Determination of the Complex Shear Modulus of an Adhesive Layer by the Composite Beam Test Method*

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The complex shear modulus of a thin adhesive layer may be determined from the torsional resonance test on a composite beam with metal facings and an adhesive core. In order to determine this shear modulus from such method (Composite Beam Test Method), it is necessary to know the torsional rigidity of a three-layer composite beam. This was analyzed exactly, and a method to determine the complex shear modulus was presented herein using the result of analysis. An error analysis is essential in this method for reliable determination of characteristics of the extremely thin adhesive layer, and it was performed in the present paper. The test beam configuration which minimized the resultant error of the shear modulus were discussed theoretically based on the results of error analysis. Validity of the present method was verified by some torsional tests which were carried out on the symmetrical elastic beams bonded by two types of structural adhesives at several temperatures.

Key Words: Elasticity, Adhesive Layer, Complex Shear Modulus, Composite Beam Test Method, Torsional Rigidity, Error Analysis

1. Introduction

Static and dynamic moduli of a structural adhesive are important factors evaluating the mechanical characteristics of primary adhesive bonding structures in various environments. The method of determination for these moduli, however, has not been established yet. The main reasons for the fact result from that a precise detection of the displacement for extremely thin adhesive layers is a difficult task, and that the singularity of deformations⁽¹⁾ in the adhesive layer arises from the constraint by rigid adherends. On the other hand, the measured values by large scale specimens made of adhesive only have not been proved to correspond to those of the thin layers⁽²⁾ according to the scale size effect⁽³⁾.

Over the previous three decades, the inverse methods to determine the complex moduli of viscoelastic materials by utilizing the theoretical relationships which were developed to evaluate the damp-

ing characteristics of the elastic/viscoelastic composite layer beams, the so-called Composite Beam Test Method or Coated Beam Test Method, have been presented by a few authors. For example, Oberst et al.⁽⁴⁾ and Nashif^{(5),(6)} studied bending resonance tests for two- and three-layer beams, respectively. Johnson and Woolf⁽⁷⁾ studied torsional resonance tests for two-layer beams, and Pritz⁽⁸⁾ investigated the resultant error for these methods by applying the theoretical results of Oberst et al. and Nashif.

However, the composite beam test method has not been applied to determine the complex moduli of a thin adhesive layer, except for the investigation by Jones and Parin⁽⁹⁾ concerning soft adhesive layers.

In the present paper, therefore, this method is applied to symmetrical three-layer beams with a thin structural adhesive core by means of the torsional resonance test. The stress components and torsional rigidity of the beam are analyzed exactly herein, by extending Johnson-Woolf's analytical procedure⁽⁷⁾ to this problem. A method for determining the complex shear modulus is presented, and the complex shear modulus of two types of structural epoxy resin adhesives is also determined by the present method.

* Received 23rd June, 1987. Paper No. 86-0841 B

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Moreover, the controlling factors of the resultant error of the determined values and the optimum configuration of the test beams in reducing the resultant error to a minimum are discussed from the results of error analysis performed.

2. Stress analysis and torsional rigidity of a three-layer beam

In bending deformation of multi-layer composite beams, both axial and shear stresses are produced in the adhesive layer. Therefore, the exact theoretical relationship, which determines either Young's modulus or shear modulus of an adhesive layer from bending resonance test only, can not be obtained. Jones and Parin⁽⁹⁾ investigated the complex Young's modulus for a soft adhesive from a bending resonance test, and discussed the limitations of this method by using the relationships developed by Mead and Markus⁽¹⁰⁾. However, such bending method, as Jones and Parin had indicated, could not be applied to structural adhesives with high rigidity due to the assumption taken in the analysis.

In this paper, consequently, a torsional test was employed because the theoretical analysis for the composite beam did not require any analytical assumptions which would restrict practical measurements.

