A Probabilistic Study on Fatigue Life Distribution of a Metallic Material with Surface Defects Considering the Number of Initiated Cracks*

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It is well-known that the fatigue failure of metallic materials occurs due to the initiation of cracks and their subsequent growth. However, the number of crack nuclei is not provided deterministically. In engineering applications, this number is randomly variable and often significant, and should be taken into account in the evaluation of crack coalescence. In the present study, based on the numbers of initiated cracks, a statistical evaluation was made of the coalescence of cracks as a function of the number of stress cycles, and the fatigue life distribution was probabilistically derived from the above evaluation assuming that the failure occurs immediately after the coalescence of cracks. The analytical results obtained are in good agreement with experimental results of the fatigue life distribution.

Key Words: Fatigue, Number of Initiated Cracks, Crack Coalescence, Probabilistic Treatment, Distribution of Life

1. Introduction

Metallic materials used for structural members and machine parts have, in many cases, various types of surface defects which act as crack starters under cyclic loading. Due to the number of surface defects generally present in a material, the possibility exists that several cracks can form in a specimen. Of course, the crack initiation life at each defect usually has a large scatter due to the variation of defect sensitivity⁽¹⁾⁻⁽⁵⁾. This fact indicates that the number of cracks depends on the loading cycles and should thus be evaluated probabilistically for a given loading cycle.

The authors previously examined the fatigue life distribution of a round bar specimen having three small drilled holes a set distance apart, and fatigue life distribution was theoretically derived by assuming

that the final failure occurred immediately after the first coalescence of cracks. The analytical result thus obtained was in good agreement with the experimental fatigue life distribution⁽⁶⁾. In this study, the number of cracks was deterministically provided in advance, because fatigue cracks took place at all the drilled holes. However, the number of crack nuclei is often not given deterministically in engineering applications. In these cases, the number of initiated cracks is a random variable, and this should be taken into account in the evaluation of crack coalescence.

In the present study, in order to investigate this probabilistic feature, uniform rotating bending fatigue tests were newly carried out on S50C steel specimens having twelve small holes drilled on the cylindrical surface. It was then found that the number of fatigue cracks had a certain variation at the cyclic stage of final failure. The fatigue failure similarly occurred in this program, immediately after the first coalescence of cracks took place at a location on the specimen surface.

Based on this experimental evidence, fatigue life distribution is analytically derived by evaluating the probability of crack coalescence at any stage of cyclic

* Received 6 th October, 1987. Paper No. 86-0850 A

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loading. It was found that the experimental fatigue life distribution was well-explained by the present analysis. The major aim of this study is to develop the theory in the previous paper⁽⁶⁾ as to be applicable in the case where the number of cracks is not provided deterministically.

2. Specimen and Experimental Procedure

The material used in the present study was S50C carbon steel. Its chemical composition and mechanical properties are listed in Tables 1 and 2, respectively. Specimens were machined into the configuration shown in Fig. 1(a). All specimens were polished with emery papers to a final grade of 2000. The final finish of the specimen surface was accomplished by buff-polishing.

Three small holes (diameter: 0.3 mm, depth: 0.3 mm) were drilled at equi-distance points on the specimen surface in each of four sections as shown in Fig. 1(b). Accordingly, each specimen had twelve surface defects, which acted as crack initiators. This type of specimen is hereafter referred to as a drilled specimen, whereas the specimen without any defect is referred to as a smooth specimen.

Fatigue tests were carried out by a uniform rotating bending testing machine⁽⁷⁾ at a testing speed of approximately 3600 rpm. The initiation and propagation of fatigue cracks was monitored with the aid of an optical microscope attached to the machine, periodically interrupting the test. The exact crack length was measured by taking the replica around each hole and observing this replica with another optical microscope at a magnification of 100 $\times$. Crack length l was defined as the total length of the surface crack, includ-

ing the hole diameter. Fatigue tests were also performed on smooth specimens for the sake of comparison.

