Fretting Fatigue Analysis Using Fracture Mechanics

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In this paper, an estimation method for fretting fatigue strength is proposed. Fretting fatigue cracks initiate very early in the fatigue life of metals, which is dominated by the propagation process of small cracks. Characteristics of fretting fatigue cracks are analysed using stress intensity factors at the tip of cracks growing from the contact edges. The stress intensity factors are calculated from the contact pressure and tangential stress distributions on the contact surfaces. Fretting fatigue limits were estimated by comparing the stress intensity factor ranges with the threshold stress intensity factor range of the material. These calculated results were then compared with experimental results of fretting fatigue tests.

Key Words : Fretting Fatigue, Fatigue Limit, Fracture Mechanics, Stress Intensity Factor, Non-Propagating Crack

1. Introduction

Fretting can occur when a pair of structural elements are in contact under a normal load with cyclic stress applied to one element resulting in relative displacement of the contact surface. This condition occurs in bolted or riveted joints⁽¹⁾⁽²⁾, in shrink fitted shafts⁽³⁾⁻⁽⁷⁾, in the blade dovetail region of turbo machinery⁽⁸⁾⁻⁽¹⁰⁾, and elsewhere. Under the above mentioned fretting conditions, fatigue strength decreases to less than one-third of that under non-fretting conditions. The reason for this strength reduction is thought to be the concentrations of contact pressure and tangential stress which occur at the contact edge where the fretting fatigue crack initiates and propagates. Stress analysis and fracture mechanics analysis of the contacting structures has not been done yet because of the difficulty of accurately

estimating contact conditions such as frictional coefficients, and of stress intensity factor analysis of edge cracks subjected to contact forces near the crack.

In this paper, the contact pressure distributions and the tangential stress distributions are analysed by a Finite Element Method program, primarily developed for the analysis of contact problems⁽¹¹⁾, and the stress intensity factors of cracks at the contact edges are calculated using the Rooke and Jones' equations⁽¹²⁾⁽¹³⁾. Using the calculated results of the stress intensity factors, the crack propagation characteristics and thresholds of fretting fatigue are discussed and are compared with the experimental results.

a : Crack length

R : Stress intensity factor ratio, K_{min}/K_{max}

K_{IP} : Stress intensity factor for concentrated normal force P

K_{IQ} : Stress intensity factor for concentrated tangential force Q

$K_{I(p)}$: Stress intensity factor for distributed contact pressure p

$K_{I(q)}$: Stress intensity factor for distributed tangential stress q

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$K_{I(\sigma_0)}$: Stress intensity factor for axial nominal stress σ_0

ΔK_{th} : Threshold stress intensity factor range

$\Delta K_{th}(R=0)$: Threshold stress intensity factor range for $R=0$

$\Delta K_{th}(a=\infty)$: Threshold stress intensity factor range for long crack

2. Fracture Mechanics Analysis

2.1 Fundamental equations of stress intensity factors

Rooke and Jones⁽¹²⁾ derived the stress intensity factor for a crack in the edge of a half-plane subjected to point forces, both normal force, P , and tangential force, Q , acting at a point on the edge of the half-plane (See Fig. 1). In this derivation, they used the stress intensity factor equation for a crack subjected to distributed forces acting on the crack surface and the stress distribution at the imaginary crack site due to the point forces acting at the same point on the edge of the non-cracked half-plane. The final results of stress intensity factors for a crack in the edge of a half-plane subjected to point forces are as follows:

$$K_{I,P} = \frac{P}{\sqrt{\pi a}} (1 - \xi^2) \left[0.824 + 0.00637\xi - 0.843\xi^2 + 15.41\xi^3 - 53.38\xi^4 + 59.74\xi^5 - 21.83\xi^6 \right] \quad (1)$$

$$K_{I,Q} = \frac{Q}{\sqrt{\pi a}} (1 - \xi^2) \left[1.2943 + 0.0044\xi + 0.1289\xi^2 + 10.89\xi^3 - 22.14\xi^4 + 10.96\xi^5 \right] \quad (2)$$

$$\xi = x/(x+a) \quad (3)$$

In these equations, K_{IP} is the mode-I stress intensity factor due to normal force P , K_{IQ} is that due to

