Stress Intensity Factors of Surface Crack with Undulated Front*

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The stress intensity factor (SIF) of a surface crack having an undulated crack front, which was defined using a cosine function based on a semi-circular crack, was evaluated by finite element analysis. The average SIF along the crack front is almost the same if the area of crack is the same. Furthermore, crack growth simulations were conducted. The undulated front crack tended to become semi-elliptical in shape during growth. Although the undulated front crack takes a relatively small SIF compared with the semi-circular crack, the growth rate was faster. This incoherence was brought about by the difference in the crack front length.

Key Words: Stress Intensity Factor, Finite Element Analysis, Crack Propagation, Undulated Crack Front, Mesh Generator

1. Introduction

The stress intensity factor (SIF) is used as the crack driving force in growth predictions of fatigue and stress corrosion cracking (SCC)^{(1)–(3)}. In particular, the SIF of semi-elliptical cracks is often used for the prediction because the cracks are modeled as semi-elliptical surface ones even if they are not semi-elliptical in shape^{(4),(5)}. Since the cracks tend to become semi-elliptical under simple loading conditions^{(6),(7)}, prediction based on the semi-elliptical shape gives reasonable solution. However, the crack shape may deviate from the semi-ellipse due to various factors. For example, if two semi-elliptical surface cracks coalesce, the resultant crack forms a dented portion at the contact point and becomes semi-elliptical in shape as it grows^{(8)–(10)}. The shape of microstructurally small cracks also becomes complex due to inhomogeneity of material properties caused by microstructures such as grains or grain boundaries⁽¹¹⁾. In the case of intergranular stress corrosion cracking, the crack changes its direction of progress at grain boundary triple points and the crack driving force changes depending on the crack kink angle^{(12)–(15)}. The change in the SIF brings about the change in the crack shape.

Even if the crack shape deviates from the semi-ellipse, the SIF of the dented portion becomes relatively large and the crack returns to the smooth shape. Thus, the

growth behavior of surface cracks with undulated front is intricate and difficult to evaluate, although there are some studies in which the growth of a dented surface crack was simulated in order to clarify the transitional behavior in the coalescence of two semi-elliptical surface cracks^{(16)–(20)}. To assess the growth of an undulated front crack and to determine how slow (or fast) the crack grows compared with a semi-elliptical crack, the SIF must be calculated. The maximum SIF of an arbitrary-shaped surface crack was analyzed by Murakami et al. using the body force method, and an empirical equation was proposed based on the crack area⁽¹⁶⁾. With this maximum SIF, it is possible to obtain a conservative prediction of the crack growth. However, in order to understand the influence of the crack shape and to evaluate the exact crack growth rate, the SIF along the crack front is required.

In this study, the SIF of a surface crack with an undulated crack front was evaluated by finite element analysis (FEA). The undulated front shape was determined using a cosine function based on a semi-circular crack. By changing the magnitude and frequency of the cosine function, the SIF for various crack shapes was evaluated. The finite element meshes were generated using a newly developed mesh generator, capable of producing meshes of arbitrary-shaped cracks. Then, crack growth simulations were performed in order to evaluate the influence of the crack shape on growth behavior.

2. Procedure of Analysis

The shape of the undulated crack front was defined based on a semi-circular crack, the depth of which is c_0 , as

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Fig. 1 Shape of undulated crack front

Fig. 2 Definition of variation dc for different V value

shown in Fig. 1. The variation of crack tips from the semi-circular line, dc , is determined using the cosine function as follows:

$$dc = -r_s \cos(4pV) \quad (1)$$

where V is frequency, r_s is the amplitude of cosine curve and p is the angle defined in Fig. 1. The V corresponds to half the number of bulges of the crack front. The front crosses perpendicularly to the x -axis and y -axis. Figure 2 shows parameter dc for different value of V . The SIF is evaluated in the case of $V = 1 \sim 4$ and $r_s/c_0 = 0.1 \sim 0.4$ in addition to the semi-elliptical case of $a/c = 0.5, 0.75$ and 1 , where a and c are crack depth and half surface length normalized by the following equation:

$$\sqrt{ca} = c_0. \quad (2)$$

This normalization makes the crack area the same as a semi-circular crack having depth c_0 .

The SIF of the undulated front cracks and semi-elliptical cracks were evaluated using FEA. The cracks were located at the center of a plate subjected to uniform remote tensile stress of σ_0 . The model geometry is shown in Fig. 3. The thickness, t , half-length, B , and half-width of the plate, W , were $t/c_0 = 10$, $B/c_0 = 10$ and $W/c_0 = 5$.