3. Experimental Results

3.1 S-N characteristics

S-N properties on smooth and drilled specimens are depicted in Fig. 2, where the hollow symbols indicate the experimental results on smooth specimens and the solid ones the results on drilled specimens. The fatigue limit for the former type of specimen is approximately 304 MPa. In order to examine the fatigue life distribution on drilled specimens, fatigue tests were repeatedly performed at $\sigma=255$ MPa on twenty specimens. In this type of specimen, the procedure of analysis focussed on the fatigue life distribution at a definite stress level, and the fatigue limit was not determined. However, a significant decrease of the fatigue limit is expected from the trend of the results at $\sigma=245$ and 225 MPa.

Drilled specimens had three small holes on each of four sections I, II, III and IV, as shown in Fig. 1(b). Fatigue cracks took place at some holes, and propagated gradually as a result of cyclic loading. Once the adjacent cracks in a critical section began to coalesce with each other, the specimen soon failed in this section. It was noted that all specimens failed during this process, and that failure was attributed to the propagation of at least two cracks. Frequency of failure⁽⁸⁾ for each section is depicted in Fig. 3. It is found that the frequency is almost uniform over all sections. Therefore, the frequency distribution is

Table 1 Chemical composition of sample material

Material	Chemical composition (%)							
	C	Mn	Si	P	S	Cu	Ni	Cr
S50C	0.53	0.79	0.32	0.024	0.023	0.04	0.02	0.10

Fig. 1 Specimen

Table 2 Heat treatment and mechanical properties of sample material

Heat treatment	Yield point σ_s (MPa)	Tensile strength σ_B (MPa)	Elongation ϕ (%)	Reduction of area ψ (%)	Vickers hardness Hv
830°C, 1 hr, A.C.	442	738	21.3	51.2	208

Fig. 2 S-N curve

assumed to be uniform for the sake of convenience.

3.2 Distribution of crack initiation life and final life

In this study, crack initiation life N_0 is defined as the number of stress cycles until the crack length l reaches $l_0=0.5$ mm. Experimental distributions of this crack initiation life N_0 and the final fatigue life N_f are plotted on Weibull probability paper in Fig. 4. The ordinate, cumulative probability $F(N)$, is calculated as $F(N_i)=i/(n+1)$, where N_i indicates the i 'th shortest life in n total data.

Since each specimen had twelve holes on the surface, twenty specimens assigned to fatigue life tests provided $12 \times 20 = 240$ data on the crack initiation life. When cracks do not take place at some holes until the final failure of the specimen, such holes give the censored data on crack initiation life⁽⁹⁾. As shown by the solid symbols in Fig. 4, the final fatigue life has a considerable scatter. Therefore, the above data on crack initiation life provide a multi-censored sample having the size of 240. If the shortest fatigue life among twenty specimens ($N_f=4.157 \times 10^5$ cycles) is taken as the unified censoring time N_c , crack initiation lives obtained at drilled holes on some specimens become longer than this censoring time, occasionally.

Fig. 3 Frequency of failure on each section

Fig. 4 Distributions of crack initiation life and final life

In such cases, let us replace all the measured data in $N_0 > N_c$ by the censoring time N_c . Based on this replacement, one can obtain the single-censored sample which is composed of 155 definite life data and 85 censored data. The number of censored data "85" is indicated around the censored symbol in Fig. 4.

The solid line passing through the hollow symbols represents the distribution function in the following Weibull-type distribution fitted by linear regression technique:

$$F_0(N_0) = 1 - \exp\left\{-\left(\frac{N_0}{b}\right)^a\right\}, \quad (1)$$

where a and b are shape and scale parameters, their values of which are indicated in the figure. The other curve passing through the solid symbols represents the Weibull-type distribution function on final fatigue life having three parameters including the location parameter c . Respective parameters are estimated by means of the correlation coefficient method⁽¹⁰⁾, and their values are also listed in the figure. Since the final failure takes place immediately after the first coalescence of fatigue cracks, this curve can directly provide the distribution function of the crack coalescence life⁽¹¹⁾.

The median of the final fatigue life, $\tilde{N}_f$, is determined from the intersection of the distribution curve and the horizontal line of $F(N)=50\%$. This value is thus given as $\tilde{N}_f=6.26 \times 10^5$. The cycle ratio $n=N/\tilde{N}_f$ is newly introduced to represent the stage of cyclic loading, and this ratio will be often used in the latter stages of this analysis.