tangential force Q , and a is crack length. When the edge crack is subjected to distributed contact pressure, p , and distributed tangential stress, q , instead of point forces, P and Q , the stress intensity factors due to these distributed stresses can be obtained by substituting pdx and qdx for P and Q , respectively, and integrating Eqs. (1) and (2) along the x direction as follows (See Fig. 2):

$$K_{I(P)} = \int \frac{p}{\sqrt{\pi a}} (1 - \xi^2) \left[0.824 + 0.0637\xi - 0.843\xi^2 + 15.41\xi^3 - 53.38\xi^4 + 59.74\xi^5 - 21.83\xi^6 \right] dx \quad (4)$$

$$K_{I(Q)} = \int \frac{q}{\sqrt{\pi a}} (1 - \xi^2) \left[1.2943 + 0.0044\xi + 0.1289\xi^2 + 10.89\xi^3 - 22.14\xi^4 + 10.96\xi^5 \right] dx \quad (5)$$

The stress intensity factor for a crack in the edge of a half-plane subject to nominal stress, σ_0 , can be calculated by the following equation:

$$K_{I(\sigma_0)} = 1.122\sigma_0\sqrt{\pi a} \quad (6)$$

The total stress intensity factors for fretting conditions can be calculated by adding Eqs. (4), (5) and (6).

In this study, the calculation of the mode-II stress intensity factor is omitted as it is considered that the crack propagation characteristics will be dominated mainly by mode-I.

2.2 Stress analysis of a contact region with a crack and calculation of stress intensity factors

The transmitting characteristics of the frictional force between a specimen and a pad is the most important factor in stress analysis; thus, the finite element program which was developed for the contact problem⁽¹¹⁾ is used in this study. In this finite element analysis, the nodes in the contact region are made on both the specimen and the pad, and relative displacement and contact pressure between these node pairs can be calculated using this program. The dimensions of the analytical model, shown in Fig. 3, are the same as the experimental model. In this model, the crack is assumed to initiate at the contact edge and grow normally to the contact surface. The finite element mesh is shown in Fig. 4. This is a quarter section of

Fig. 1 Cracked plate subjected to contact forces

Fig. 2 Cracked plate subjected to distributed contact stresses

Fig. 3 Model of contacted specimen

the analytical model. Uniform pressure, P_o , on the top of a pad and uniform axial load, σ_o , on the edge of the specimen are applied in this model.

Fig. 4 Finite element model of contacted specimen

All nodes on the horizontal axis are constrained in the vertical direction and all nodes on the vertical axis are constrained in the horizontal direction. The frictional coefficient on the contact surface was found to be 0.7 on the basis of observed results of contact surface damage and relative displacement at the contact edge in a fretting test⁽⁷⁾⁽⁸⁾.

Calculated results of contact pressure and tangential stress distributions near the contact edges are shown in Fig. 5 and Fig. 6, respectively. In each figure, the nominal contact pressure, P_o , is 196 MPa and the crack lengths are 0.1 mm and 0.5 mm respectively. The concentration of contact pressure and tangential stress at the contact edge decreases with the growth of the crack. The reason for this is the reduction in

Fig. 5 Contact pressure and tangential stress distributions ($P_o=196$ MPa, $a=0.1$ mm)

Fig. 6 Contact pressure and tangential stress distributions ($P_o=196$ MPa, $a=0.5$ mm)

stiffness near the contact edge caused by the crack. Also, the concentration of the contact pressure and tangential stress at the contact edge are decreased by adding the axial tensile stress, σ_o . The reason for this is the shrinkage of the specimen in the vertical direction, especially near the contact edge, due to the axial tensile stress, σ_o .

The stress intensity factors for a crack in the edge of a half-plane subjected to the above contact conditions are calculated by using Eqs. (4), (5) and (6) as follows:

$$K_I' = K_{I(p)} + K_{I(q)} + K_{I(\sigma_o)} \quad (7)$$

For the finite width specimen model of this analysis, the stress intensity factors can be calculated by using modification coefficient, $f(a/w)$, as follows:

$$f(a/w) = \frac{(1 + 0.122 \cos^4 \frac{\pi a}{w})}{1.122} \sqrt{\frac{w}{\pi a} \tan \frac{\pi a}{w}} \quad (8)$$

$$K_I = f(a/w) K_I' \quad (9)$$

where W is the width of the specimen.

