Crack Identification of Plates Using Genetic Algorithm

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In this paper, a method for identifying of a crack in a plate that uses a genetic algorithm (GA) based on changes in natural frequencies is presented. To calculate the natural frequencies of the cracked plates, a FEM (Finite Element Method) program, which is based on the BFM (Bogner, Fox and Schmidt) model, is developed since the accuracy of the forward solver is important. In the analysis, two types of cracks, i.e., internal and edge cracks are considered. To identify the crack location and the depth from frequency measurements, the width and position of the crack in a plate are coded into a fixed-length binary digit string. Using GA, the square sum of residuals between the measured data and the calculated data is minimized in the identification process and thus the crack is identified. To avoid a high calculation cost, the response surface method (RSM) is also adopted in the minimizing process. The combination of GA and RSM makes the identification more effective and robust. The applicability of the proposed method is confirmed by the results of numerical simulation.

Key Words: Inverse Problem, Plate, Crack Identification, Finite Element Method, Natural Frequency, Genetic Algorithm, Numerical Analysis

1. Introduction

Crack generation in structures, such as machine structures, may cause unexpected failures and accidents. Since crack generation results particularly from fatigue in materials and unexpected overloads, the preemptive detection of cracks in structures is crucially important for maintaining their safety. The authors have proposed an inverse analysis method that uses both natural frequency changes caused by cracks and a genetic algorithm (GA)(1)–(7), and have demonstrated that the method is very accurate in identifying the locations and sizes of cracks (hereinafter referred to as “crack parameters”) in beam structures. These studies mainly dealt with cracks in beam structures, whereas few studies have addressed the inverse analysis of cracks in plate structures(8). In this study, we present an inverse analysis method of cracks in plates by applying the previously proposed method to cracked plates employing the finite element method (FEM) based on BFS elements(9). First, an element stiffness matrix and a consistent mass matrix for a BFS element are derived, and a finite element analysis program is written. Then, inverse analysis of a crack in plates is performed using both this FEM program and a GA. In this inverse analysis, the natural frequency data generated by the FEM program are used as search data. Finally, several examples of numerical analysis are shown, and the applicability of the inverse analysis, based on a GA, to internal and edge cracks in plates is demonstrated.

2. Vibration Analysis Using FEM

2.1 BFS element

As shown in Fig. 1, a four-node rectangular element with lengths of $2a_0$ and $2b_0$ and a thickness of $h$ is considered. In the BFS element, each node is assumed to have four degrees of freedom: deflection $w_i$, deflection angles in the $x$ and $y$ directions $\theta_{xi}$ and $\theta_{yi}$, and torsional angle $\theta_{xy}$. elements(9). First, an element stiffness matrix and a consistent mass matrix for a BFS element are derived, and a finite element analysis program is written. Then, inverse analysis of a crack in plates is performed using both this FEM program and a GA. In this inverse analysis, the natural frequency data generated by the FEM program are used as search data. Finally, several examples of numerical analysis are shown, and the applicability of the inverse analysis, based on a GA, to internal and edge cracks in plates is demonstrated.

Fig. 1 BFS element
(404)

\[
\begin{align*}
\delta_i = [N_i][\delta], \\
\end{align*}
\]

where \( N_i \) is a shape function and \( \delta \) is a nodal displacement vector. Using a coefficient matrix \([B]\) for the strain-displacement relation and an elastic coefficient matrix \([D]\) [10], an element stiffness matrix \([K_e]\) is expressed as

\[
[K_e] = \int_\Delta [B]^T[D][B]dxdy \\
= a_0b_0 \int_{\xi=1}^{\xi=1} \int_{\eta=1}^{\eta=1} [B]^T[D][B]d\xi d\eta.
\]

Using the interpolation function \([N_i]\) in Eq. (1), a consistent mass matrix \([M_e]\) is expressed as

\[
[M_e] = \int_\Delta [N_i]^T[N_i]dxdy \\
= \rho a_0b_0 \int_{\xi=1}^{\xi=1} \int_{\eta=1}^{\eta=1} [N_i]^T[N_i]d\xi d\eta,
\]

where \( \rho \) is the density of the material. Therefore, to calculate the natural frequency \( \omega \) of the plate, a following conventional characteristic equation obtained by superposing individual element stiffness and consistent mass matrices should be solved:

\[
([K] - \omega^2[M])[\delta] = 0,
\]

where \([K]\), \([M]\) and \([\delta]\) represent the global stiffness matrix, the global mass matrix, and the global nodal displacement vector, respectively. In this study, this FEM program was made using Visual C++.