3.3 Distribution of the number of initiated cracks

The distribution characteristics of crack initiation life N_0 are given by Eq. (1); this fact suggests that the number of initiated cracks until a definite stress cycle should be evaluated probabilistically. In other words, the number of initiated cracks is also a random variable. Figure 5 indicates $G(m)$, the cumulative distribution of the number of initiated cracks m , at six stages of cyclic loading, $n=0.2, 0.4, 0.6, 0.8, 1.0, 1.2$.

Fig. 5 Distribution of the number of initiated cracks

and 1.2.

The distribution of the number of initiated cracks at a given number of stress cycles N can now be theoretically derived: the probability density function of the number of initiated cracks m is given by

$$g(m) = \binom{z}{m} F_0(N)^m \{1 - F_0(N)\}^{z-m}, \quad (2)$$

where $F_0(N)$ is the distribution function of crack initiation life and z the total number of surface defects ($z=12$). Hence, $G(m)$ is given by the following summation,

$$G(m) = \sum_{i=0}^m g(i). \quad (3)$$

At the noted six stages of cyclic loadings, $g(m)$ was calculated using Eq. (2), after which $G(m)$ was obtained using Eq. (3). The distribution functions thus obtained are plotted by solid curves in Fig. 5. It is found that the respective curves are in good agreement with the corresponding experimental results.

3.4 Distribution of crack length

Propagation behavior of the cracks emanating from each of the drilled holes was observed on twenty specimens. From such results, one can obtain the distribution characteristics of crack length l at any stage of cyclic loading⁽¹²⁾⁻⁽¹⁴⁾. Figures 6 and 7 indicate such distributions obtained at five stages of $n=0.6, 0.7, 0.8, 0.9$ and 1.0 , where Fig. 6 is plotted on Weibull probability paper and the 7 on log-normal probability paper. No significant difference on the goodness of fit was seen between both distributions in Weibull and log-normal types, that is, distribution characteristics of the crack length are equally well approximated by each. It is found that the distribution pattern is always represented by a straight line in the range of $0.6 \leq n \leq 1.0$. In other words, the lower bound of the distribution remains $l=0$, regardless of the distribution type, in subsequence of the cyclic loading.

Fig. 6 Distribution of crack lengths (Weibull probability paper)

This aspect is attributed to the following facts. The specimen has a number of crack nuclei ($z=12$), and the crack initiation life has a large scatter as shown in Fig. 4. Hence, it is not expected that fatigue cracks will occur at all defects (drilled holes) by a certain stage of cyclic loading. However, after this stage of cyclic loading, new cracks may occur. Thus, the minimum length of the initiated cracks remains $l=0$, regardless of the stage of cyclic loading.

Since a Weibull-type distribution function is frequently used to represent the distribution characteristics of the crack length⁽¹⁵⁾⁽¹⁶⁾, the following distribution function is used in the present analysis to calculate the probability of crack coalescence.

$$F_l(l) = 1 - \exp\left\{-\left(\frac{l}{\beta}\right)^\alpha\right\}. \quad (4)$$

Solid lines in Fig. 6 indicate distribution functions of Eq. (4), where the two parameters α and β can be determined by linear regression technique.

Variation of these parameters subsequent to cyclic loading is shown in Fig. 8, where the cycle ratio n is taken as the abscissa. In the semi-logarithmic plots shown in Fig. 8, excellent linear relationships are found in both $\alpha-n$ and $\beta-n$ relations. Solid lines in this figure are assumed to represent the dependency of the cycle ratio, n , on α and β , and these are used in the numerical calculation in the next section. Thus, distribution characteristics of the crack length can be quantitatively provided at any stage of cyclic loading.

4. Analysis and Discussion

If a cylindrical surface having twelve defects as shown in Fig. 1(b) is unrolled into a plane, we have a two-dimensional configuration of surface defects as shown in Fig. 9. Fatigue cracks will then occur at the

Fig. 7 Distribution of crack lengths (log-normal probability paper)

defects illustrated by solid marks in the lattice, and will propagate in the circumferential direction. We assume that the fatigue crack propagates symmetrically on both sides of the drilled hole.