A stress analysis and calculation of torsional rigidity of a three-layer beam with rectangular cross section, as shown in Fig. 1, are presented in the following. The beam consists of three layers having length l , width b , and thickness a_i with shear modulus G_i (subscript i refers to each layer). Both sides of the beam are made of elastic layers and the core is an adhesive one. The origin of coordinates is placed at the center of the core. Let τ_{ixz} , τ_{iyz} and w_i denote the shear stresses and the displacement in the z -direction in part i , and α is the twisted angle per unit length resulting from a constant twisting moment T . The Saint Venant semi-inverse method in torsion problems is readily extended to the present beam. Introducing the well-known torsion function $\phi_i(x, y)$, the fundamental equations of elasticity and the boundary conditions then provide Eqs. (1), (2) and (3).

$$\frac{\partial^2 \phi_i}{\partial x^2} + \frac{\partial^2 \phi_i}{\partial y^2} = 0 \quad (1)$$

$$\left. \begin{aligned} \sigma_{ix} = \sigma_{iy} = \sigma_{iz} = \tau_{ixy} = 0 \\ \tau_{ixz} = G_i \alpha \left(\frac{\partial \phi_i}{\partial x} - y \right), \tau_{iyz} = G_i \alpha \left(\frac{\partial \phi_i}{\partial y} + x \right) \end{aligned} \right\} \quad (2)$$

$$\left. \begin{aligned} \tau_{1yz} = \tau_{2yz} = \tau_{3yz} = 0 \quad (y = \pm b/2) \\ \tau_{1xz} = 0 \quad (x = -a_1 - a_2/2) \\ \tau_{3xz} = 0 \quad (x = a_3 + a_2/2) \\ \tau_{1xz} = \tau_{2xz}, w_1 = w_2 \quad (x = -a_2/2) \\ \tau_{3xz} = \tau_{2xz}, w_3 = w_2 \quad (x = a_2/2) \end{aligned} \right\} \quad (3)$$

Here, following Johnson-Woolf's analytical procedure⁽⁷⁾, we take a function $f_i(x, y)$ defined by

$$f_i(x, y) = 1 + \frac{\partial^2 \phi_i}{\partial x \partial y} \quad (4)$$

in order to simplify the boundary condition treatment. We can readily show that Eqs. (1) to (3) yield the following Eqs. (5) to (7).

$$\frac{\partial^2 f_i}{\partial x^2} + \frac{\partial^2 f_i}{\partial y^2} = 0 \quad (5)$$

$$\frac{\partial \tau_{ixz}}{\partial y} = G_i \alpha (f_i - 2), \frac{\partial \tau_{iyz}}{\partial x} = G_i \alpha f_i \quad (6)$$

$$\left. \begin{aligned} f_1 = f_2 = f_3 = 0 \quad (y = \pm b/2) \\ f_1 = 2 \quad (x = -a_1 - a_2/2) \\ f_3 = 2 \quad (x = a_3 + a_2/2) \\ f_1 - 2 = \varepsilon(f_2 - 2), \frac{\partial f_1}{\partial x} = \frac{\partial f_2}{\partial x} \quad (x = -a_2/2) \\ f_3 - 2 = \varepsilon(f_2 - 2), \frac{\partial f_3}{\partial x} = \frac{\partial f_2}{\partial x} \quad (x = a_2/2) \end{aligned} \right\} \quad (7)$$

where $\varepsilon = G_2/G_1$.

