The Effect of Mean Stress and Strain on Crack Growth and Crack Closure in Low-Cycle Fatigue*

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JIS SNCM 439, S 35 C and SUS 304 steels were low-cycle fatigued with mean stress and strain under load-and displacement-controlled conditions. The cyclic J integral range, ΔJ , was not effective for the evaluation of the crack growth rate under test conditions accompanied by ratchet deformation. Therefore, the parameter of $\Delta J / \{1 - (\phi_c^{\min} / \phi_c^{\max})\}^{0.2}$, which controlled the acceleration of the crack growth rate due to ratchet deformation as well as the crack growth rate under a fully reversed cyclic strain, was usefully proposed here, where $\phi_c^{\min}$ and $\phi_c^{\max}$ were the minimum and maximum crack-center opening displacements. The crack tip opening displacement range $\Delta \phi_{t(200)}$ was also useful parameter for controlling the fatigue crack growth rate under these test conditions. The $\Delta \phi_{t(200)}$ was correlated with ΔJ by the equation, $\Delta \phi_{t(200)} = A(\Delta J)^q$. In general, the power exponents, q , were smaller than the values of $1/(n'+1)$ or $1/(n+1)$ estimated by assuming the HRR singular field around the fatigue crack tip, where n' and n were the cyclic and monotonic strain hardening exponents of materials

Key Words: Metallic Materials, Low-Cycle Fatigue, Fatigue Crack Growth, Mean Stress, Mean Strain, Cyclic J -Integral Range, Crack Opening Displacement

1. Introduction

Crack growth rates in low-cycle fatigue have been successfully analyzed in terms of the elastic-plastic fracture mechanics parameters^{(1)~(7)}; the crack opening and closure being the significant fracture mechanics factors at that time^{(4)~(6)}. It is mentioned here that opening and closure at the crack tip are elucidated much less under a low-cycle fatigue condition⁽⁴⁾ than in high-cycle fatigue⁽⁸⁾.

In a low-cycle fatigue test, ratchet deformation is induced under a load-controlled condition, being enhanced by cyclic loading with tensile mean stress. Opening and closure at the crack tip are significantly influenced by this ratchet deformation. Moreover, ratchet deformation causes an acceleration of the fatigue crack growth rate, which cannot be controlled

by the cyclic J -integral range, $\Delta J^{(5)(9)(10)}$.

The present paper aims to clarify the effect of mean stress and strain on crack opening and closure behavior, and to find an effective parameter to evaluate the crack growth rate under test conditions accompanied by ratchet deformation.

2. Materials Tested, Experimental Procedures and Analyses

The materials used were JIS SNCM 439 alloy steel, S 35 C medium carbon steel and SUS 304 stainless steel. Table (1) shows the static mechanical properties of the materials heat-treated in the conditions presented in Table (2).

Figure 1 shows a center-cracked-type test specimen. Two small and shallow holes across the crack were prepared at locations separated by 2 mm, as shown in the magnification in Fig. 1. The opening displacement at the center of the crack was measured by setting up a linear differential transformer-type extensometer between these two holes.

Two types of push-pull low-cycle fatigue tests were performed using a closed-loop-type test system; either axial displacement between two reference

* Received 26th November, 1987. Paper No. 86-0948 A

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points, which are 12 mm from each other on the specimen, or axial load was controlled with a given mean strain, R_ϵ or a given mean stress, $R\sigma$, respectively, during cyclic deformation.

The crack size and the crack tip opening displacement were measured by using photographs from an optical microscope with a magnification of 100. At proper intervals the test machine was manipulated by hand, and the crack tip opening displacements, Φ_t , were measured at several loading points including the tensile and compressive ends of the stress-strain hysteresis loop. The displacement between the two holes mentioned above was used to determine the crack center opening displacement, Φ_c .

