Analysis of the Localized Deformation of Sheet Metals Under Uniaxial Tension*

Manabu GOTOH**, Hayato AMAKI***
and Kôzô TANAKA**

The localized deformation of sheet strips of mild steel and aluminum with various thicknesses, subjected to uniaxial tension, are experimentally investigated by the use of the Moiré fringe method. These results are compared with theoretical ones calculated by the use of the elastic-plastic finite element method, considering the effect of large deformation on the basis of J_2 -flow theory and Gotoh's vertex theory. The upper limit of thickness to the development of the shear-band type local necking is found to be dependent on the material properties. Gotoh's vertex theory predicts localized necking in a diffusely necked thin strip, thus clearly verifying its effectiveness over a large deformation range.

Key Words: Plasticity, Strain Localization, Uniaxial Tension, Moiré Pattern Method, Nonuniform Deformation, Mild Steel, Thickness Dependence, Large Deformation, FEM

1. Introduction

Material behavior under uniaxial tension is the most fundamental pattern of deformation, however, few detailed studies have been so far carried out involving the effect of such complicated factors, as the discontinuous deformation in small strain regions immediately after the yielding of mild steel, and nonuniform deformation in large strain regions after the onset of diffuse necking. Nonuniform, large deformation in such necking regions gives basic and important information to instability problems such as bifurcation analysis, and embodies the complicated behavior of any material with arbitrary configuration under other types of loadings. Moreover, the investigations on the thickness-dependence of nonuniform deformation due to the development of necking and on the effectiveness of the constitutive equation used in its

analysis will give a clearer understanding of the process of strain localization.

In this paper, we make an attempt to establish the analytical basis for arbitrary deformations of sheet metals, by focusing our attention on the clarification of complicated phenomena appearing in the sheet specimens deformed under uniaxial tension. To this end, sheet strips of mild steel and aluminum with various thicknesses subjected to uniaxial tension were studied to observe how deformation develops, and the strain distribution pattern developing after the onset of diffuse necking was analyzed by using the Moiré fringe method. The effect of sheet thickness on the appearance of the shear-band type local necking was also investigated. Furthermore, a numerical simulation of necking deformation for the case of plane stress state by the use of the elastic-plastic FEM (finite element method) was performed considering the large deformation effect, and compared to the experimental result. In the calculations, the J_2 -flow theory ($J_2 F$) was used as well as a new vertex hardening theory previously proposed by one of the authors (Gotoh)⁽¹⁾⁻⁽³⁾ as the constitutive equation to verify the effect of the vertex on inducing the shear-

* Received 12th March, 1988. Paper No. 86-1230 A

** Department of Mechanical Engineering, Gifu University, 1-1 Yanagido, Gifu, 501-11, Japan

*** Daidô Steel Co., Ltd., Ishihara 500, Shibukawa, 377, Japan

band type local necking.

2. Experimental Method

In order to investigate the dependence of localized deformation under uniaxial tension on sheet thickness, uniaxial tension tests were performed on sheet strip specimens with the same width of 12.5 mm and various thicknesses of 1, 2, 3, 4 and 6 mm. The material employed was a drawn low-carbon steel bar (S 20 C), whose chemical composition is given in Table 1. The tested strip specimens were made by machining along the drawing direction of the bar, to obtain samples of a rectangular cross-sectional shape and the various above mentioned thicknesses, and then were annealed in an electric furnace at 780°C for 1 hour. Table 2 summarizes the uniaxial tensile properties of the annealed S 20 C strips for each thickness.

First, following the procedure of the Moiré method, the standard square grids were initially printed on the surface of the undeformed specimen, and the plate with the reference square grids was mounted in the Moiré camera apparatus. The spacings of all grids were 250 μ /in. Then, the superposition of the reference and specimen grids was realized on the surface of the analyzer plate, onto which the image of specimen grids was projected through the lens of the Moiré camera apparatus. Thirdly, the initial Moiré fringes were made to appear by giving an appropriately slight mismatch and misalignment (θ_0) between the above reference and specimen grids. When the external load was applied, the specimen grids deformed together with the underlying specimen, therefore we could take photographs of a series of Moiré patterns to show the development of necking in the strip specimen, produced by the interference of

the reference and the deforming specimen grids. The negative films of the fringe patterns thus obtained at several stages of deformation were enlarged and printed for the numerical fringe analysis. Then, by measuring the fringe spacings of these enlarged Moiré patterns, we could determine the two-dimensional strain distribution along the surface of the deformed specimen at various stages of deformation by the use of the following formula of finite Euler-Almansi strain derived previously by the authors⁽⁴⁾.