The calculated results of stress intensity factors using the above equations are shown in Fig. 7. In this figure, it is shown that the stress intensity factors at zero-axial loading, $\sigma_o = 0$, are negative. The reason for this is the closure of the crack due to the contact pressure acting near the crack. It is shown that the stress intensity factor in a short crack region is considerably higher than the stress intensity factor when there is no contact pressure, which is calculated by Eq. (6). The reason for this is that the tangential stress near the crack acts effectively to propagate the crack, especially when the crack is short.

2.3 Fretting fatigue threshold analysis

Generally, the threshold stress intensity range, ΔK_{th} , is influenced by the stress intensity factor ratio, R , where

$$R = K_{min}/K_{max} \quad (10)$$

and by crack length, a , especially in the short crack region. To estimate the fretting fatigue limit by using fracture mechanics, the precise determination of ΔK_{th} is indispensable. In this paper, the influence of stress intensity factor ratio R , and crack length, a , on ΔK_{th} are considered.

The relation between ΔK_{th} and stress intensity factor ratio, R , is determined using various experimental results⁽¹⁶⁾ as follows:

$$\Delta K_{th(R)} = \Delta K_{th(R=0)} (1-R)^{0.7} \quad (R < 0) \quad (11)$$

$$\Delta K_{th(R)} = \Delta K_{th(R=0)} (1-R)^{(0.5 + \log(1-R)/10)} \quad (R > 0) \quad (11)'$$

To predict ΔK_{th} in the short crack region, Ohuchi⁽¹⁷⁾, Tanaka⁽¹⁸⁾ and El Haddad⁽¹⁹⁾ derived the following equation:

$$\Delta K_{th(a)} = \Delta K_{th(a=\infty)} \sqrt{\frac{a}{a+a_0}} \quad (12)$$

In this equation, the critical crack length, a , is determined by using the fatigue limit of a plain specimen, σ_{wo} , and the threshold stress intensity factor range for a long crack $\Delta K_{th}(a=\infty)$ as follows:

$$a_0 = \left(\frac{\Delta K_{th}(a=\infty)}{\Delta \sigma_{wo}} \right)^2 \cdot \frac{1}{\pi} \quad (13)$$

Using Eqs. (11) and (12), the threshold stress intensity factor range, ΔK_{th} , considering both R and a is derived as:

$$\Delta K_{th(R,a)} = \Delta K_{th(R=0,a=\infty)} (1-R)^{0.7} \sqrt{\frac{a}{a+a_0}} \quad (R < 0) \quad (14)$$

$$\Delta K_{th(R,a)} = \Delta K_{th(R=0,a=\infty)} (1-R)^{(0.5 + \log(1-R)/10)} \times \sqrt{\frac{a}{a+a_0}} \quad (R > 0) \quad (14)'$$

Using the calculated results of stress intensity factor, K , for the fretting conditions (See Fig. 7), the relation between K and nominal stress, σ_o , can be obtained, as shown in Fig. 8, and can be approximately expressed as:

$$K = -A + (\sigma_o - B) \cdot C \quad (15)$$

In this equation, A and B are constants for a given contact pressure, P_o . The relation between C and crack length, a , is shown in Fig. 9. By using this equation, the stress intensity factor range, ΔK , and stress intensity factor ratio can be calculated as follows:

$$\Delta K = \Delta \sigma_o C \quad (16)$$

$$R = \frac{-A + (\sigma_{min} - B)C}{-A + (\sigma_{max} - B)C} \quad (17)$$

The threshold stress intensity factor range, ΔK_{th} , for the fretting condition can be calculated by substituting Eq. (17) into Eq. (14) or Eq. (14)'. The comparisons of ΔK with ΔK_{th} are shown in Figs. 10, 11 and 12.

Fig. 7 Stress intensity factor vs crack length for various nominal stresses σ_o ($P_o = 196$ MPa)

In these figures, the mean of the axial nominal stress, σ_m , is 0 and the amplitudes of the axial nominal stresses, σ_a , are 140 MPa, 142 MPa, and 143 MPa, respectively. As shown in Fig. 10 ($\sigma_a=140$ MPa), the small crack initiated by the fretting damage arrested

Fig. 8 Stress intensity factor K vs nominal stress σ_o ($P_o=196$ MPa)