In the above transformations, the elastic layers should have the same thicknesses and material properties ($a_1 = a_3$, $G_1 = G_3$) in order to avoid shifting of the center of twist caused by the beam geometries or rigidities of composed materials. Assuming the separation of variables in the solution of Eq. (5), the function $f_i(x, y)$ becomes

$$f_i(x, y) = \sum_{n=0}^{\infty} \left(p_i \cosh \frac{k_n}{b} x + q_i \sinh \frac{k_n}{b} x \right) \cos \frac{k_n}{b} y \quad (8)$$

where $k_n = (2n+1)\pi$. The coefficients p_i and q_i in Eq. (8) are determined by solving simultaneous algebraic equations after substituting Eq. (8) into Eq. (7); these equations are easily derived from consideration of the orthogonality of trigonometric functions.

$$\left. \begin{aligned} p_1 = p_3 = \frac{[1 + (1 - \varepsilon)\{x_2(y_2 X_1 + x_2 Y_1) - y_2^2\}]\theta_n}{\varepsilon y_2 Y_1 + x_2 X_1} \\ q_1 = -q_3 = \frac{x_2(1 - \varepsilon)(x_2 X_1 + y_2 Y_1 - y_2)\theta_n}{\varepsilon y_2 Y_1 + x_2 X_1} \\ p_2 = \frac{[1 - (1 - \varepsilon)Y_1]\theta_n}{\varepsilon y_2 Y_1 + x_2 X_1}, q_2 = 0 \end{aligned} \right\} \quad (9)$$

Fig. 1 Analytical model of three-layer beam

where $x_i = \sinh(k_n R_i/2)$, $y_i = \cosh(k_n R_i/2)$, $X_i = \sinh(k_n R_i)$, $Y_i = \cosh(k_n R_i)$, $\theta_n = 8(-1)^n/k_n$ and $R_i = a_i/b$.

Denoting δ_i as the values of the x -coordinate at the outside surface or the interfaces in the beam cross section, the shear stress components τ_{ixz} and τ_{iyz} are obtained by integrating Eq. (6) using Eqs. (8) and (9) over each layer cross section. Then,

$$\left. \begin{aligned} \tau_{ixz} &= \tau_{ixz}(x, 0) + G_i \alpha \int_0^y \{f_i(x, s) - 2\} ds \\ \tau_{iyz} &= \tau_{iyz}(\delta_i, y) + G_i \alpha \int_{\delta_i}^x f_i(s, f) ds \end{aligned} \right\} \quad (10)$$

Here $\tau_{ixz}(x, 0)$ is determined from the geometrical symmetry of the beam with respect to the x -coordinate axis, and $\tau_{iyz}(\delta_i, y)$ is determined by suitably integrating the equilibrium equation in the z -direction.

$$\left. \begin{aligned} \tau_{ixz}(x, 0) &= 0 \\ \tau_{iyz}(\delta_i, y) &= - \int_{-b/2}^y \left\{ \frac{\partial \tau_{ixz}(\delta_i, s)}{\partial x} \right\} ds \end{aligned} \right\} \quad (11)$$

Finally, the shear stress components in each layer are obtained from Eqs. (10) and (11) as follows:

$$\left. \begin{aligned} \frac{\tau_{ixz}}{G_i \alpha} &= \sum_{n=0}^{\infty} \frac{b}{k_n} \left(p_i \cosh \frac{k_n}{b} x \right. \\ &\quad \left. + q_i \sinh \frac{k_n}{b} x \right) \sin \frac{k_n}{b} y - 2y \\ \frac{\tau_{iyz}}{G_i \alpha} &= \sum_{n=0}^{\infty} \frac{b}{k_n} \left(p_i \sinh \frac{k_n}{b} x \right. \\ &\quad \left. + q_i \cosh \frac{k_n}{b} x \right) \cos \frac{k_n}{b} y \end{aligned} \right\} \quad (12)$$

The resultant twisting moment T may be defined in terms of the shear stresses as the surface integral over each layer cross section (D_i):

$$T = \sum_{i=1}^3 T_i = \sum_{i=1}^3 \int_{D_i} (x \tau_{iyz} - y \tau_{ixz}) dD_i \quad (13)$$