The cyclic J -integral range, ΔJ , was estimated using the following equation proposed by Dowling et al.,^{(1),(3)} which is based on the concept of Rice et al.⁽¹¹⁾

$$\Delta J = (\Delta K_{\text{eff}})^2 / E + A / 2bB, \quad (1)$$

where E , b , B and A are the Young's modulus, the ligament width and thickness of the specimen, and the strain energy corresponding to the area of the load-displacement hysteresis loop above the crack closure point. Incidentally, the effective stress intensity factor range, ΔK_{eff} , is expressed by the equation

$$\Delta K_{\text{eff}} = \Delta \sigma_{\text{eff}} \sqrt{\pi a} \{1 + 0.128(a/W) - 0.288(a/W)^2 + 1.523(a/W)^3\}, \quad (2)$$

where a is half the crack length, and $\Delta \sigma_{\text{eff}}$ is given by $\Delta P_{\text{eff}} / 2WB$, denoting the effective load range above the crack closure point and the specimen width as ΔP_{eff} and $2W$, respectively. It is noted here that eq. (1) was used to estimate ΔJ only for a crack of $a/W \geq 0.4$ in size.

The crack tip opening displacement, Φ_t , was

measured at a position of 200 μm behind the crack tip; this is denoted as $\Phi_{t(200)}$. The difference between $\Phi_{t(200)}$ at the maximum and minimum loads is referred to as the range of crack tip opening displacement, $\Delta \Phi_{t(200)}$.

The stress and strain ratios are defined as $R\sigma = P_{\text{min}} / P_{\text{max}}$ and $R_\epsilon = \delta_{\text{min}} / \delta_{\text{max}}$, where P_{min} and P_{max} are the minimum and the maximum loads, and δ_{min} and δ_{max} are the minimum and the maximum displacements, respectively. The crack opening load range ratio, U , is given by the equation

$$U = (P_{\text{max}} - P_{\text{op}}) / (P_{\text{max}} - P_{\text{min}}), \quad (3)$$

denoting the crack opening load as P_{op} .

3. Test Results and Discussion

3.1 Crack opening and closure behaviors

Figures 2(a) and (b) show the change in the crack opening load range ratio, U , with an increase in half crack length, a , in SNCM 439 alloy steel and SUS 304 stainless steel low-cycle fatigued under displacement-controlled condition. The crack opening load range ratio decreases as the crack length increases in both materials. The relation between U and a is influenced by the strain ratio, R_ϵ , in SNCM 439 steel. This, however, reaches a stable state almost independent of the strain ratio under the test conditions of R_ϵ ranging from 0 to 2/3. On the other hand, SUS 304 stainless steel, which is lower in strength and higher in ductility than SNCM 439 steel, has a fairly stable relation between U and a for all strain ratios from -1 to 2/3, and this is hardly affected by the strain ratio. Such material and strain ratio dependencies of this relation are closely associated with the scale of plastic deformation produced in the specimen: plastic deformation develops easily in materials lower in strength and higher in ductility, under a test condition of a higher strain range and a higher strain ratio as controlled in the test. Such materials and test conditions easily lead to the stabilization of the relation. It should be noted that the stabilized relation is almost the same in both materials, being insensitive to the kind of materials.

Table 1 Static mechanical properties of materials tested

Material	Yield stress, MPa	Ultimate strength, MPa	Elongation, %	Reduction of area, %	Fracture ductility	Hardness Hv (300 gr.)
SNCM439	866	963	18	64	1.02	300
S35C	393	602	30	59	0.89	204
SUS304	252*	657	80	80	1.61	184

* 0.2% proof stress

Table 2 Heat-treatments performed for test materials

Material	Heat Treatment
SNCM 439	Heated at 855°C for 30min → Air cooled,
	Heated at 845°C for 30min → Oil quenched and then
	Heated at 635°C for 1h → Water quenched
S 35 C	Heated at 865°C for 30min → Air cooled
SUS 304	Heated at 1060°C for 1h → Water toughened

Fig. 1 Shape and dimensions of the specimen

Figure 3 shows U versus a under the load-controlled test condition of $R\sigma = -1.0$. Incidentally, crack closure was not detected on the load-displacement hysteresis loop in all materials of SUS 304, S 35 C and SNCM 439 steels under a test condition of $R\sigma \geq 0$. According to Fig. 3, U increases monotonously with an increase in a and finally reaches unity in SUS 304 steel. On the other hand, U is almost constant around the value of $0.6 \sim 0.7$ until the crack extends to a given size, and then it increases steeply toward unity in S 35 C and SNCM 439 steels. The larger scale yielding produced around the crack tip causes the steeper increase in U , as shown in this figure.