$$\left. \begin{aligned} e_{11} &= p_0 \{ 1/\delta_{yy} - 1/(\delta_{yy})_0 \} - p_0^2 \{ [1/\delta_{yy} - 1/(\delta_{yy})_0]^2 + \{ -1/\delta_{xy} + 1/(\delta_{xy})_0 \}^2 \} / 2 \\ e_{22} &= p_0 \{ \pm 1/\delta_{xx} - 1/(\delta_{xx})_0 \} - p_0^2 \{ [\pm 1/\delta_{xx} - 1/(\delta_{xx})_0]^2 + \{ 1/\delta_{yx} - 1/(\delta_{yx})_0 \}^2 \} / 2 \end{aligned} \right\} \quad (1)$$

Equation (1) is for the case of the initial Moiré pattern with right-up fringes of X -Moiré and right-down fringes of Y -Moiré ($\cos \theta_0 > p_0/p$, $+\theta_0$), and the strains e_{11} and e_{22} are the axial and lateral components of Euler-Almansi's strain tensor, respectively. The constants p and p_0 are the pitches of the reference grids and the initial specimen grids, respectively. The symbol δ_{ij} indicates the distance between two adjacent fringes of the I -Moiré pattern with respect to the j -direction and the subscript 0 denotes the values measured for the initial Moiré pattern in the undeformed state. Furthermore, in the above equations, the upper of the double signs is valid for the case of the right-up fringes of X -Moiré in the deformed state under loading, and the lower for right-down fringes of Y -Moiré.

In addition to the above tested material of S 20 C, we also carry out the similar experiments for aluminum and another mild steel, S 25 C, in order to investigate how the thickness-dependence of local necking phenomenon is affected by material properties.

3. Numerical Simulation

The numerical analysis of localized deformation of a sheet metal under uniaxial tension was performed by making use of FEM on the assumption of the plane

Table 1 Chemical composition of an annealed S 20 C sheet

C	Si	Mn	P	S	Cu	Ni	Cr
0.20	0.23	0.45	0.020	0.014	0.010	0.04	0.04

Table 2 Uniaxial tensile properties of annealed S 20 C sheet

Thickness	Direction of fracture to the axis	Direction of localized necking to the axis	n-value	r-value	Elongation (%)	Ultimate tensile strength (MPa)
1 mm	72.7°	71.5°	0.20	1.7	25.6	383
2 mm	73.5°	72.7°	0.22	1.6	34.7	383
3 mm	—	—	0.20	1.8	37.7	389
4 mm	—	—	0.23	1.8	39.9	396
6 mm	—	—	0.26	1.5	40.8	380

stress state. In the calculations, we utilized the Mises yield criterion, the hardening law of Eq. (2) which will be mentioned later, and two kinds of constitutive equations. One of the adopted constitutive equations is the J_2 -flow theory of plasticity (J2F), and the other one is a linearized constitutive equation of vertex theory⁽¹⁾⁽²⁾ (in what follows, M. G. C. E.) with vertex effect on the subsequent yield surface at the loading point, previously proposed by one of the authors (Gotoh). Furthermore, we adopted the large deformation formulation of FEM to account for large strains and large rotations (see e. g., (5)), utilizing the linear triangular elements.

The work-hardening characteristic equation used here is expressed as follows:

$$\left. \begin{aligned} \bar{\sigma} &= E\bar{\varepsilon} & ; & \bar{\sigma} \leq \sigma_Y \\ \bar{\sigma} &= C(\varepsilon_0 + \bar{\varepsilon})^n & ; & \bar{\sigma} > \sigma_Y \end{aligned} \right\} \quad (2)$$

where $\bar{\sigma}$ is the equivalent stress, $\bar{\varepsilon}$ is the equivalent strain, and σ_Y is the initial yield stress for uniaxial tension. Material constants employed are the average values of those obtained by the uniaxial tensile tests of S 20 C steel sheets of various thicknesses used for the experiment; they are as follows: $C=1\,005$ MPa, $n=0.22$, $\varepsilon_0=0.001\,28$, $\sigma_Y=232$ MPa, $E=181$ GPa, and Poisson ratio $\nu=0.3$.