Integrating Eq. (13) after substituting Eqs. (9) and (12) into Eq. (13), the torsional rigidity of each layer C_i ($C_i = T_i/\alpha$) and the resultant torsional rigidity C ($C = \sum_{i=1}^3 C_i$) of the three-layer beam are given by

$$C_1 = C_3 = G_1 b^4 \left\{ \left(\frac{R_1}{3} - \sum_{n=0}^{\infty} \frac{64 t_1}{k_n^5} \right) + \epsilon R_2 \sum_{n=0}^{\infty} \frac{8 T_1 (t_1 + t_2)}{k_n^4 (\epsilon + t_2 T_1)} + \epsilon \sum_{n=0}^{\infty} \frac{32 t_1 T_1 (t_1 + t_2)}{k_n^5 (\epsilon + t_2 T_1)} \right\} \quad (14)$$

$$C_2 = G_1 b^4 \left\{ \epsilon \left(\frac{R_2}{3} - \sum_{n=0}^{\infty} \frac{64 t_2}{k_n^5} \right) - \epsilon R_2 \sum_{n=0}^{\infty} \frac{16 T_1 (t_1 + t_2)}{k_n^4 (\epsilon + t_2 T_1)} + \epsilon \sum_{n=0}^{\infty} \frac{64 t_2 T_1 (t_1 + t_2)}{k_n^5 (\epsilon + t_2 T_1)} \right\} \quad (15)$$

$$C = \sum_{i=1}^3 C_i = G_1 b^4 \left\{ 2 \left(\frac{R_1}{3} - \sum_{n=0}^{\infty} \frac{64 t_1}{k_n^5} \right) + \epsilon \left(\frac{R_2}{3} - \sum_{n=0}^{\infty} \frac{64 t_2}{k_n^5} \right) + \epsilon \sum_{n=0}^{\infty} \frac{64 T_1 (t_1 + t_2)^2}{k_n^5 (\epsilon + t_2 T_1)} \right\} \quad (16)$$

where $t_i = \tanh(k_n R_i/2)$, $T_i = \tanh(k_n R_i)$.

In Eq. (16), if we take a shear modulus ratio ϵ as unity, and replace R_1 and R_2 with $R_1 = R_2 = R/3$, or $R_1 = R/2$, $R_2 = 0$, then Eq. (16) coincides with the well-

known torsional rigidity of a homogeneous rectangular beam. The first terms in Eqs. (14) and (15) represent the torsional rigidity of an elastic layer $E/2$ and an adhesive layer V , respectively, and the sum of the second and third terms represents the increments of torsional rigidities $\Delta E/2$ and ΔV in each layer whose deformation is constrained by bonding. In the same manner, the first term and the second term in Eq. (16) represent the torsional rigidity of elastic layers E and adhesive layer V , respectively, and the third term represents the increment $\Delta E + \Delta V$ of the torsional rigidity of a three-layer beam by the constraint.

The torsional rigidity of a symmetrical three-layer beam with constant elastic layer thicknesses is calculated in Fig. 2. In the case of the rigid adhesive ($\epsilon = 10^{-1}$), the torsional rigidity increases with the core thickness, but in the case of the soft adhesives ($\epsilon = 10^{-2}, 10^{-3}$ and 10^{-4}), the torsional rigidity decreases with increase of the core thickness. These different characteristics of torsional rigidity may be explained in that the constraining force between elastic layers and an adhesive one decreases with increase of soft adhesive core thickness.

The quantitative relationships between the torsional rigidity C and its components E , ΔE and ΔV are calculated in Fig. 3. It has been characterized that the torsional rigidity of the beam is considerably influenced by its increment ΔE due to the constraint. That is, the torsional rigidity with a rigid adhesive core depends on the increments ΔE of the elastic layers, and that with a soft adhesive core depends on the torsional rigidity of the elastic layers E owing to decrease of the constraining force in the elastic layers. Torsional rigidity V of the adhesive layer and its increment ΔV have little effects on the whole tor-

Fig. 2 Torsional rigidity of the symmetrical three-layer beam

sional rigidity, except for the case that either its rigidity or thickness is extremely large (V is not illustrated in Fig. 3 as its value is small enough). In the above calculations in Eqs.(14) to (16), computation was made up to the 51 terms.

