In-Plane Displacement Measurement Using Digital Image Correlation with Lens Distortion Correction

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Two-dimensional displacement measurement using digital image correlation with lens distortion correction is described in this paper. A single cross-grating is used as a calibration reference. Using two-dimensional Fourier transform, the phases of the grating pattern are analyzed and lens distortion distribution is obtained from the unwrapped phase maps. After detecting lens distortion, the coefficients of lens distortion are determined using the least-squares method. Then, the displacement distributions without the lens distortion are obtained. The effectiveness of the method is demonstrated by applying the proposed method to the rigid body translation test and the uniaxial tension test. The results show that the proposed distortion correction method removes the effect of lens distortion from the measured displacements. By the proposed method, accurate measurements can be performed even if images are deformed by lens distortion.

Key Words: Displacement, Digital Image Correlation, Lens Distortion Correction, Deformation Measurement, Image Processing, Optical Methods, Experimental Stress Analysis

1. Introduction

Displacement measurement technique based on digital image correlation, which can yield the surface deformation by comparison of digital images of undeformed and deformed configurations, has been developed and improved by several researchers(1)–(8). Since this method does not need a complicated optical system, the measurement can be performed simply and easily. In addition, unlike other methods which utilize the interference of light waves, phase analysis of the fringe pattern and the subsequent phase unwrapping process are not required. Thus, many applications of this method to various problems can be found, such as in studies of high-temperature deformation measurement(9), dynamic crack propagation(10), shape memory alloy(11),(12), time-dependent materials(13), and inverse stress analysis(14). In addition to the simplicity of measurements, the recent progress of high-resolution handy digital cameras with zoom lenses has made effective use of the method to various applications. The use of the method not only for experimental studies but also in daily inspections of large structures is also expected.

In digital image correlation, a telephoto lens should be used to minimize any change in magnification due to small out-of-plane displacement of a specimen. However, a wide-angle lens and a zoom lens are sometimes useful for certain applications. It is well known that in a low-cost optical system, a wide-angle lens or a zoom lens suffers from lens aberrations. In digital image correlation, measurement error due to geometrical lens aberration, that is, lens distortion, is easily introduced because the movement of a specimen surface is detected from images. The influence of lens distortion can be neglected by using an appropriate combination of a CCD camera (size and resolution of CCD) and a lens. However, this is not always possible, or the cost of a distortion-free system is too high. In this case, lens distortion correction must be introduced to overcome this problem. Camera calibration including distortion correction is considered to be an important issue in the field of computer vision. Various lens distortion correction methods can be found in the literature(15)–(22) and patents. Some of these methods can be applied to distortion correction in digital image correlation. However, the measurement procedure and/or the subsequent data processing for distortion detection and correction are complicated because most of these methods are calibration meth-
ods for three-dimensional computer vision. In addition, since some methods involve geometrical image processing techniques such as the detection of straight line edges or targets, the accuracy is not very high.

In the present paper, a simple method of lens distortion correction is proposed in order to improve the measurement accuracy of digital image correlation for two-dimensional displacement measurement. A single two-dimensional cross-grating is used as a calibration reference. It is known that the phase of the grating can be analyzed accurately using Fourier transform. Mori-moto and coworkers developed a two-dimensional displacement distribution measurement method by analyzing a two-dimensional grating pattern using Fourier transform. Utilizing this method, the phases of the grating pattern are analyzed. Then, the lens distortion distribution is determined from the unwrapped phase maps. After detecting the lens distortion, the coefficients of the lens distortion are determined by the least-squares method. Then, the corrected displacement distributions are obtained. The effectiveness of the method is demonstrated by applying the proposed method to rigid body translation and uniaxial tension tests. The results show that the proposed distortion correction method removes the effect of lens distortion from the measured displacements. The proposed distortion detection method is fundamentally accurate compared with conventional edge detection methods because phase analysis is performed. It is emphasized that accurate measurements can be accomplished even if images are deformed by lens distortion.