2.2 Automatic mesh and crack generation

The inverse analysis method proposed herein makes use of the natural frequencies of plates computed by the FEM program. As detailed in section 3.3, the values of the natural frequencies need to be determined under various conditions of crack parameters when we employ a GA. Therefore, numerous natural frequency data at the positions of lattice points that represent any crack parameters need to be analyzed in advance. In order to readily perform FEM analysis of a cracked plate of any size at any location, an automatic quadrilateral mesh generator was made with reference to the literature [11]. This program, which was written in Visual Basic, can easily generate a crack by mouse dragging as well as generate meshes interactively, as shown in Fig. 2. A crack was generated using a procedure in which new nodes are created near original nodes so as to create a small gap between the neighboring elements. Figure 3 shows an internal crack and an edge crack, both of which were generated in accordance with this procedure. Using input data created in this way, FEM natural frequency analysis was performed on the basis of BFS element described in section 2.1, allowing the natural frequencies to be calculated.

3. Crack Identification Using Genetic Algorithm and Response Surface Method

The inverse problem considered in this paper is to estimate the crack parameters on the basis of the measurement of various-order natural frequencies. In this study, various-order natural frequencies of cracked plates were adopted as observation data for crack estimation. The GA, which requires function values only and has the advantage of being able to deal with multiple peaks, was adopted as an estimation technique on the basis of our previous study [7]. For the sake of simplicity, natural frequency values obtained by a forward analysis based on the FEM were used as the measured values \( f_i \) (in Eq. (5) described below) for the natural frequencies of the cracked plates.

3.1 Crack searching procedure using GA

In this section, we describe our crack searching procedure. As shown in Fig. 6, only cracks oriented parallel to the \( y \)-axis, which have a larger effect on bending vibration, are considered in this study. First, crack positions \( a_1 \) and \( b_1 \) and crack size \( c \) are set as unknown parameters for crack searching (see Figs. 4 and 6). Each parameter is expressed as an 8-bit binary string of “1 s” and “0 s”, and a 24-bit string is provided to each individual as genetic
This means that each individual has crack parameters. Then, various operations based on the GA, such as crossover, selection and mutation, are repeatedly performed among these individuals over a certain number of generations. In this study, an algorithm that gradually decreases its mutation rate with increasing number of generations was employed. The initial mutation rate was set at 0.9 and then gradually decreased. When the elapsed number of generations was half of a given number of generations, the mutation rate had declined to 0.01. The mutation rate was subsequently held constant. This technique enabled us to narrow a solution with diversity to a true solution.

3.2 Error evaluation function and fitness value

In this study, we focus our attention on the difference in natural frequencies as a function of crack location and size, and therefore fitness values, used for the evaluation of individuals, must be determined. When the $i$-th natural frequency, the $i$-th measured natural frequency and the number of natural frequencies for crack searching are denoted by $f_i$, $\bar{f}_i$ and $n$, respectively, error evaluation function $E_n$ is expressed as

$$E_n = \frac{1}{n} \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left( \frac{f_i}{\bar{f}_i} - 1 \right)^2}. \tag{5}$$

Therefore, the error evaluation function $E_n$ will reach its minimum value at the true crack position and size. The reciprocal of Eq. (5) is defined as fitness.

$$fitness = \frac{1}{\frac{1}{n} \sqrt{\sum_{i=1}^{n} \left( \frac{f_i}{\bar{f}_i} - 1 \right)^2}}. \tag{6}$$

The fitness function will reach its maximum value at the true crack position and size.

3.3 Approximation of natural frequency

In order to estimate the crack parameters using the GA described above, we must now compute the natural frequencies by FEM. However, this method requires eigenvalue analysis by FEM for each individual and if we employ this approach, it would require an enormous calculation time.

In our study, we focus on the fact that natural frequency is a function of three variables: two crack positions and size, and we analyze the natural frequency of each order at each of the appropriate lattice points in advance. On the basis of these natural frequency values, the natural frequency at any point is evaluated at given parameters using the following interpolation function.

