surface begins to slip at smaller $W_L$, and is not the determining factor. Using $\mu = 0.25$ and $\beta = 51^\circ$ in Eq. (12),

$$W_L = 0.631 F_A$$ (12)'

The largest transverse force 4.21 kN shown in Fig. 5 is smaller than $W_L = 0.631, F_A = 23.3$ kN, and the nut seat would not begin to slip at once. However, the value 4.21 kN, 1/5.53 of $W_L$, cannot be disregarded. Further, the large fluctuation of the transverse force easily induces slipping and loosening.

6. Conclusions

Stress and deformation

(1) The stress in the position of bolt hole is larger than that in the lowermost part of the wheel being affected by the load. [Fig. 4 (a)]

(2) The deformation of wheel due to bolt tightening forces is far larger than that due to the load $W_W$ and the moment. [Fig. 6 (a)]

(3) The part of the wheel at the border of the hub hole inside the bolt hole pitch diameter is greatly deformed. [Fig. 6 (b)]

(4) The fact that the part of the wheel at the border of the hub hole inside the bolt hole pitch diameter is greatly deformed is caused by the bolt tightening force. (Fig. 8)

Transverse force of the wheel bolt

(5) The transverse force of the wheel bolt is almost only upward and fluctuates greatly during the revolution of the wheel. [Fig. 4 (b)]

(6) The transverse force is almost only upward and is the smallest in the lowest position, nearest to the load, and is largest in the top position, farthest from the load. [Fig. 4 (b)]
(7) The average of the transverse force of a whole revolution is larger than the share of the load. (Fig. 5)

(8) The transverse force due to bolt tightening forces acts in the radial direction. [Fig. 7 (b)]

(9) The transverse force due to bolt tightening forces amounts to half the load. [Fig. 7 (b), chapter 4]

(10) The finding in (9) explains the reason for (7), that the average of the transverse force is larger than the share of the load.

(11) The finding in (5) and (6) explain the fact mentioned in (8).

Concerning the possibility of loosening of wheel nuts

(12) The force acting here may not immediately cause slipping of the nut seat. However, the influence of the force cannot be ignored.

(13) The large fluctuation in the transverse force appears to promote slipping and loosening.

References