In addition to the above constants, for the case of M. G. C. E., we must specify the value of the constant ρ which governs the evolution rate of the vertex on the subsequent yield surface accompanying with the progress of plastic deformation. The half angle of the pointed vertex of the current loading surface denoted by Θ_0 is formulated as follows⁽³⁾:

$$\left. \begin{aligned} \Theta_0 &= \pi/2 - \rho \bar{\varepsilon}^2 & ; & \bar{\varepsilon} = \int d\bar{\varepsilon}^p \\ d\bar{\varepsilon}^p &= \text{equivalent plastic strain increment} \end{aligned} \right\} \quad (3)$$

In this study, we adopted the value of ρ to be 0.5 for convenience.

4. Results and Discussions

4.1 Dependence of necking pattern on sheet thickness

It is easily seen from the tensile properties of S20C, as shown in Table 2, that the total elongation of this material tends to become larger with increasing thickness. Also, it is confirmed for the n -value, the r -value and the tensile strength that the individual values obtained for each sheet thickness are almost comparable, with a slight scattering. Through the experimental observations on the necking behavior of all specimens, the following dependence of necking phenomenon on the sheet thickness has been verified to exist.

First, from experiments on the mild steel of S20C, the shear-band type local necking is observed to occur, the orientation being inclined by 72.7° and 73.5°

to the tensile axis for the specimens of 1 and 2 mm thickness, respectively. On the other hand, for the specimens of 3, 4 and 6 mm thickness, the so-called local neck does not appear. Even in the case where the shear-band type local neck appears, its inclination angle to the tensile axis is increased and the band of necking itself becomes more blunt as the thickness increases.

The local neck is said to be a typical phenomenon in thin plates. Hill's classical theory predicts that it will appear in the direction of $54^\circ 44'$ to the tensile axis⁽⁶⁾. However, in this experiment, a much larger inclination angle was observed. According to the experimental results obtained for the specimens much thicker than 3 mm, the effect of the sheet thickness on the strain localization could not be neglected, and thus the plane stress state could no longer be assumed. This necessitated that a three-dimensional analysis was carried out for such cases, though it was beyond the scope of the present study. Moreover, it should be noted that the localized necking was influenced by the sheet thickness in the following manner even for the case of thinner sheets such as those of 1 and 2 mm thickness. From the experimental observation, it was found that the annealed steel of S 20 C employed here had a thickness limit between 2 and 3 mm with 12.5 mm width to the appearance of the local neck (the ratio of the thickness to the width $=0.16\sim0.24$). In similar experiments with other materials, it was also demonstrated that the limiting thicknesses for the mild steel of S 25 C and soft aluminum were found to be between 3 and 4 mm (the above ratio $=0.24\sim0.32$). These results reveal that the limiting thickness of sheet metal to the occurrence of localized necking depends a fair amount on the material properties.

Next, in order to look into the characteristic features of deformation, a few examples of the load-displacement curves are shown in Fig. 1 for the mild steel S 20 C with thickness of 1, 3 and 6 mm. It can be seen by comparison of the above three curves that the deformation patterns are almost the same up to the maximum load point. After the attainment of the maximum load, however, the thicker sheet undergoes larger deformation before the steep drop in the load occurs, and thus attains greater total elongation. This fact is also realized in Table 2 where we see that the total elongation increases with increasing sheet thickness. From the comparison of such elongations obtained at a similar stage of the localized necking process (e. g., when the ratio of the minimum width in the necked portion to the initial width of the specimen is about 0.78), we can also see that the elongation becomes much larger with increasing thickness, though the specified load ($P/P_{\max}$) takes almost a

constant value. These results mean that the greater the thickness, the more stably the sheet specimen is elongated with the developing neck, exhibiting greater deformability or ductility owing to the three-dimensional effect of the greater thickness.

(a) Thickness of 1 mm (S 20 C)

(b) Thickness of 3 mm (S 20 C)

(c) Thickness of 6 mm (S 20 C)

Fig. 1 Load-displacement curves (experiment)

Furthermore, it was found that in the case of 1 and 2 mm thickness, separation of the specimen due to fracture occurred along a specific direction relative to the width direction, which exhibited the role of the shear-band type local neck mentioned above. On the other hand, separation of the specimen with more than 3 mm thickness no longer showed any inclination to the width's direction. Further observation on the lateral edges of the fractured specimens with 4 mm or less thickness showed that the separation took place along a plane inclined at a certain angle to the thickness direction. However, it was seen that the specimen with 6 mm thickness experiences the cup-and-cone-type fracture observed commonly in the standard tensile test of a round bar.