Fig. 3 Geometry of cracked plate and boundary conditions

Fig. 4 Finite element mesh for semi-circular crack and undulated front crack

These sizes are large enough so that the crack is considered to exist in an infinite plate⁽²¹⁾. Due to the symmetries of the problem, only one quarter of the plate was modeled by finite elements. The meshes for the semi-circular crack and undulated surface cracks of $(V, r_s/c_0) = (2, 0.1)$ and $(3, 0.1)$ are shown in Fig. 4. The model consists of 42 064 8-noded solid elements for the semi-circular crack. A newly developed mesh generating system was used in order to generate the finite element meshes. By a programmed procedure using a command language equipped in commercial software PATRAN Ver. 2003r3, this system can automatically generate the mesh for an arbitrary-

shaped crack based on crack tip coordinates.

The commercial code, ABAQUS Ver. 6.3, is applied for FEA. In this code, the SIF is calculated along the crack front by the energy release rate, which is obtained by the virtual crack extension integral method. The SIF is evaluated at each node from the J-integral value of the second, third and fourth path around the crack tip nodes. The finite element models have 73 nodes along the crack front and distances between each node are the same. Poisson's ratio of the material is taken to be 0.3.

3. Results and Discussion

3.1 Accuracy of the analysis

The evaluated SIF of mode I, K_I , along the crack front of a semi-circular surface crack is shown in Fig. 5, where SIF is normalized as

$$F_1 = \frac{K_I}{\sigma_o \sqrt{\pi c_o}}. \quad (3)$$

The SIF obtained in other studies^{(22)–(24)} is also shown in Fig. 5. The present results agree well with those obtained in the references, confirming the validity of the finite element mesh generated by the mesh generating system and evaluation procedure.

3.2 SIF along the crack front

Figure 6 shows the SIF along the crack front in the case of $r_s/c_o = 0.1$ under various V conditions together with the values for $r_s/c_o = 0$, namely a semi-circular crack. The SIF takes various values along the crack front corresponding to the bulges and dents. The SIF becomes large at the dent portion and small at the peak of the bulges. The center of the fluctuation of the SIF is almost equivalent to the SIF of the semi-circular crack, which takes almost the same value from the surface ($p = 0^\circ$) to the deepest point ($p = 90^\circ$). The magnitude of the fluctuation becomes large as the number of bulges, V , increases.

Fig. 5 Stress intensity factor of semi-circular crack

3.3 Average and maximum SIF along the crack front

Figure 7 shows the average F_1 along the crack front, F_{ave} , which is calculated according to the following equation:

$$F_{ave} = \frac{1}{T} \int F_1 d\Gamma. \quad (4)$$

Here, Γ denotes the path along the crack front and T denotes the length of the crack front, which is calculated by the following equation:

$$T = \int_0^\pi (c_o + dr) dp \quad (5)$$

where $(c_o + dr)$ corresponds to distance from the center (point of $x = 0, y = 0$ in Fig. 1) at each point. The averaged F_1 is almost constant in every calculation condition although it becomes small as the amplitude of bulge, r_s , and number of bulges, V , increases. Since the area of the crack is the same for all conditions, as pointed out

Fig. 6 Normalized stress intensity factor ($r_s/c_o = 0.1$)

Fig. 7 Average stress intensity factor along the crack front for various shapes of crack front

Fig. 8 Change in the average stress intensity factor, F_{ave} , with parameter $T/(\pi c_0)$

Fig. 9 Relationship between maximum stress intensity factor, F_{max} , and degree of bulge of crack

by Murakami and Nemat-Nasser⁽¹⁶⁾, the SIF is related to the area of crack. The length T represents the degree of complexity of the crack front shape. The normalized T , $T/(\pi c_0)$, becomes 1 for the semi-circular cracks. Figure 8 shows the relationship between F_{ave} and parameter $T/(\pi c_0)$. F_{ave} shows good correlation with parameter T and is approximated by the following equation:

$$F_{ave} = -0.14 \frac{T}{\pi c_0} + 0.82. \quad (6)$$

This equation means that the average SIF decreases as the shape of the crack front becomes more complex. If the crack growth rate is determined by the SIF, the crack has a different growth rate along the front corresponding to the SIF at each position. For the parameter representing the driving force of the crack, the average SIF rather than the maximum one is more relevant.

The degree of variation of SIF along the undulated crack front is evaluated using the ratio of the maximum F_1 in the crack tips, F_{max} , and F_{ave} . Figure 9 shows the relationship between the parameter F_{max}/F_{ave} and magnitude of the bulge, r_s/c_0 . As shown in Fig. 6, the maximum

SIF increases as the number of bulges, V , increases. If the crack propagates along the grain boundaries and is kinked by 60° , which is a typical case in SCC of alloy 600 in high temperature water⁽²⁾, the SIF of the kinked crack tip is reduced to the 65% of the no-kink case⁽²⁵⁾. In this case, the growth rate at the kinked crack tip slows down and the front would become a dented shape. However, the SIF of the dented portion becomes large and, according to Fig. 9, the reduction in SIF due to the crack kink of 60° can be balanced when the dent of crack front r_s/c_0 is 0.24, because the SIF of dent portion is about $F_{max}/F_{ave} = 1/0.65$, in $V = 1$ cases. The number of dents required for obtaining a balance with the reduced SIF decreases with the number of bulges, which can be regarded as the number of crystal grains along the crack front, and the crack front shape becomes smooth from the macroscopic point of view.