2. Digital Image Correlation with Lens Distortion Correction

2.1 Outline of digital image correlation

In the digital image correlation technique, random speckle-like black and white dot patterns are observed on specimen surfaces. These image patterns, one before and another after deformation, are digitized and stored in a computer. The digitized images are compared to match subsets from one image to another. Typically, a subset such as 20×20 or 30×30 pixels from the undeformed image is chosen and its location is then found in the deformed image. Once the location of this subset in the deformed image is found, the displacements of this subset can be obtained. For the best estimate of the displacements, a normalized cross-correlation coefficient, \( S \), defined below, is used:

\[
S(x, y; u_x, u_y, \frac{\partial u_x}{\partial x}, \frac{\partial u_x}{\partial y}, \frac{\partial u_y}{\partial x}, \frac{\partial u_y}{\partial y}) = \frac{1}{\sqrt{\sum I_u(x, y)^2 \sum I_u(x', y')^2}} \frac{\sum I_u(x, y) I_u(x', y')}{
\]

Here, \( u_x \) and \( u_y \) are the displacement components at the center of the subset, \( I_u \) and \( I_d \) represent the gray levels of the undeformed and deformed images, respectively, and \( x \) and \( y \) are the coordinates of a point on the subset before deformation and \( x' \) and \( y' \) are those after deformation. The coordinates \( x' \) and \( y' \) after deformation are related to the coordinates \( x \) and \( y \) before deformation by

\[
x' = x + u_x + \frac{\partial u_x}{\partial x} \Delta x + \frac{\partial u_y}{\partial y} \Delta y,
\]

\[
y' = y + u_y + \frac{\partial u_x}{\partial x} \Delta x + \frac{\partial u_y}{\partial y} \Delta y,
\]

where \( \Delta x \) and \( \Delta y \) are the \( x \)- and \( y \)-directional components of the distance from the center of the subset to the considered point, respectively.

The correlation coefficient \( S \) is a function of the displacement components \( u_x \) and \( u_y \), and the displacement gradients \( \partial u_x/\partial x, \partial u_x/\partial y, \partial u_y/\partial x \) and \( \partial u_y/\partial y \). Therefore, the displacements are determined by searching for the best set of displacements and displacement gradients, that minimize the correlation coefficient \( S \). In this process, the approximate displacements are first estimated within the accuracy of one pixel with zero gradients. After the first estimation, the process varies to search for both the displacements and the displacement gradients by the Newton-Raphson method. In this step, to increase the resolution of the correlation method, an interpolation scheme, such as a bilinear interpolation method, is used to reconstruct a continuous intensity instead of a discrete intensity pattern. The details of digital image correlation with Newton-Raphson optimization are given in Refs. (2), (3), (27) and (28).

2.2 Distortion models

The image distortion model is usually given as a map from the distorted image coordinates to the undistorted image coordinates. The coordinates \( x' \) and \( y' \) of a point on a distorted image can be expressed as

\[
x' = x + \alpha_x, \quad y' = y + \alpha_y,
\]

where \( x \) and \( y \) are the distortion-free image coordinates and \( \alpha_x \) and \( \alpha_y \) express the amount of distortion along the \( x \) and \( y \) directions, respectively. The image distortion model can be decomposed into two terms, that is, radial distortion and tangential distortion. As shown in Fig. 1 (a), radial distortion is deformation of an image along the direction from the center of distortion to the considered point. On the other hand, tangential distortion is deformation perpendicular to this direction. It can be found that for many applications, tangential distortion need not be considered because the amount of tangential distortion is very small compared with that of radial distortion. Figure 1 (b) shows the effect of radial distortion. Radial distortion causes an inward and outward displacement of a given image point from its ideal location, as shown in this figure. A negative radial displacement of the image points
The amount of radial distortion $\alpha_r$ can be expressed as 
\[\alpha_r = k_1 r^3 + k_2 r^5 + k_3 r^7 + \ldots, \] (4)
where $r$ is the radial distance from the center of the distortion to the image points, and $k_1, k_2, k_3, \ldots$ are the coefficients of radial distortion. Assuming that the terms of higher order than three are negligible, Eq. (4) can be rewritten as
\[\alpha_r = k_1 r^3. \] (5)

At each image point represented by polar coordinates, the radial distortion corresponds to the distortion along the radial direction. The image point can also be expressed in terms of Cartesian coordinates. Then, the amount of radial distortion on the Cartesian coordinates can be represented by
\[\alpha_x = k_1 x \left( x^2 + y^2 \right); \]
\[\alpha_y = k_1 y \left( x^2 + y^2 \right). \] (6)

It is seen from Eqs. (5) and (6) that the amount of distortion is proportional to the cube of the radial distance $r$. Then, the distortion can be corrected by knowing the coefficient $k_1$.

When decentering distortion and thin prism distortion are presented, tangential distortion should be taken into account in distortion correction(15), (16). Decentering distortion arises from the decentering of lens elements. On the other hand, thin prism distortion arises from imperfection in the lens design and the manufacturing as well as the camera assembly. In this case, the amount of distortion is expressed as
\[\alpha_x = (g_1 + g_3) x^2 + g_4 xy + g_1 y^2 + k_1 x \left( x^2 + y^2 \right), \]
\[\alpha_y = g_2 x^2 + g_1 xy + (g_2 + g_4) y^2 + k_1 y \left( x^2 + y^2 \right). \] (7)
where $g_1, g_2, g_3, g_4$ are the coefficients of tangential distortion. As a result, five coefficients are required for distortion correction when tangential distortion as well as radial distortion are considered.