On the drilled specimen in this program, the crack coalescence⁽⁶⁾⁽¹⁷⁾ takes place only if two or three cracks occur in the same section. When the crack initiates at only one defect, crack coalescence cannot be expected in this section. Thus, the probability of crack coalescence is evaluated in two cases where two and three cracks occur in a definite section, after which the probability is again evaluated considering the crack coalescence in all four sections.

4.1 In the case where three cracks occur in a definite section

The probability of crack coalescence in this case was previously derived⁽⁶⁾. When the lengths of three cracks at a given stage of cyclic loading are denoted by l_1 , l_2 and l_3 , the probability that crack coalescence does not occur at this cyclic stage is provided by

$$H = 3 \left[\int_0^r f(l_3) \int_{l_3}^{2r-l_3} f(l_2) F(2r-l_2) dl_2 dl_3 - \int_0^r f(l_3) F(l_3) F(2r-l_3) dl_3 + \int_0^r f(l_3) \{F(l_3)\}^2 dl_3 \right] \quad (5)$$

In Eq. (5), $f(l)$ and $F(l)$ are the probability density and distribution functions of the crack length l , and r is the distance between adjacent drilled holes, which is given by one third of the circumference of the section. Therefore, the probability that the cracks coalesce somewhere in the definite section is given as follows:

$$P_{c3} = 1 - H. \quad (6)$$

4.2 In the case where two cracks occur in a definite section

Let the lengths of two initiated cracks be l_1 and l_2 , and their sum be $s = l_1 + l_2$. Then, crack coalescence

will occur if $s \geq 2r$. As the crack lengths l_1 and l_2 are assumed to be statistically independent of each other, the probability density function of s is given through convolution integral⁽¹⁸⁾ as follows:

$$f_s(s) = \int_0^s f(l_1) f(s-l_1) dl_1 = \int_0^s f(s-l_2) f(l_2) dl_2, \quad (7)$$

and its distribution function is given by

$$F_s(s) = \int_0^s f_s(s) ds. \quad (8)$$

Consequently, the probability that crack coalescence does not occur is directly given by the following expression, provided that only two cracks take place in a section;

$$K = F_s(2r), \quad (9)$$

and the probability that crack coalescence takes place is

$$P_{c2} = 1 - K. \quad (10)$$

4.3 Probability of crack coalescence considering all four sections

As the total number of defects is denoted by z and the number of initiated cracks by m , crack coalescence can occur only in the case of $m \geq 2$. In the cases of $m=0$ and 1, crack coalescence cannot take place on the specimen. Furthermore, even in the case of $m \geq 2$, more than two cracks must initiate in the same section to cause the cracks to coalesce. Therefore, in the case of $m \geq 2$, it is important to properly choose the number m of crack-induced defects among z total defects, and this problem should be evaluated probabilistically.

Now let us first examine the number of feasible cases in which m of z defects results in crack initiation. The possible range of m is $2 \leq m \leq 12$, since $z=12$ in this program. In this condition, there are a number of ways to choose the m defects. Possible cases are illustrated in Fig. 10 such that the number of cracked defects decreases from column I to IV in Fig. 9 and the cracked defects are assigned from the top to the

Fig. 8 Dependencies of shape and scale parameters of Weibull distribution on cycle ratio

Fig. 9 Two-dimensional configuration of surface defects

bottom. Lattice points of $3 \times 4 = 12$ indicate the drilled defects and the marks of $\times$ indicate the defect-initiated cracks. Four-digit indices such as [2000], [1100] and [3000] indicate the number of cracks initiated on each column of I, II, III and IV. These indices will be often used in the following analysis. Of course, in an actual evaluation of the number of feasible cases, we should further consider how to choose a significant column among I, II, III and IV, and how to choose a significant defect in a definite column. Therefore, the number of possible cases is, in actual fact, considerably greater than that of the typical example illustrated in Fig. 10. Namely, the concept of combination should be taken into account in the exact evaluation of the number of possible cases.

Based on this point, the exact number of possible cases is analysed for each m -value.