First, we consider an eight-node hexahedron element with lengths of $2a_0$, $2b_0$ and $2c_0$ as shown in Fig. 5. A point $(x_0, y_0, z_0)$ in a global coordinate is assumed to be the center position and a following dimensionless local coordinate system given by $(\xi, \eta, \zeta)$ which has its origin at $(x_0, y_0, z_0)$ is used.

$$\xi = \frac{x - x_0}{a_0}, \quad \eta = \frac{y - y_0}{b_0}, \quad \zeta = \frac{z - z_0}{c_0} \tag{7}$$

Hence, each coordinate ranges from $-1$ to $1$. Assuming a linear change along each edge, a natural frequency $f(\xi, \eta, \zeta)$ at any point $(\xi, \eta, \zeta)$ in an element is expressed by the following interpolation function.

$$f(\xi, \eta, \zeta) = a_1 + a_2 \xi + a_3 \eta + a_4 \zeta + a_5 \xi \eta + a_6 \eta \zeta + a_7 \zeta \xi + a_8 \xi \eta \zeta. \tag{8}$$

The eight constants $a_i (i = 1 \sim 8)$ in Eq. (8) are determined using the values of $f(\xi, \eta, \zeta)$ at any node $(\xi, \eta, \zeta)$ in an element is expressed by the following interpolation function:

$$f(\xi, \eta, \zeta) = \frac{1}{8} \sum_{i=1}^{8} (1 + \xi) (1 + \eta) (1 + \zeta) f_i. \tag{9}$$
where \((\xi, \eta, \zeta)\) are coordinates at each node \((i = 1, 2, \sim, 8)\); and \(\xi, \eta\) represent nondimensional crack positions and \(\zeta\) represents a nondimensional crack size. Natural frequency values at all crack parameters are calculated using the above interpolation function, and then fitness values defined by Eq. (6) are calculated on the basis of natural frequency values. This technique enables us to significantly reduce the time required to calculate the natural frequency and helps rapid identification. Except that unknown parameters are derived by an interpolation from points on a lattice, the above technique is similar to conventional response surface methodology (RSM), where unknown parameters are evaluated using surface approximation.

4. Inverse Analysis

4.1 Natural frequency analysis

Before inverse analysis, the accuracies of natural frequencies obtained by BFS elements are investigated. In the plate shown in Fig. 6, the natural frequency of the plate, simply supported on all edges, having an internal crack in the center with \(c/b = 0.5\), is addressed. The following notations are introduced: the length, width, and thickness of the plate are \(a, b\) and \(h\); the positions of crack are represented by \(a_1\) and \(b_1\); the size of the crack is \(c\); the crack is oriented parallel to the \(y\)-axis and penetrates through the plate. The physical values, e.g., Young’s modulus \(E\), shown in the figure were used.

Analysis was performed under the following conditions: \(a/b = 1\), \(b = a = 0.4\) [m], and \(h = 0.001\) [m]. The computed results were compared with those obtained by Yamada et al. using the boundary element method (13) and the results derived using series solutions by Stahl et al. (14), as shown in Fig. 7. The figure indicates that the results of our analysis are fully in agreement with the results of both Yamada et al. and Stahl et al. Taking the above numerical results into consideration, the plate is divided into 400 rectangular elements in the subsequent calculation, and the resulting natural frequency values are employed in the inverse analysis.
Next, the effects of cracks on the natural frequency of the plate are investigated to perform inverse analysis in advance. Crack position $a_1$, $b_1$ and crack size $c$ (see Fig. 6) are expressed as $a_1/a$, $b_1/b$ and $c/b$ in nondimensional form, respectively. It should be noted that the crack position of the edge crack is expressed as $b_1/b = 0$.

Figures 8–10 show the relationship between natural frequency and crack positions $a_1/a$ and $b_1/b$ at a constant crack size of $c/b = 0.1$. Figures 8 and 9 show the analyzed results of the first and third natural frequencies of a cantilever plate. Figure 10 shows the analyzed results of the first natural frequencies of a plate clamped at both ends. These figures indicate that edge cracks correspond to significant decreases in natural frequency. Such decreases also occurred for a different aspect ratio $a/b$ of the plate.

4.2 Results of crack identification

On the basis of a preliminary calculation, the parameters of GA were set as follows: an initial crossover rate of 0.5 and a population of 150. The number of generations was set at 150. Identified solutions were deduced from the crack parameters of the individual that has the maximum fitness value during the calculation period up to the last generation.