4.2 Strain distribution and its progression

Figures 2-5 show a few examples of the strain distributions around the necked portions, which are determined by the Moiré fringe method with respect to the mild steel (S 20 C), for a specimen of 1 mm thickness as a typical example which breaks upon the appearance of the shear-band type local neck, and for the specimen of 4 mm thickness as an example which does not exhibit such a necking mode.

Here, we employ the Euler-Almansi's strain e as mentioned earlier. As can be seen from Figs. 2-5, the contours of equi-strain are drawn in intervals of approximately .02 which is considered to be fine enough to see the detailed strain distribution in such a large strain region as the necking portion. It is noteworthy that we were able to measure so large a strain

(a) $B/B_0=0.828$, $U/L_0=0.231$, $P/P_{max}=0.985$ (b) $B/B_0=0.758$, $U/L_0=0.187$, $P/P_{max}=0.917$

Fig. 2 Contours of axial strain e_{11} around the necked portion (thickness = 1 mm; experiment)

as up to around 0.7, though it is not illustrated here. Some examples of the photographs of Moiré patterns are shown in Figs. 6 and 7, corresponding to each deformation state in Figs. 2-5, where the right-hand and left-hand photographs in each figure show X -Moiré and Y -Moiré patterns, respectively.

Experimental observation shows that the diffuse neck development accompanied with the change of strain distribution inevitably causes high strain in the necking region. As the deformation proceeds, further neck development leads to the progressive strain

concentration at the central portion of the specimen around the neck region, remaining in the region outside the neck to undergo very small and decreasing strain increments. We also notice from Figs. 2-5 that low strain regions exist slight to the left and right of the place where the severely diffuse necking develops, and that in the early stage of deformation, the higher e_{11} -strain region spreads along the width direction and the higher e_{22} -strain region along the axial direction.

As for the early diffuse necking, the distribution patterns of the strains e_{11} and e_{22} have little dependence on the thickness of the specimen. However, as the necking deformation proceeds with further extension testing, the difference between the strain distributions in the thinner specimens (1 and 2 mm thickness) and those in the thicker specimens (3, 4 and 6 mm thickness) becomes more distinguishable, owing to the fact that the shear-band type local neck develops only in the thinner sheets. Namely, the distributions of both e_{11} and e_{22} in the thinner specimens clearly exhibit the high-level contours concentrating in the bands lying along the directions inclined by a certain angle to the tensile axis as the deformation proceeds (see Figs. 2 and 4). After further deformation as in the visible local necks evolve as two intersecting bands, the letter X , with the angles given in Table 2; then fracture occurs along one of the two bands, though we cannot obtain the corresponding Moiré fringes because of the difficulties resulting from such severely concentrated deformation.

The strain distributions and thus the Moiré patterns are not necessarily symmetric with respect to the X - and Y -axes, primarily because the initial plastic properties of the material are not completely homogeneous throughout the specimen, and secondly

(a) $B/B_0=0.875$, $U/L_0=0.231$, $P/P_{\max}=0.985$ (b) $B/B_0=0.775$, $U/L_0=0.259$, $P/P_{\max}=0.933$

Fig. 3 Contours of axial strain e_{11} around the necked portion (thickness = 4 mm; experiment)

(a) $B/B_0=0.828$ (b) $B/B_0=0.758$

Fig. 4 Contours of transverse strain e_{22} around the necked portion (thickness = 1 mm; experiment)

(a) $B/B_0=0.875$ (b) $B/B_0=0.775$

Fig. 5 Contours of transverse strain e_{22} around the necked portion (thickness = 4 mm; experiment)

because the initial geometry of the specimen is not complete, and the loading is also not completely symmetric.

4.3 Results of numerical simulation

In what follows, the numerical results are shown only for the specific case of the strip sheet with 12.5 mm width, 37.5 mm length and 1 mm thickness subjected to uniaxial tension under the condition of built-in ends. Therefore, the present numerical analysis is exclusively aimed at simulating the deformation process in the developing diffuse neck. Figures 8-12 illus-

trate the results obtained numerically by assuming the plane stress state using both J 2 F and M. G. C. E. as the constitutive equations.