[in the case of $m=2$]

The total number of cases where fatigue cracks occur at any two defects is given by

$$\binom{12}{2} = \frac{12!}{2!10!} = 66.$$

The number of cases in [2000] is provided by

$$\binom{4}{1} \binom{3}{2} = 4 \cdot 3 = 12,$$

and the number of cases in [1100] by

$$\binom{4}{2} \binom{3}{1} \binom{3}{1} = 54.$$

Hence, the respective probabilities that the cases [2000] and [1100] will take place are given as follows:

$$P_{2000} = 12/66, P_{1100} = 54/66.$$

[in the case of $m=3$]

The total number of cases is similarly given by

$$\binom{12}{3} = 220.$$

The number of cases in [3000] is

$$\binom{4}{1} = 4,$$

the number of cases in [2100] is

$$\binom{4}{1} \binom{3}{1} \binom{3}{2} \binom{3}{1} = 108$$

and the number in [1110] is

$$\binom{4}{3} \binom{3}{1} \binom{3}{1} \binom{3}{1} = 108.$$

Hence, the probabilities of the respective cases [3000], [2100] and [1110] are provided as follows:

$$P_{3000} = 4/220, P_{2100} = 108/220, P_{1110} = 108/220.$$

Similarly, one can derive such probabilities in each case of $m=4, 5, \dots$ and 12. Thus, we obtain the following results:

[in the case of $m=4$]

$$P_{3100} = 36/495, P_{2200} = 54/495,$$

$$P_{2100} = 324/495, P_{1111} = 81/495$$

[in the case of $m=5$]

$$P_{3200} = 36/792, P_{3110} = 108/792,$$

$$P_{2210} = 324/792, P_{2111} = 324/792$$

[in the case of $m=6$]

$$P_{3300} = 6/924, P_{3210} = 216/924, P_{3111} = 108/924,$$

$$P_{2220} = 108/924, P_{2211} = 486/924$$

[in the case of $m=7$]

$$P_{3310} = 36/792, P_{3220} = 108/792,$$

$$P_{3211} = 324/792, P_{2221} = 324/792$$

[in the case of $m=8$]

$$P_{3320} = 36/495, P_{3311} = 54/495,$$

$$P_{3221} = 324/495, P_{2222} = 81/495$$

[in the case of $m=9$]

$$P_{3330} = 4/220, P_{3321} = 108/220,$$

$$P_{3222} = 108/220$$

[in the case of $m=10$]

$$P_{3331} = 12/66, P_{3322} = 54/66$$

[in the case of $m=11$]

$$P_{3332} = 12/12 = 1$$

[in the case of $m=12$]

$$P_{3333} = 1.$$

The probability that the individual event of $[ijkl]$ takes place in a specimen, P_{ijkl} , was thus obtained for all the values of m in $2 \leq m \leq 12$. As described previously, it is obvious that the crack coalescence occurs only when two or three cracks take place in the same section. In each case, the probability that the cracks

Fig. 10 Illustration of feasible cases in which m of z defects results in crack initiation

do not coalesce at the given stage of cyclic loading is provided by Eq.(5) or Eq.(9). Consequently, the probability, F_m that crack coalescence takes place for an individual value of m is provided as follows:

[in the cases of $m=0,1$]

Since crack coalescence cannot take place, we have

$$F_0 = F_1 = 0. \quad (11)$$

In the case of $2 \leq m \leq 12$, the probability of crack coalescence under the event of $[ijkl]$, F_{ijkl} , is first evaluated and the probability of the event $[ijkl]$ itself is then taken account into the calculation of the final probability F_m .

[in the case of $m=2$]

Since

$$F_{2000} = 1 - K \text{ and } F_{1100} = 0 \text{ (no coalescence),}$$

we have

$$F_2 = F_{2000} \cdot P_{2000} \quad (12)$$

[in the case of $m=3$]

Similarly,

$$F_{3000} = 1 - H, F_{2100} = 1 - K, F_{1110} = 0 \text{ (no coalescence)}$$

and

$$F_3 = F_{3000} \cdot P_{3000} + F_{2100} \cdot P_{2100} \quad (13)$$

[in the case of $m=4$]

$$F_{3100} = 1 - H, F_{2200} = 1 - K^2,$$

$$F_{2110} = 1 - K, F_{1111} = 0 \text{ (no coalescence)}$$

and

$$F_4 = F_{3100} \cdot P_{3100} + F_{2200} \cdot P_{2200} + F_{2110} \cdot P_{2110}. \quad (14)$$

In a similar way, one can obtain the probabilities $F_5 \sim F_{12}$.