2.3 Distortion correction using Fourier transform

Figure 2 shows the two-dimensional cross-grating pattern used as the reference of the distortion correction. By phase analysis of this grating pattern, lens distortion correction for digital image correlation is proposed in the present study. It is known that the distribution of the phases of the grating or an interference fringe pattern can be analyzed using Fourier transform(23) – (26). In this study, the two-dimensional displacement measurement method proposed by Morimoto and coworkers(25), (26) is adopted as the distortion detection method.

Figure 3 shows the procedure for the distortion detection using two-dimensional Fourier transform. The grating pattern shown in Fig. 2 is deformed by lens distortion; then, the deformed grating pattern shown in Fig. 3 (a) is recorded as a digital image. Figure 3 (b) represents the two-dimensional Fourier spectrum of the deformed grating pattern. The horizontal axis shows the $x$-directional frequency $\omega_x$, and the vertical axis shows the $y$-directional frequency $\omega_y$. The brightness corresponds to the power of the frequencies. By extracting and centering the $x$-directional first harmonic from the spectrum and perform-
ing the inverse Fourier transform, the wrapped phase distribution that shows the $x$-directional displacement distribution of the grating pattern is obtained, as shown in Fig. 3 (c). The $y$-directional displacement distribution shown in Fig. 3 (d) is obtained in a similar manner. These phase maps are wrapped in the range of $-\pi$ to $\pi$. Thus, by performing spatial phase unwrapping, the unwrapped phase maps shown in Fig. 3 (e) and (f) is obtained. The displacement distributions $\beta_x$ and $\beta_y$ of the grating are calculated from the unwrapped phases as

$$\beta_x = \frac{\delta_x}{2\pi\omega_{x0}},$$
$$\beta_y = \frac{\delta_y}{2\pi\omega_{y0}},$$

(8)

where $\delta_x$ and $\delta_y$ are the unwrapped phase values and $\omega_{x0}$ and $\omega_{y0}$ are the frequencies of the undeformed grating along the $x$ and $y$ directions, respectively. These displacements represent the deformation of the grating pattern, that is, the lens distortion when the original grating patterns along the $x$ and $y$ directions coincide with the horizontal and vertical axes of the camera. In this case, displacements $\beta_x$ and $\beta_y$ in Eq. (8) are equivalent to distortion components $\alpha_x$ and $\alpha_y$.

In actuality, it is difficult to adjust the alignment of the grating pattern with the camera perfectly. In addition, an error is introduced when the shift of the first harmonic to the center cannot be performed accurately. The misalignment of the grating pattern with the camera represents the rigid body rotation of the grating. On the other hand, the miscentering of the first harmonic is equivalent to the de-
formation of the uniaxial tension. These are represented as plane surfaces, whereas the displacements corresponding to the distortion are represented as curved surfaces. Therefore, the displacements obtained by Fourier transform can be expressed as

\[\beta_x = a_1x + a_2y + a_3 + a_4,\]
\[\beta_y = a_4x + a_5y + a_6 + a_7,\]

where \(a_1, a_2, a_3, a_4, a_5, a_6\), and \(a_7\) are the coefficients that represent miscentering and misalignment, and \(a_4\) and \(a_5\) are the distortion components that are expressed by Eq. (6) or (7). Therefore, the coefficient \(k_1\) of the radial distortion or the coefficients \(k_1, g_1, g_2, g_3\), and \(g_4\) of the radial and tangential distortions are determined from the results of Fourier transform (Eq. (8)). Here, the displacements obtained by Fourier transform are the values at the distorted coordinates \(x'\) and \(y'\). On the other hand, the distortion model is a function of the distortion-free coordinates \(x\) and \(y\). Therefore, the coefficients cannot be determined directly. Thus, the least-squares method with an iterative procedure is proposed for the determination of the coefficients in this study. First, the coordinates \(x'\) and \(y'\) of the points with lens distortion are substituted into the right-hand side of Eq. (9) instead of the distortion-free coordinates \(x\) and \(y\) as the initial values. Then, the approximated values of the coefficients are determined in a least-squares sense as

\[g = (x^T x)^{-1} x^T b,\]

where \(g\) is the coefficient matrix

\[g^T = [g_1 \ g_2 \ g_3 \ g_4 \ k_1 \ a_1 \ a_2 \ a_3 \ a_4 \ a_5 \ a_6],\]