Figure 4 shows the change in the hysteresis loop shape and the development of ratchet deformation with crack extension under the load-controlled condition of $R\sigma = -1.0$. Additionally, the appearance around the crack tip is also presented in this figure. According to the figure, the width of the hysteresis loop and the accumulation of the unidirectional tensile deformation become large as the crack extends. These lower the crack closure point, P_{cl} . As the result,

P_{cl} approaches the minimum load point, P_{min} .

3.2 Crack growth rate, da/dN , related to cyclic J -integral range, ΔJ , and crack tip opening displacement range, $\Delta \Phi_t$.

Figures 5(a)~(c) show da/dN plotted against the cyclic J -integral range, ΔJ , in SNCM 439, S 35 C and SUS 304 steels. Since the da/dN versus ΔJ relation is hardly affected by the strain range and strain ratio controlled in the tests, all these data are marked

Fig. 2 The change in the crack opening load range ratio with an increase in the crack length under a displacement-controlled test condition

Fig. 3 The change in the crack opening load range ratio with an increase in the crack length under a load-controlled test condition

Fig. 4 The changes in the hysteresis loop shape and crack tip opening displacement, and development of ratchet deformation accompanied by extension of the crack in the load-controlled test with $R\sigma = -1.0$

by solid circles, not distinguishing any one from the other.

The data of da/dN versus ΔJ obtained under the load-controlled condition with various stress ratios nearly fall on the line constituted by those obtained under displacement-controlled condition in SNCM 439 steel. This is also the case for the load-controlled test with a stress ratio of $R\sigma = -1.0$ in S 35 C and SUS 304 steels. The data obtained in the load-controlled test with $R\sigma = 0$, however, tends to separate from the displacement-controlled test data toward the higher crack growth rate side in these two materials. It should be noted that such a separation occurs for data under the test condition satisfying $U \approx 1.0$. Thus, the limitation of the ΔJ -approach is revealed in the evaluation of the crack growth rate under a test condition accompanied by enhanced ratchet deformation, as suggested by Hoshide et al.⁽⁵⁾, Ando et al.⁽⁹⁾ and Jono et al.⁽¹⁰⁾. Hoshide et al.⁽⁵⁾ proposed that such a limitation of the ΔJ -approach appears when the maximum net cross-sectional stress, σ_{net}^{max} reaches the static yield stress, σ_y , as given by the following equation

$$\sigma_y = \sigma_{net}^{max} = \sigma^{max} / \{1 - (a/W)\}, \quad (4)$$

where σ^{max} is the maximum nominal stress imposed on the specimen. The data points satisfying the test condition of $\sigma_{net}^{max} \geq \sigma_y$ are marked with a slash in Figs. 5 to 8.

Ando et al.⁽⁹⁾ and Jono et al.⁽¹⁰⁾ successfully analyzed the acceleration of the crack growth rate due to

ratchet deformation using the concept of the maximum J -integral, J_{max} . This method, however, necessitated certain constants depending upon the specimen shape and the kind of materials, and included some ambiguities.