If the probabilities $F_0, F_1, F_2, \dots$ and F_{12} are obtained in the above procedure, the probability that crack coalescence takes place anywhere on an actual specimen is given by the following summation:

$$F = \sum_{m=0}^{12} F_m \cdot g(m), \quad (15)$$

where $g(m)$ is already provided by Eq.(2).

By assuming that the distribution of crack length is a Weibull type and the n -dependency of its two parameters is represented by solid lines in Fig. 8, one can finally obtain the probability of Eq.(15) as a function of the number of loading cycles. In the present study, this result can directly provide the fatigue life distribution as mentioned previously.

The analytical fatigue life distribution thus obtained is plotted by solid lines on each of the Weibull and log-normal probability papers in Fig. 11⁽¹⁹⁾⁻⁽²²⁾, where the experimental result of the fatigue life distribution is also replotted for the sake of comparison. The theoretical distribution is in good agreement with the experimental one in each figure, though the scatter of the calculated distribution is somewhat smaller than that of the experimental result. This

slight difference would come from the fact that the distribution function $G(m)$ calculated by Eq.(3) has a smaller scatter than that in the experimental distribution as shown in Fig. 5. The small number of specimens used in this program is another possible source of this difference. For further clarification on this point, a larger number of specimens must be tested.

In this study, location and shape of surface defects were deterministically provided in advance, but the practical defects of engineering materials could not be discussed deterministically with respect to both the location and the shape. These defects are randomly spaced inside the material or on the specimen surface, and the size and shape of such defects fluctuate significantly in actual cases. Mutual interaction of the propagation behavior of adjacent cracks would thus influence the analytical result. These problems should be resolved step by step in future studies.

5. Conclusions

Round bar specimens having twelve drilled holes on the surface were prepared, and uniform rotating bending fatigue tests were carried out on these specimens together with smooth specimens. Initiation and propagation behavior of cracks emanating from sur-

Fig. 11 Comparison between theoretical and experimental fatigue life distributions

face defects were observed in detail, and distribution characteristics of the final fatigue life were also examined for the drilled specimens. Based on these experimental results, a theoretical interpretation was made from the probabilistic viewpoint. The main results obtained in this study are summarized as follows.

(1) The drilled specimen has three defects on each of four sections, and fatigue cracks take place at some defects and propagate as a result of cyclic loading. Once the adjacent cracks coalesce, the specimen soon fails in this section. Failure frequency is almost uniform over the above-mentioned four sections.

(2) Distribution characteristics of the number of initiated cracks at a definite stage of cyclic loading are analytically given by a binomial distribution. The distribution property of the crack length is equally well-approximated by distributions of both Weibull and log-normal type. It is also found that the lower bound of the distribution remains $l=0$ subsequent to cyclic loading. This aspect is attributed to the fact that a new crack can take place at any stage of the cyclic loading due to a large scatter of the crack initiation life.

(3) Combining the distribution of the number of initiated cracks and the crack length distribution, the probability of crack coalescence is derived as a function of the number of loading cycles. The relationship between this probability and the number of cycles can directly provide the fatigue life distribution of the drilled specimens. It is found that the analytical result is in good agreement with the experimental fatigue life distribution.