and \(x\) and \(b\) are

\[
x = \begin{bmatrix}
x_1^2 + y_1^2 & 0 & x_1 & y_1 & (x_1^2 + y_1^2) & x_1 & y_1 & 1 & 0 & 0 & 0 \\
x_2^2 + y_2^2 & 0 & x_2 & y_2 & (x_2^2 + y_2^2) & x_2 & y_2 & 1 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
x_n^2 + y_n^2 & 0 & x_n & y_n & (x_n^2 + y_n^2) & x_n & y_n & 1 & 0 & 0 & 0 \\
0 & x_1^2 + y_1^2 & x_1 & y_1 & (x_1^2 + y_1^2) & 0 & 0 & 0 & x_1 & y_1 & 1 \\
0 & x_2^2 + y_2^2 & x_2 & y_2 & (x_2^2 + y_2^2) & 0 & 0 & 0 & x_2 & y_2 & 1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & x_n^2 + y_n^2 & x_n & y_n & (x_n^2 + y_n^2) & 0 & 0 & 0 & x_n & y_n & 1 
\end{bmatrix},
\]

\[b^T = [\beta_{x1}, \beta_{x2}, \ldots, \beta_{xn}, \beta_{y1}, \beta_{y2}, \ldots, \beta_{ym}],\]

where the subscript represents the index of the data points and \(n\) gives the total number of data points. In the case that the tangential distortion need not be considered, Eqs. (11), (12) and (13) become simpler. On the other hand, when the center of distortion is different from the center of images, the position of the center of distortion, as well as the coefficients, can be simultaneously determined by the method of nonlinear least squares, similar to the determination method of the stress intensity factor and crack tip location(29). Next, the approximated coordinates \(x\) and \(y\) of the points without lens distortion are calculated using the approximated value of the coefficients. Then, the approximated coordinates are used for the recalibration of the coefficients. This procedure is repeated until the coefficients and the coordinates converge. Finally, the distortion coefficients \(k_1, g_1, g_2, g_3\), and \(g_4\) are obtained by adopting the convergent value. About four or five iterations are usually needed to obtain the convergent values. Then, the distortion components \(a_x\) and \(a_y\) are obtained using Eq. (6) or (7).

Using the distortion components \(a_x\) and \(a_y\), the corrected coordinates \(x\) and \(y\) are determined using Eq. (3). Here, the iterative procedure is again used to determine the corrected coordinates \(x\) and \(y\) from the distorted coordinates \(x'\) and \(y'\). Finally, the displacements \(u_x\) and \(u_y\) without lens distortion are obtained from the corrected coordinates \(x\) and \(y\).

3. Experimental Verifications and Results

3.1 Rigid body translation test

First, a rigid body translation test is performed in order to validate the proposed method. A sample made of an aluminum plate is mounted on a linear translation stage. A random black and white pattern on the surface of the sample is created by spray painting. The sample on the stage is illuminated by a white light source (halogen lamp). Then, the sample is translated along the \(x\) (horizontal) direction from 0 to 5 mm at the increment of 0.5 mm on a linear translation stage with the resolution of 0.01 mm. A digital single-lens reflex camera (3.554 \times 2.336 pixels \times 24-bit) with a zoom lens is used to record the random pattern before and after the translation. It is known that the size of the random pattern should be selected to oversample the intensity pattern obtained using several sensors for accurate measurement(27). In this experiment, each random pattern is oversampled by 10 ~ 40 pixels. After the images are acquired, they are converted to 8-bit images and analyzed by digital image correlation.

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A cross-grating pattern, of which pitches are 1.5 mm for both $x$ and $y$ directions, is also recorded using the same experimental setup for distortion correction. Then, the amount of distortion is determined by the proposed method. Here, the mixed radix fast Fourier transform (MR-FFT)\cite{30}, which can be applied to an arbitrary-length data series, is used for the phase analysis.

Figure 4 shows the random pattern and the cross-grating pattern obtained in this experiment. The imaging area is about $150 \times 100\, \text{mm}^2$. Thus, the length of 1 mm corresponds to about 24 pixels on these images. It is observed that the grating pattern near the edge of the image is deformed by lens distortion. Displacements $u_x$ and $u_y$ of the rigid body translation of 5 mm along the $x$ direction obtained by digital image correlation are shown in Fig. 5. It should be noted that the contour maps in Fig. 5 must be uniform because this figure shows the results of the rigid body translation test. However, displacement distributions due to the lens distortion are observed in Fig. 5. Using the distortion correction method proposed in this paper, the distortion in the displacement distributions is corrected. Figure 6 shows the results of the radial and tangential distortion corrections. Here, it is assumed that the center of distortion is located at the center of the image. Almost uniform displacement distributions are observed in both of the $x$- and $y$-directional displacements. That is, the distortions in the displacements in Fig. 5 are appropriately corrected by the proposed method.