3.4 Crack growth simulations

In the simulation, the crack growth rate (da/dt) was determined according to the power law of the SIF expressed by the following equation:

$$\frac{da}{dt} = DK_I^m, \quad (7)$$

where D and m are material constants and $D = 5.24 \times 10^{-14}$ and $m = 3.46$, which are obtained by experiments of stress corrosion cracking using alloy 600 under a primary water environment at 633 K⁽²⁾, were adopted. The growth rate is given in m/sec and K in MPa $\sqrt{m}$. The crack extension length, Δa , is calculated by:

$$\Delta a = D(F_1 \sigma_0 \sqrt{\pi c_0}) \Delta t, \quad (8)$$

where Δt is the interval of the step, which is controlled so that the maximum is 10 h and the change of crack shape per step does not become extremely large. The crack front points, at which the SIF was calculated, were moved perpendicularly to the crack front by length of Δa . The simulations were conducted for three cases: the initial crack shapes were semi-circular, semi-elliptical of $a/c = 0.5$ and undulated front crack of $V = 2$, $r_s/c_0 = 0.2$. During the simulation, the boundary sizes, W , B and t were kept constant against the crack size c_0 . The simulations were carried out under uniform tensile stress conditions of $\sigma_0 = 200$ MPa.

Figure 10 shows the loci of cracks obtained by the simulations. The semi-circular cracks grow similarly, although the semi-elliptical and undulated front cracks change to a semi-circular shape. In Case 3 (undulated front crack of $V = 2$, $r_s/c_0 = 0.2$), at an early stage of the simulation, the dented portions grow at a relatively fast rate despite there being little growth at the bulge portions. Figure 11 shows the relationship between time and equivalent crack size c_e normalized by initial crack size $c_{0(initial)}$. The c_e corresponds to the depth of the semi-circular crack having the same area, and $c_e = 1$ at the initial conditions of all cases. The growth rate of Case 3 is the fastest of the three cases, although the difference between the three

Fig. 10 Loci of crack front obtained by simulations (loci are drawn every three step)

Fig. 11 Crack growth behaviors obtained by simulations

Fig. 12 Change in the stress intensity factor during simulations

cases is very small.

The change in the SIF during the simulations is shown in Fig. 12. The average SIF, F_{ave} , is relatively small at the initial state in Case 3 as discussed for Fig. 8. However, it increases and approaches the value of Cases 1 and 2. At the same time, as shown in Fig. 12(b), the fluctuation in the SIF along the crack front decreases and approaches zero. When the simulation was continued until the value of $(F_{\text{max}} - F_{\text{min}})/F_{\text{ave}}$ decreased to 0.028, the crack shape converged to the ratio, a/c , of 0.88. Despite the average SIF being relatively small, in Case 3 the crack growth is faster than in the other cases. This incoherence was brought about by the difference in the crack front length, T . The increase in crack area size depends on the crack front length and consequently the growth rate of Case 3 became the fastest although the average SIF was relatively small. For the precise growth prediction, the parameter F_{ave} is more relevant than F_{max} , because the F_{max} represents only a part of the crack driving force.

The cracks may change in shape due to various factors, although they tend to become semi-elliptical surface cracks. However, the change in crack shape brings a posi-

tive effect on the crack growth rather than a negative effect. Even if the crack has undulated front, prediction based on the semi-circular crack, which has the same area, provides a good approximation. In the case of coalescence of semi-elliptical surface cracks, the change rate of the area of a coalesced crack during growth is supposed to be the same as that of the semi-circular crack of the same size, although the crack shape varies from dented arc to semi-ellipse.

4. Summary and Conclusions

The SIF of surface cracks having an undulated front, which were defined by a cosine function, were evaluated by FEA. The calculations were performed for various crack shape conditions by changing the magnitude and frequency of the undulation. Crack growth simulations were conducted in order to evaluate the influence of the crack shape on the growth behavior. The conclusions obtained are as follows:

(1) The average of the SIF along the crack front takes almost the same value under the same crack area conditions, although it tends to decrease as the complexity of the front increases.

(2) The fluctuation of the SIF along the crack front becomes larger as the magnitude of the undulation of the front increases and the frequency of the undulation decreases.

(3) The undulated front crack tends to become semi-elliptical in shape during growth.

(4) The decrease in the average SIF along the crack front due to complex shape does not slow down the crack growth, which is measured by the change in area size.

(5) The change in area size during growth of undulated front cracks can be estimated by replacing with a simple shape crack of the same size.

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