To perform crack detection based on the technique shown in section 3.3, natural frequency values at lattice points in each element need to be analyzed in advance. The values of various-order natural frequencies were calculated for a total of 396 cracks created by dividing the position of crack end $x$ into 11 at intervals of 0.1 (i.e., at $a_1/a = 0.00, 0.05, 0.10~0.90$ (step 0.1)), the position of crack end $y$ into 6 at intervals of 0.1 (i.e., $b_1/b = 0.00, 0.05, 0.10~0.40$ (step 0.1)), and crack size at each position $(x, y)$ into 6 at intervals of 0.1 (i.e., $c/b = 0.00~0.50$). On the basis of these lattice point data, the values of various-order natural frequencies at all crack parameters were calculated using Eq. (9). Subsequently, calculation of fitness values using Eq. (6) and inverse analysis using the GA were performed. The number of natural frequencies used for identification was set at 4: the first to fourth natural frequencies. In consideration of the symmetric property of the shape, a crack searching was performed within the range of $0 \leq b_1/b \leq 0.5$.

Figures 11 and 12 show the results of the inverse analysis of cracks with a size of $c/b = 0.15$ in a plate with an aspect ratio of $a/b = 1$, a length of $a = b = 0.4$ [m], and a thickness of $h = 0.001$ [m]. Figures 11 and 12 show the results for edge cracks and internal cracks, respectively. The dashed lines in the figures represent actual crack positions and sizes, and the solid lines represent identified crack positions and sizes. Although there are some minor exceptions, the edge cracks are distinguished from the internal cracks, indicating an excellent identification almost throughout the range.

Figure 13 shows an example of the relationship between generation and the fitness values (maximum and average) of each generation in the inverse analysis results shown in Fig. 12. The figure indicates that both fitness values increase with an increased generation number to about the 50th generation, while the maximum fitness value is almost unchanged after the 50th generation.

Figures 14 and 15 show the results of the inverse analysis of cracks on a plate clamped at both ends under the same calculation conditions as in Figs. 11 and 12. In consideration of the symmetric property of the shape,
a crack searching was performed within the ranges of $0.0 \leq a_1/a \leq 0.5$ and $0.0 \leq b_1/b \leq 0.5$. Although some poor identification accuracies are recognized, a satisfactory identification was demonstrated throughout the range. Only the results for cracks with a size of $c/b = 0.15$ are shown here, but the identification of cracks of other sizes give also almost equally good results. However, cracks with smaller sizes were less accurately identified. This is because smaller cracks cause smaller changes in natural frequency, as seen in the case of the identification of cracks in a beam structure\(^{\text{6}}\).

Next, the results with a crack size of $c/b = 0.15$ in plates with different aspect ratios $a/b$ are described below. Figures 16 and 17 show the identified results of cracks in a cantilever plate and a plate clamped at both ends, both with an aspect ratio of $a/b = 0.5$. Figures 18 and 19 show...
the identified results of $a/b = 2.0$ under the same conditions as in Figs. 16 and 17. The figures demonstrate a good identification of cracks throughout the range. However, decreases in accuracy can be seen in some locations. This is because there is a problem with the approximation accuracy of the interpolation function expressed by Eq. (9), and because rapid changes in natural frequency cannot be resolved due to coarse lattice-point intervals. In the results of the inverse analysis of cracks in the plate clamped at both ends with an aspect ratio of $a/b = 0.15$ shown in Fig. 16, satisfactory identification accuracies were not obtained for four natural frequencies for a similar reason to that for the decreases in accuracy described above. The figure shows the results obtained when the number of natural frequencies used for identification was increased to 6.

In the above, only the case in which the size of the crack $c/b = 0.15$ was shown, but we have actually carried out analysis for $c/b = 0.05, 0.25, 0.35, 0.45$ other than $c/b = 0.15$. Two examples for $c/b = 0.05, 0.25$ are shown in Figs. 20 and 21. It can be seen that the identification accuracy is improved when the crack size increases and the accuracy reversely decreases, when the crack size decreases.

We have proposed herein an identification method for cracks in plates with different boundary conditions and aspect ratios. From the computed results, it is shown that 4 – 6 natural frequencies are sufficient to identify the three unknown parameters (positions $(x, y)$ and size $c/b$).

### 5. Conclusions

Inverse analysis of cracks in plates was performed using both GA and FEM. The following conclusions were obtained.

1. We have proposed an inverse analysis method for cracked plates using both GA and FEM, and have demonstrated the effectiveness of this method through several examples of numerical analysis.

2. The results showed that this method provides close to satisfactory identification of cracks in plates with different boundary conditions and aspect ratios.

### References