Figure 7(a) shows the $x$-directional displacements in pixels with the amount of the translation at the points A (near left and top corner), B (near left edge) and C (center) shown in Fig. 4(a). That is, the abscissa shows the amount of given translation and the ordinate represents the measured displacements. The displacements at points A and B differ from the displacement at point C. The differences in the displacements between points A and C (A–C), and points B and C (B–C) are shown in Fig. 7(b). A large difference, of a maximum of about 5 pixels, between the center point and the edge of the image is observed when the translation is 5 mm. The radial and tangential distortion correction using Eq. (7) and the radial distortion correction using Eq. (6) are performed and compared. Figure 8 shows the correction results of radial and tangential distortions. It is observed that almost the same displacements are obtained at points A, B and C. The differences in the displacements among these points are reduced to less than
0.5 pixels. That is, the difference of 5 pixels in displacements produced by the lens distortion is reduced to less than 0.5 pixels by the proposed method. Almost the same results are obtained when the radial distortion correction is performed, as shown in Fig. 9. As a result, the correction of the radial distortion is sufficient and the tangential distortion correction is not required for the optical system and the camera used in this study.

The results of the rigid body translation test show that the proposed correction method is effective in displacement measurements by digital image correlation.

### 3.2 Uniaxial tension test

The second test is a displacement measurement of a plate under tension. The plate specimen made of SS400 steel, 140 mm in width, 700 mm in length and 12 mm in thickness, shown in Fig. 10, is adopted. Tensile load is applied at the upper and lower holes of the specimen. Then, the images of the upper center area of the specimen before and after deformation are recorded. The strains along the tensile direction are also measured with strain gauges on the back of the specimen.
Figure 11 shows the contour maps of the displacement distribution before distortion correction under the load of 240 kN. The contours of the displacements must be straight because simple tension is applied to the plate specimen. However, the unnatural distributions, that is, curved contours, are observed in both of \(x\)- and \(y\)-directional displacements. The corrected displacement distributions are shown in Fig. 12. Straight contour lines are observed in this figure, that is, the lens distortion was corrected by the proposed method. In addition, it is observed in Fig. 12 that the specimen exhibits rigid body rotation.

The strain distributions are calculated from the displacements in Figs. 11 and 12 using a moving least-squares method. Here, the values of the displacement at nine points on and around the observed point are fitted to a second-order polynomial surface to calculate the strain. Figure 13(a) shows the strain distribution \(\varepsilon_{yy}\) along the \(y\) direction obtained from the displacement \(u_y\) without the distortion correction in Fig. 11(b). A uniform strain distribution must be obtained because the applied load is uniaxial tension. As shown in this figure, the strain obtained from the uncorrected displacement shows an unusual distribution under uniaxial tension. On the other hand, the strain distribution in Fig. 13(b), obtained from the corrected displacement, is almost uniform. That is, the effectiveness of the proposed distortion correction method is also validated by the strain distribution. The average value of strain in Fig. 13(b) is estimated as \(1009 \times 10^{-6}\).
Fig. 12 Contour maps of displacements (a) $u_x$ and (b) $u_y$ with distortion correction (uniaxial tension test)

Fig. 13 Contour maps of strain $\varepsilon_{yy}$ obtained from displacement (a) without distortion correction and (b) with distortion correction

and the standard deviation as $500 \times 10^{-6}$. These values show fairly good agreement with the values of the strain of $708 \times 10^{-6} - 894 \times 10^{-6}$ obtained using strain gauges. It is emphasized that the proposed distortion correction for digital image correlation is also validated by the uniaxial tension test.

4. Conclusions

Displacement measurement using digital image correlation with lens distortion correction was described. Lens distortion was detected by phase analysis of the cross-grating pattern using Fourier transform. Then, the coefficients of lens distortion were determined using the least-squares method. Finally, the displacement distributions without lens distortion were obtained. The effectiveness of the method was demonstrated by applying the proposed method to the rigid body translation test and the uniaxial tension test. The results showed that the proposed distortion correction method removes the effect of lens distortion from the measured displacements. With the proposed method, accurate measurements can be achieved even if images are deformed by lens distortion. An optical system with distortion, such as a low-cost camera system and a zoom lens, can be utilized in displacement measurements with digital image correlation.

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