References

- (1) Tanaka, T., Sakai, T. and Okada, K., Statistical Aspect of Fatigue Crack Propagation from Micro Surface Defects, Proc. 25th Jpn. Congr. Mat. Res., (1982), p. 100.
- (2) Sakai, T., Ueno, A., Fujitani, K. and Tanaka, T., A Statistical Study on Fatigue Life Distributions of SCM 435 Round Bar Specimens Having Surface Defects, Eng. Fract. Mech., Vol. 24, No. 5 (1986), p. 759.
- (3) Harada, T., Ishii, A., Ochi, Y. and Sasaki, S., Prepr. of Jpn. Soc. Mech. Eng., (in Japanese), No. 830-10 (1983-10), p. 228.
- (4) Ichikawa, M., Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 50, No. 456, A (1984), p. 1435.
- (5) Ichikawa, M., Takura, T. and Tanaka, S., Statistical Investigation of Fatigue Life by Separating Crack Initiation and Propagation Lives, J. Soc. Mater. Sci. Jpn., (in Japanese), Vol. 31, No. 346 (1982) p. 697.
- (6) Tanaka, T., Sakai, T. and Okada, K., A Statistical Study on Fatigue Life Distribution Based on the Coalescence of Cracks from Surface Defects, Bull. JSME, Vol. 28, No. 236 (1985), p. 209.
- (7) JSMS Handbook of Testing of Metallic Materials (in Japanese), (1977), p. 254, Yoken-do Press.
- (8) Akita, S., Prepr. of 6th Symp. Reliability Eng. of JSMS, (in Japanese), (1984-12), p. 57.
- (9) Sakai, T. and Tanaka, T., On Parameter Estimation of Life Distribution Near Fatigue Limit, J. Soc. Mater. Sci. Jpn., (in Japanese), Vol. 30, No. 339 (1981), p. 1211.
- (10) Sakai, T. and Tanaka, T., Estimation of Three Parameters of Weibull Distribution in Relation to Parameter Estimation of Fatigue Life Distribution, Proc. 24th Jpn. Congr. Mat. Res., (1981), p. 141.
- (11) Tanaka, T., Sakai, T. and Okada, K., Study on Coalescence Probability of Cracks Originating from Randomly Spaced Surface Defects and Associated Fatigue Life Distribution, Bull. JSME, Vol. 29, No. 249 (1986), p. 658.
- (12) Tokaji, K., Ando, Z. and Mizutani, H., On Statistical Property of Fatigue Crack Length of High Strength Steel, J. Soc. Mater. Sci. Jpn., (in Japanese), Vol. 31, No. 351 (1982), p. 1204.
- (13) Ide, R., Tsunenari, T. and Horikawa, T., Prepr. of 16th Symp. Fatigue of JSMS, (in Japanese), (1982-11), p. 16.
- (14) Sasaki, S., Proc. 30th Jpn. Nat. Symp. Strength, Fracture and Fatigue, (1985-4), p. 15.
- (15) Tokaji, k. and Ando, Z., J. Soc. Mater. Sci. Jpn., (in Japanese), Vol. 31, No. 344 (1982), p. 520.
- (16) Ishihara, S., Shiozawa, K. and Miyao, K., Distribution of Corrosion Fatigue Crack Lengths in Carbon Steel, Bull. JSME, Vol. 28, No. 236 (1985), p. 185.
- (17) Kitagawa, H., Yuuki, R., Takahashi, S. and Chang -Min Suh, Prepr. of Jpn. Soc. Mech. Eng., (in Japanese), No. 790-9 (1979-9), p. 25.
- (18) Gibra, I. N., Probability and Statistical Inference for Scientists and Engineers, (1973), p. 86, Prentice-Hall.
- (19) Tanaka, T., Iwaya, T. and Sakai, T., Statistical Distributions of Fatigue Life and Fatigue Strength of Low Carbon Steel in Long Life Region, Proc. 26th Jpn. Congr. Mat. Res., (1983), p. 141.
- (20) Yokobori, T., Yokobori, T. and Awaji, H., Prepr. of Jpn. Soc. Mech. Eng., (in Japanese), No. 830-2 (1983-3), p. 152.
- (21) Shimokawa, T. and Hamaguchi, Y., Relationship Between Fatigue Life Distribution, Notch Configuration, and $S-N$ Curve of a 2024-T4 Aluminum Alloy, Trans. ASME, MT, Vol. 107 (1985), p. 214.
- (22) Nishijima, S. and Jono, M., J. Soc. Mater. Sci. Jpn., (in Japanese), Vol. 31, No. 343 (1982), p. 409.