3. Analytical approach on dynamic torsion test

In the torsion test, energy dissipation from the clamped end of a test beam is also a problem. However, if we consider the test beam to be sufficient long, thus enabling such energy dissipation to be relatively small enough, it is better to conduct tests on a cantilever beam, as is the normal procedure. The test beam having a inertia bar at its free end, as shown in Fig. 4, is excited to induce a free damped torsional vibration whose frequency and logarithmic decrement are measured.

The equation of motion for a homogeneous uniform cross section bar may also be applied to the three-layer beams within the relevant low frequency range. Then, the following frequency equation is obtained by solving the equation of motion.

$$\beta \tan \beta = J \quad (17)$$

where $\beta = \omega l \sqrt{(J_b/C^*)}$ and $J = IJ_b/J_m$.

In Eq.(17) J_b and J_m are polar moments of inertia of the beam per unit length and inertia bar respectively, and ω is natural angular frequency of this torsional system. C^* is the complex torsional rigidity of the beam, which is obtained directly from the static torsional rigidity in Eq.(16) by replacing the shear modulus of the core G_2 with the complex shear modulus of the adhesive core $G_2^* = G_2(1 + j\eta)$. The real part G_2 is referred to as the dynamic modulus, and the imaginary part in parentheses η as loss factor. This treatment, known as the correspondence principle,

may be justified from linear viscoelasticity theory. The solution of Eq.(17) can be obtained by Taylor series expansion if we make the following approximation: $\tan \beta = \beta + \beta^3/3^{(11)}$. Considering up to the second-order terms in the series, the first- and k th-order approximate solutions are given as follows:

$$\left. \begin{aligned} \beta_1 &= \sqrt{J(1-J/3)}, \\ \beta_k &= (k-1)\pi + J(1-J/3)/(k-1)\pi \end{aligned} \right\} \quad (18)$$

The comparison of the solutions of Eq.(18) with the exact numerical calculations of Eq.(17) yields little difference for the first-order solution, and becomes even less significant in the higher order solutions. Moreover, if a large moment of inertia bar is utilized, the approximation is sufficiently satisfied in the first-order solution (if $J > 0.45$, the difference becomes less than 0.5%). Therefore, the first- and k th-order complex angular frequency ω_1^* , ω_k^* and the resultant loss factor η_c of the three-layer beam are given by Eqs.(19) to (21) as follows:

$$\dot{\omega}_1^{*2} = \frac{C^* J}{J_b l^2} \left(1 - \frac{J}{3}\right) \quad (19)$$

$$\omega_k^{*2} = \frac{C^* J}{J_b l^2} \left\{ (k-1)\pi + \frac{J(1-J/3)}{(k-1)\pi} \right\}^2_{k=2,3,\dots} \quad (20)$$

$$\eta_c = \left\{ \frac{\text{Im}(\omega_k^{*2})}{\text{Re}(\omega_k^{*2})} \right\}_{k=1,2,\dots} = \frac{C_I}{C_R} \quad (21)$$

where C_R and C_I are the real and imaginary parts of C^* .

Considering the fundamental mode torsional vibration test, then, the dynamic modulus and loss factor of an adhesive layer are formulated by the following equations (these equations in higher order are given by Eqs.(20) and (21) in the same manner).