Fig. 9 Relation between C and crack length a

Fig. 10 Comparison of K with K_{th} ($P_o=196$ MPa)

at a length of $45\mu\text{m}$, and as shown in Fig. 12 ($\sigma_a=143$ MPa), the crack initiated by the fretting damage grew slowly, and finally the specimen broke. Figure 11 shows the critical condition where the crack arrested at the length of $85\mu\text{m}$; however, above this stress amplitude ($\sigma_a=142$ MPa), the crack grew continuously. This critical condition is the fretting fatigue limit. The calculated result of the fretting fatigue limit for various mean stresses, σ_m , is shown in Fig. 13 by a solid line. This estimated propagating or arresting behavior of fretting fatigue cracks agreed well with the results obtained by Satoh⁽²⁰⁾. From these calculated results, it is found that the fretting fatigue

Fig. 11 Comparison of ΔK with ΔK_{th} ($P_o=196$ MPa)

Fig. 12 Comparison of K with K_{th} ($P_o=196$ MPa)

Fig. 13 Fretting fatigue limit vs mean stress

strength is very sensitive to the mean stress⁽²¹⁾.

3. Fretting Fatigue Test

3.1 Experimental procedure

The fretting fatigue test apparatus is shown in Fig. 14. The contact pressure, $P_o=196$ MPa, is increased by tightening the four screws shown in the figure and is measured with strain gauges mounted at the center of the specimen (Strain gauge A in Fig. 14.) ; and fluctuating axial stress, σ_a , is applied using a closed loop servo controlled electro-hydraulic test machine with a load capacity of 100 kN. The test specimen and fretting pads are made respectively from Ni-Mo-V steel and low alloy steel. The frictional coefficient of the contact surface is estimated by measuring the results of strain hysteresis near the contact edge with strain gauge B as shown in Fig. 14. Using this method, the frictional coefficient at the location of fretting damage was found to be 0.7.

3.2 Experimental results

Experimental results of fatigue strength of plain specimens (no fretting) and fretting specimens are shown in Fig. 15. It is found that, at zero mean stress, the fretting fatigue strength is reduced to about one third of that of the no-fretting condition. For a mean

stress of 95 MPa, the fretting fatigue limit is reduced from 120 MPa to about 78 MPa. The fretting fatigue limits obtained by these experimental results are shown in Fig. 13 by solid dots. These experimental results agree well with the analytically estimated results. The observed results of a fretting-damaged surface by scanning electron microscope are shown in Fig. 16. The loading condition of this surface was as follows:

Contact pressure	$P_o=196$ MPa
Mean stress	$\sigma_m=0$
Stress amplitude	$\sigma_a=98$ MPa
Number of cycles	$N=2.4 \times 10^7$ (non-break)

The heavily damaged region near the contact edge corresponds to the contact pressure and tangential stress concentrating regions which are shown in Fig. 5. The non-propagating crack at the contact edge is shown in Fig. 16 (b). A sectional view of these cracks observed by a scanning electron microscope is shown in Fig. 17. This crack length corresponds well with the estimated non-propagating crack length shown in Figs. 10 and 11. Accordingly, the suitability of the estimation method for fretting fatigue limit by using the ΔK_{th} of a small crack is confirmed.

Fig. 14 Fretting fatigue test apparatus

Fig. 15 Effect of mean stress on fretting fatigue strength

Fig. 16 Fretting-damaged surface and non-propagating crack

Fig. 17 Non-propagating crack length

4. Conclusions

Fretting fatigue strength is analysed by FEM stress analysis, fracture mechanics analysis and fretting fatigue tests. The results are summarized below:

(1) Contact pressure and tangential stress distributions can be obtained by FEM stress analysis for given contact conditions. Using these calculated contact pressure and tangential stress distribution results, the stress intensity factors for cracks at the contact edges can be obtained.

(2) The contact pressure loading near the crack causes the crack to close and therefore makes the stress intensity factor negative. The tangential stress working near the crack makes the stress intensity factor high especially for small cracks.

(3) The fretting fatigue limit is estimated by comparing the calculated results of the stress intensity factor with the threshold stress intensity factor in terms of both crack length, a , and stress intensity factor ratio, R . In this way, the influence of mean stress, σ_m , on the fretting fatigue limit can be estimated.

(4) These analytical fretting fatigue limit results were confirmed by fretting fatigue tests.

(5) The existence of a non-propagating crack can be observed in the specimen which was subjected to a load lower than the fatigue limit. By this result, the suitability of the estimated mechanism of crack initiation and crack propagation was obtained.

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