The progress of ratchet deformation is directly reflected on the crack opening and closure behaviors. Accordingly, the parameter expressed by the term

$$\Delta J / \{1 - (\phi_c^{min} / \phi_c^{max})\}^{0.2}, \quad (5)$$

was proposed in the present study, where ϕ_c^{min} and ϕ_c^{max} are the minimum and the maximum crack-center opening displacements. The crack growth rates are plotted against ΔJ modified by the crack opening displacements as in eq. (5) in Figs. 6(a)-(c). The data marked with a slash correspond to the data discriminated from others by the same mark as in Figs. 5(a)-(c). The slash-marked data, which are separated from the strain-controlled test data in Figs. 5(b) and (c), move into the band constituted by the strain-controlled test data in Figs. 6(b) and (c). Incidentally, crack closure occurs under the test condition where ratchet deformation does not develop greatly. In such a case, the proposed parameter is equal to ΔJ since ϕ_c^{min} becomes zero. Thus, it is evident from Fig. 6 that the parameter $\Delta J / \{1 - (\phi_c^{min} / \phi_c^{max})\}^{0.2}$ can be useful factor for evaluating the fatigue crack growth rate under the test condition accompanied by large-scale yielding and ratchet deformation.

Fig. 5 Crack growth rates related to cyclic J -integral range, ΔJ , in load- and displacement-controlled tests; a) SNCM 439, b) S 35 C and c) SUS 304 steels. The data pointed out by the arrows in Figs. (a)~(c) correspond to those in Figs. 6(a)~(c), respectively. The slashed data were obtained under a test condition of $\sigma_{net}^{max} \geq \sigma_y$.

The crack growth rates are plotted against the local deformation parameter, $\Delta\Phi_{t(200)}$, in Figs. 7 (a)~(c). The da/dN versus $\Delta\Phi_{t(200)}$ relations are denoted by the solid circles in the displacement-controlled test. Least-squares regression analyses were carried out on these data, and then the dotted

lines were determined. Meanwhile, the load-controlled test data are discriminated by open or semifilled circles, depending upon the stress ratios employed. The data points numbered as 1, 2, 3, and 4 in Fig. 7(c) deviate greatly from the displacement-controlled test data toward the higher crack growth

Fig. 6 Crack growth rates plotted against the modified ΔJ -parameter, $\Delta J / [1 - (\Phi_c^{\min} / \Phi_c^{\max})^{0.2}]$; a) SNCM 439, b) S 35 C and c) SUS 304 steels. The data pointed out by the arrows in Figs. (a)~(c) correspond to those in Figs. 5 (a)~(c), respectively. The slashed data were obtained under a test condition of $\sigma_{\text{net}}^{\max} \geq \sigma_y$.

Fig. 7 Crack growth rates plotted against the crack tip opening displacement range, $\Delta\Phi_{t(200)}$; a) SNCM 439, b) S 35 C and c) SUS 304 steels. The slashed data were obtained under a test condition of $\sigma_{\text{net}}^{\max} \geq \sigma_y$.

rate side in SUS 304 stainless steel. The numbers 1 and 2 are used for distinguishing the data under the test condition accompanied by well-developed ratchet deformation and the numbers 3 and 4 for distinguishing those under the gross-yield condition from the remaining data. Thus, the marked acceleration of the crack growth rate due to ratchet deformation and gross yielding is not controlled by the crack tip opening displacement range $\Delta\Phi_{t(200)}$ in some cases. In general, however, the correlation is quite good between da/dN and $\Delta\Phi_{t(200)}$. Least-squares regression analyses were carried out on both the load- and displacement-controlled test data, leaving the difference in the test mode out of consideration. The solid lines show the da/dN versus $\Delta\Phi_{t(200)}$ relations determined in this way in Figs. 7(a)~(c). A small difference is found in the slopes of the log da/dN versus log $\Delta\Phi_{t(200)}$ relations between the dotted and the solid lines; the slopes of the dotted and solid lines are 1.99, 2.95, and 2.12, and 1.72, 2.65 and 1.74 in SNCM 439, S 35 C and SUS 304 steels, respectively.

Incidentally, Hoshida et al.⁽⁶⁾ reported that the slopes of the log da/dN versus log $\Delta\Phi_{t(250)}$ plots were about 2.2~2.3 for carbon steels. Thus, the slopes in the log da/dN versus log $\Delta\Phi_{t(200)}$ plots obtained in the present study are fairly close to these values.