$$G_2 = \left\{ \frac{J_b l^2 \omega_1^2}{b^4 J(1-J/3)} - G_1 S_1 \right\} \frac{1}{S_2} \quad (22)$$

$$\eta = \frac{S_1 + \epsilon S_2}{\epsilon S_3} \eta_c \quad (23)$$

where

Fig. 3 Components of torsional rigidity of the three-layer beam

Fig. 4 Geometry of the test specimen

$$S_1 = 2 \left(\frac{R_1}{3} - \sum_{n=0}^{\infty} \frac{64 t_1}{k_n^3} \right)$$

$$S_2 = \frac{R_2}{3} - \sum_{n=0}^{\infty} \frac{64 t_2}{k_n^3} + \sum_{n=0}^{\infty} \frac{64 T_1 (t_1 + t_2)^2}{k_n^3 (\varepsilon + t_2 T_1)}$$

$$S_3 = \frac{R_2}{3} - \sum_{n=0}^{\infty} \frac{64 t_2}{k_n^3} + \sum_{n=0}^{\infty} \frac{64 t_2 T_1^2 (t_1 + t_2)^2}{k_n^3 (\varepsilon + t_2 T_1)^2}$$

Therefore, using Eqs.(22) and (23), the complex shear modulus can be determined by the measured values of ω_1 and η_c from the resonant torsional test, since the geometrical dimensions, the mass of the test specimen including the inertia bar and the elastic layer shear modulus G_1 are previously known.

4. Experimental Procedure

4.1 Specimens

The elastic beam specimens were made of stainless steel(SUS 420) with dimensions ($a_i \times b \times l$) of 1 mm $\times$ 10 mm $\times$ 160 mm. The specimens were prepared by roughening the surfaces with sandpaper (# 240) and cleaning with acetone. The inertia bar (6 mm $\times$ 10 mm $\times$ 80 mm) was made of the same materials as the beam specimens. Two types of commercially available structural epoxy adhesives were used. One of them was Araldite-standard (CIBA-GEIGY Co.) and the other was Araldite-rapid (the same Company). These adhesives were of two-component type, which utilized an epoxy resin and a hardening agent in weight ratio of 1 : 1. The test specimens bonded by Araldite-standard (abbreviated as specimen S) were cured for three hours by a hot-press at 80°C under a constant pressure. The other test specimens bonded by Araldite-rapid (specimen R) were cured for 24 hours at room temperature, compressed between two plates having ground surfaces.

The thicknesses of adhesive layer required were arranged by means of block gage settings, and the mean thickness was averaged from the values obtained by subtracting the elastic layer thickness from the resultant beam thickness at the same ten

Table 1 Thicknesses of test specimens

Specimen No.	a_1 (mm)	$\Delta a_1/a_1$ (%)	a_2 (μ m)	$\Delta a_2/a_2$ (%)
R-1	1.012	0.3	164	6.7
R-2	1.008	0.4	170	4.7
R-3	1.012	0.3	259	8.5
R-4	1.003	0.6	263	4.6
R-5	1.008	0.5	271	7.7
S-1	0.996	0.5	150	22.7
S-2	0.996	0.6	60	33.0
S-3	0.995	1.0	58	18.6

points along the test specimen. Test specimens with several adhesive layer thicknesses were prepared. Layer thicknesses of the test specimens and their deviations are given in Table 1.

4.2 Method of Measurement

Free end of the test specimen was mounted on a rigid clamping device and its other free end carried the inertia bar, the center of which was supported lightly by a sharp needle support composed of a spring and a ball bearing. One end of the inertia bar was excited electromagnetically at a constant resonant amplitude (200 μ m). The resonant frequency and amplitude of free damped vibration was measured at the other end of the inertia bar by an eddy current type pick-up transducer. The torsion tests were carried out in an environmental chamber at several different temperatures, ranging from -20°C to 80°C. After decay curve of the free damped vibration was memorized in a digital wave memory unit, a numerical calculation of the dynamic shear modulus G_2 from Eq.(22) was carried out by means of the Newton-Raphson method, and loss factor η was calculated from Eq.(23), using the resonant frequency measured and resultant loss factor determined from the decay curve. In these calculations, the values of shear modulus of the beam specimens which were measured independently at the several temperatures were used.