First, it should be noted that the shear-band type local neck is clearly simulated, as seen in Fig. 8, by the use of M. G. C. E., whereas this phenomenon does not appear, at least as far as the calculation has run, if we use the conventional J 2 F. In the present calculation, the local neck is inclined about 60° from the tensile axis.

Next, Figs. 9 and 10 show the contours of the

Fig. 6 Moiré fringe patterns appearing on the necked specimen (thickness = 1 mm)

Fig. 7 Moiré fringe patterns appearing on the necked specimen (thickness = 4 mm)

axial component e_{11} of Euler-Almansi strain for J 2 F and M. G. C. E., respectively. The contours predicted by J 2 F in 9(a) are found to compare well with those determined experimentally for the specimen of 1 mm thickness at the same ratio B/B_o , which is the ratio of the minimum width in the necked region to the initial width of the specimen. On the other hand, the two sets of contours of e_{11} predicted by using M. G. C. E. are illustrated at the two deformation levels before and after the local neck appears. For the case of M. G. C. E., the local neck develops before the ratio B/B_o reduces to the value at which the diffuse necking occurs in the experiment, and thereby further deformation then localizes within the local neck band, leaving the region other than the band and the central portion of the specimen to be almost free from further straining. For the case of J 2 F, the value of e_{11} becomes much larger in the region around the central portion of the specimen than in other regions, as seen

Fig. 8 Onset and development of a local neck of shear-band type (calculation; M. G. C. E.)

in Fig. 9, and is found to be approximately equal to that obtained by the experiment. However, because of the absence of the local neck band, the strain contours do not show such inclination as those appearing in the experimental observation. On the other hand, for the case of M. G. C. E., although the pattern of strain distribution is similar to that for J 2 F, the strains are largest at the central portion of the sheet and along the local neck band. In addition, lower strain regions exist on either side of the above central portion. These features of the strain distribution are in reasonable agreement with those observed in the experiment. However, the strains in the region outside the local neck band are substantially smaller than those measured experimentally. This is because the numerically predicted local neck band begins to develop at a much earlier stage of deformation compared to the experimentally observed one. We can make the numerical results of strain much closer to the experimental ones

(a) $B/B_o=0.782$, $U/L_o=0.184$, $P/P_{\max}=0.935$

(b) $B/B_o=0.764$, $U/L_o=0.197$, $P/P_{\max}=0.918$

Fig. 9 Contours of axial strain e_{11} (calculation; J 2 F)

(a) $B/B_o=0.905$, $U/L_o=0.099$, $P/P_{\max}=0.986$

(b) $B/B_o=0.864$, $U/L_o=0.131$, $P/P_{\max}=0.699$

Fig. 10 Contours of axial strain e_{11} (calculation; M. G. C. E.)

by a more appropriate choice of the value of factor ρ in Eq. (2) which governs the evolution rate of a vertex on the subsequent yield surface. It is also found that the transverse strain e_{22} shows similar features as those for e_{11} , though the results of strain e_{22} are not presented here.

Furthermore, the contours of equivalent strain increments were obtained numerically for both J 2 F and M. G. C. E. at the same deformation levels as those in Figs. 9 and 10. From the results of J 2 F, although they are not presented here, it has been confirmed that the contours of the largest equivalent strain increment exist in the central portion of the specimen, and that the lower increment regions appear on either side of the above central portion. In addition, it was seen that at the deformation level comparable to that in Fig. 9(b), as the deformation proceeds, the development of the diffuse neck results in the localization of the equivalent strain increment within the central portion of the specimen, limiting the strain in the region outside to a very small extent. On the other hand, Fig. 11(a) for M. G. C. E. shows that the equivalent strain increment has already become greater in the region where the local neck band will develop (see Fig. 11(b)) than in other regions, though the deformation level is still comparable to that in Fig. 10(a) where the local neck band has not yet developed clearly. After the formation of the band, the equivalent strain increment, as well as the strain e_{11} mentioned previously, becomes increasingly prominent along the band and in the central portion of the specimen, limiting either side of this central portion to smaller strain increments as seen in Fig. 11(b). This clearly reveals that a strong strain localization occurs along the local neck band as the deformation proceeds.

Fig. 11 Contours of equivalent strain increment (calculation; M. G. C. E.)