All the data included in Fig. 7 are presented together in Fig. 8, distinguished only by the kind of material. Almost all the data fall within the scatter band of a factor of two, and then the following em-

Fig. 8 The da/dN versus $\Delta\Phi_{t(200)}$ plots presented together for all materials tested; the test mode is not differentiated. The slashed data were obtained under a test condition of $\sigma_{net}^{max} \geq \sigma_y$

pirical equation is derived through a least squares regression analysis for the three kinds of materials,

$$da/dN = 10^{3.65} \{\Delta\Phi_{t(200)}\}^{1.98} \quad (6)$$

It is supposed that $\Delta\Phi_{t(200)}$ is proportional to $(J/K)^{1/(n+1)}$ in material characterized by the constitutive equation of $\sigma = K(\epsilon_p)^n$, provided that the HRR singular field holds at the position of 200 μm behind the fatigue crack tip. The crack tip opening

(a)

(b)

(c)

Fig. 9 The relationship between crack tip opening displacement range and cyclic J -integral range; a) SNCM 439, b) S 35 C and c) SUS 304 steels

Table Comparison between power exponent, q , of the equation, $\Delta\Phi_{t(200)} = A(\Delta J)^q$, and $1/(n+1)$ and $1/(n'+1)$, where n and n' are the strain hardening exponents of static and cyclic stress-strain curves, respectively

	SNCM439	S35C	SUS304
q	0.764	0.609	0.662
$1/(n'+1)$	0.862	0.840	0.885
$1/(n+1)$	0.942	0.823	0.835

displacement range, $\Delta\Phi_{t(200)}$, is related to the cyclic J -integral range, ΔJ , for SNCM 439, S 35 C and SUS 304 steels tested under the load- and displacement-controlled conditions in Figs. 9 (a)~(c). We can derive the following empirical equation from these figures, independent of the stress and strain ratios,

$$\Delta\Phi_{t(200)} = A(\Delta J)^q \quad (7)$$

The exponent in eq. (7), q , is compared with $1/(n+1)$ and $1/(n'+1)$ in Table (3), where n and n' are the strain hardening exponents of the static and cyclic stress-strain curves, respectively. Table (3) shows that the values of q are slightly small, compared with the values predicted from the HRR singular field at the static crack tip.

4. Conclusions

Through-thickness center-cracked specimens of JIS SNCM 439, S 35 C and SUS 304 steels were low-cycle fatigued under the test condition with mean stress and strain. Then, fatigue crack growth was investigated in terms of the elastic-plastic fracture mechanics. The main results obtained are summarized as follows.

(1) The crack opening load range ratio, U , decreases under the displacement-controlled test condition and increases to approach unity under the load-controlled one as the crack extends. Especially, in the former, this ratio plotted against half crack length reaches a stable state independent of the strain range and strain ratio controlled in the test as large scale yielding occurs.

(2) The J -integral range, ΔJ , does not control the acceleration of the crack growth rate due to ratchet deformation. Meanwhile, the crack growth rate plotted against the parameter modified in terms of the crack opening displacements:

$$\Delta J / \{1 - (\Phi_c^{\min} / \Phi_c^{\max})\}^{0.2},$$

forms a linear line on double logarithmic scales under all the test conditions including the one producing ratchet deformation, where $\Phi_c^{\min}$ and $\Phi_c^{\max}$ are the minimum and maximum opening displacements at the

center of crack.

(3) In general, the crack tip opening displacement, $\Delta\Phi_{t(200)}$, is a dominant factor for controlling the crack growth rate in low-cycle fatigue accompanied with various stress and strain ratios.

(4) A linear relation holds between $\Delta\Phi_{t(200)}$ and ΔJ on double logarithmic scales. The power exponents, q , in the equation of $\Delta\Phi_{t(200)} = A(\Delta J)^q$ are slightly smaller than $1/(n+1)$ and $1/(n'+1)$ predicted from the HRR singular field formed around the static crack tip, where n and n' are the strain hardening exponents of the static and cyclic stress-strain curves.

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