5. Experimental Results and Discussion

5.1 Dynamic shear modulus and loss factor

The determined values of shear modulus and loss factor from the torsional resonance test are presented in Figs.5 (specimen R) and 6 (specimen S). The scatter of these measured values is sufficiently small up to room temperature. Above room temperature, the values of specimen S are regarded as stable up to the vicinity of the transition temperature (40°C), as

Fig. 5 Determined dynamic shear modulus and loss factor (Specimen R)

compared with the values of specimen R which are scattered in the transition temperature (60°C).

The different features of the two specimens seem to be caused by the difference of curing cycles, that is, specimen S was cured at high temperature (80°C) so as to obtain enhanced stability, while specimen R was cured at room temperature as a common treatment. In the high temperature range beyond the transition temperatures, the values of both specimens become more scattered. These scatters markedly depend on the inaccuracy of the experimental data measured because the adhesive core becomes softer at such high temperatures. However, if the optimum configuration of the test specimen is employed as will be described later, these scatters may be minimized. In the above calculations of G_2 and η , the time-temperature reducibility at several resonance frequencies (77 Hz-135 Hz) was not considered.

5.2 Error analysis

Since the Composite Beam Test Method is an inverse method, the accuracy of the determined complex shear modulus may be controlled by various error factors. Therefore, the error analysis on this torsional test system must be considered. This is performed for the dynamic shear modulus G_2 , based on Eq.(22).

For determining the resultant error of G_2 , let us assume G_2 to be a function of four variables such as relative layer thicknesses R_1 and R_2 , dynamic shear modulus of elastic layer G_1 and resonance angular frequency ω as follows:

$$G_2 = f(R_1, R_2, G_1, \omega)$$

The resultant error $\Delta G_2/G_2$ is obtained from total differentiation of Eq.(22) with respect to each error factor. If the error magnification factors corresponding to these error factors are denoted by symbols α_1 ,

Fig. 6 Determined dynamic shear modulus and loss factor (Specimen S)

α_2 , α_3 and α_4 , then the resultant error expressed in terms of the mean square root error formula is given as follows:

$$\frac{\Delta G_2}{G_2} = \sqrt{\left(\alpha_1 \frac{\Delta R_1}{R_1}\right)^2 + \left(\alpha_2 \frac{\Delta R_2}{R_2}\right)^2 + \left(\alpha_3 \frac{\Delta G_1}{G_1}\right)^2 + \left(\alpha_4 \frac{\Delta \omega}{\omega}\right)^2}$$

In the case of a soft adhesive ($\epsilon = 10^{-4}$), in which it is difficult to determine dynamic shear modulus, the calculated results of the resultant error of dynamic shear modulus and the magnification factors are shown in Fig. 7 as a function of adhesive core thickness. Here, the relative errors, $\Delta R_1/R_1$, $\Delta R_2/R_2$, $\Delta G_1/G_1$ and $\Delta \omega/\omega$, were assumed empirically to be 0.5%, 10%, 2% and 0.1%, respectively, in the above resultant error calculation. The calculated result in Fig. 7 indicates that the resultant error increases with increase of the adhesive core thickness. The reason for this may be explained in that the decrease of constraining force in the elastic layers owing to the increase of the soft adhesive core thickness (refer to Fig. 3) results in the decrease of the rate of variation of the resonance frequency, and such decrease of the rate of variation of the frequency makes clearly the resultant error increase. On the other hand, the extremely thin adhesive core thickness may also increase the resultant error since the rate of variation of the resonant frequency becomes small enough.

From the above consideration, it may be suggested that the optimum test beam geometry to minimize the resultant error exists at a certain thickness of adhesive layer.

The calculated resultant errors for constant thickness ratios of R_2 to R_1 are shown in Fig. 8 as a function of elastic layer thickness. It is verified that the resultant error is minimized at a certain R_1 , as in the consideration presented above.

The comparison of the resultant errors of the determined values (Figs. 5 and 6) with the calculated resultant errors (Fig. 8) are made in the following.