Finally, we illustrate the load-displacement curves in Fig. 12 obtained numerically for both J 2 F and M. G. C. E., where load and displacement are normalized by each maximum load and the initial longitudinal gage length, respectively, for comparison with the experimental results. It can now be readily seen that both load-displacement curves attain maximum values with higher loads and at much smaller displacements compared to the experimental results shown previously in Fig. 1(a). These discrepancies are of course due to the discrepancies of the boundary condition and the specimen geometry adopted in the calculation from those in the experiment. Apart from this point, we should note that there exists a significant difference between the above two predicted curves. That is to say, for J 2 F, the load decreases slowly with increasing displacement after the maximum load is reached, while M. G. C. E. predicts a rapid drop in the load owing to the local neck. This rapid drop is very similar to that seen in Fig. 1(a) of the corresponding experiment, confirming again that the formation of the local neck following the developed diffuse necking causes a severe strain localization in the very narrow band region leading to immediate fracture.

In the numerical simulation, we assumed the material to be isotropic. Furthermore, the value of ρ used for M. G. C. E. was not necessarily appropriate. These factors would have some effect on the more detailed information such as the maximum load point and the angle of the local neck to the tensile axis and so forth. However, we may say that the predicted results are in relatively good agreement with the experimental results. Above all, it is noteworthy that the local neck band is clearly simulated by the calculation, making use of M. G. C. E. instead of the conventional J 2 F.

Fig. 12 Load-displacement curves (calculation)

5. Conclusions

In order to investigate the dependence of the strain localization on the sheet thickness, by the use of the Moiré method, we observed the strain development in a large deformation range after the onset of diffuse necking in a sheet metal specimen under uniaxial tension. A numerical simulation was also performed on the neck development by the use of FEM employing two kinds of constitutive equations. The main conclusions derived from the experimental and numerical results are as follows:

(1) From the experimental results for the specimens of S 20 C, S 25 C and soft aluminum with different thicknesses, it was found that the upper limits of the thickness of the sheet specimens, in which the shear-band type local necks were observed to lie between 2 and 3mm (the ratio to the sheet width = 0.16~0.24) for the case of S 20 C, and between 3 and 4 mm (the above ratio = 0.24~0.32) for the case of S 25 C and soft aluminum. Consequently, it seems reasonable to conclude that the assumption of the pure plane stress state is valid only for the sheet with the thickness/width ratio smaller than a certain value within the range recorded above. We can also conclude that the limit thickness/width ratio to the evolution of the shear-band type local neck depends on the material properties.

(2) By the use of the Moiré method, we could draw in detail the strain distributions in the necked sheet after the onset of the diffuse necking. In this study, the longitudinal and transverse components of Euler-Almansi strain were measured up to the maximum value of around 0.7, and the strain contours at about 0.02 intervals were illustrated in the figures.

(3) The strain distributions predicted numerically using the conventional J_2 -flow theory on the

assumption that the plane stress state agrees well with the experimental results at least within the deformation range before the shear-band type local neck takes place in the experiment. In this case, however, we could not find the formation of such a local neck in the calculation. In contrast to this, the numerical results basing M. G. C. E. predicts the strain localization corresponding to the shear-band type local neck observed in the experiment of the thin sheets. This verifies that Gotoh's vertex hardening theory is effective over a wide range of deformations including special plastic flows such as local necking or shear-band formation.

References

- (1) Gotoh, M., A Class of Plastic Constitutive Equations—I. General Theory, Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 47, No. 424, A(1981), p. 1389.
- (2) Gotoh, M., A Class of Plastic Constitutive Equations—II. Discussions on the Simplest Form, Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 48, No. 428, A(1982), p. 493.
- (3) Gotoh, M., A Simple Plastic Constitutive Equation with Vertex Effect, Eng. Fract. Mech., Vol. 21, No. 4 (1985), p. 673.
- (4) Gotoh, M., Tanaka, K. and Amaki, H., On the Measurement of Finite Strain by the Moiré Method, Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 54, No. 498, A(1988), p. 360.
- (5) McMeeking, R. M. and Rice, R., Finite Element Formulations for Problems of Large Elastic-Plastic Deformation, Int. J. Solids Struct., Vol. 11 (1973), p. 601.
- (6) Hill, R., On the Discontinuous Plastic States, with Special Reference to Localized Necking in Thin Sheets, J. Mech. Phys. Solids, Vol. 1, No. 1 (1952), p. 19.