Fig. 7 Error magnification factors and resultant error

In the case of $R_2/R_1=10^{-1}$ in Fig. 8, it can be seen that if R_1 is taken a value of 10^{-1} , the calculated values of $\Delta G_2/G_2$ become smaller in the range of shear modulus ratios of $10^{-2} \sim 10^{-3}$, and become extremely larger at the ratio of 10^{-4} . That is, the calculated results suggest that the determined values may be stable in the range of ϵ of $10^{-2} \sim 10^{-3}$, and unstable at the value of ϵ of 10^{-4} . In the results of torsional tests presented in Figs. 5 and 6, the values of R_1 and R_2/R_1 were 10^{-1} and $5.8 \times 10^{-2} \sim 2.7 \times 10^{-1}$ ($a_2=60 \mu\text{m} \sim 271 \mu\text{m}$), respectively, and torsional rigidity ratios determined from specimen R and S were as follow: $\epsilon=1.8 \times 10^{-2}$ (specimen R: at -20°C) $\sim 1.5 \times 10^{-4}$ (specimen S: at 80°C). By comparing both calculated and tested results, it can be seen that the calculated results agree well with the tested results such that the determined values are stable up to the room temperature and are unstable at high temperature range.

Fig. 8 Resultant errors at constant thickness ratios

Fig. 9 Relationship between elastic layer thickness R_1 and torsional rigidity of elastic layers E , and its increment ΔE

In Fig. 7, the calculated result of the error magnification factors α_i for G_2 is presented as a typical example. The result shows that α_1 is greater than the other error magnification factors, and α_1 is over 10 times as large as α_2 . Such feature of the magnification factors suggests that the resultant error contributed by the elastic layer thicknesses is significant, and that the error contributed by the adhesive core thicknesses is significantly small. These features may be interpreted from the fact previously mentioned that the torsional rigidity of the three layered beam depended on the torsional rigidity of elastic layers E and their increments ΔE .

5.3 Optimum configuration of the test specimen

According to the previous consideration, it may be estimated that the configuration of the test specimen to minimize the resultant error strongly depends on E and ΔE . So, the relationships of E and ΔE with R_1 are presented in Fig. 9. It can be seen that the values of R_1 at the intersection of curves E and ΔE correspond clearly with the optimum values of R_1 which minimize the resultant error in Fig. 8. That is, the configuration of the test specimen, for which the torsional rigidity of elastic layers coincides with their increments, results in the minimum of the resultant error. Therefore, the optimum configuration of specimens may be determined from the above estimation.

The calculated result for the optimum configuration are shown in Fig. 10, relating R_1 with R_2 , with ϵ as a parameter. It is indicated that the elastic layer thickness should be selected in relation with the adhesive core thickness and the shear modulus ratio, and that the optimum configuration of the test specimen is such that a thinner adhesive must be selected with a thicker elastic layer, and a thicker adhesive core must correspondingly be selected with a thinner elastic layer. Therefore, in this test method, an arbitrary selection of the specimen configuration may

Fig. 10 Evaluation curve for the optimum configuration of the test specimen

cause a larger resultant error for the determination of the dynamic shear modulus.

6. Conclusions

Characteristics of the Composite Beam Test Method for determining complex shear modulus of a thin adhesive layer are very much complicated as the relaxation of constraining force in the elastic layers arises owing to varying of the adhesive layer thickness. These characteristics were revealed in this investigation. The results obtained are as follows:

(1) The relationships to determine the complex shear modulus of a thin adhesive layer were presented. Furthermore, it was shown that the determination of this modulus could be carried out directly from the torsional resonance test of the bonded beam.

(2) The effect of controlling factors on the resultant error of dynamic shear modulus was revealed, and it was made clear that the relative error of elastic layer thicknesses exerts the greatest influence on the resultant error, while that of adhesive core exerts the least influence.

(3) The optimum configuration of the test specimen minimizing the resultant error was found to exist for a given dynamic shear modulus ratio of an adhesive core to a elastic layer, and the calculated relationship providing the optimum configuration was presented.

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