Effects of Sheet Thickness on the In-Plane Stretch Forming Limit in an Aluminum Sheet*

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An investigation concerning the effects of sheet thickness on the in-plane stretch forming limit in 1100 aluminum sheets with thicknesses of 0.5 to 1.0 mm, prepared by electropolishing, has been carried out experimentally and analytically. Development of surface roughness is represented by a linear relation between surface roughness and effective strain. Little thickness and strain-ratio dependencies of the surface roughness are observed. However, the forming limit which depends on the thickness is observed, and less thickness results in a decreased forming limit. Good agreement between measured and theoretical strains from modified M-K theory is obtained. The thickness dependence of a half angle of pointed vertex, θ_0 , is introduced in Gotoh theory, assuming that the onset of localized necking in a sheet is the result of a bifurcation from a state of uniform deformation. The values of θ_0 are in the range between 70 and 90 degrees in all the materials examined and satisfy the relation $\theta_0 = \pi/2 - q_1 \bar{\epsilon}^{q_0}$, in which the coefficients q_0 and q_1 are material constants.

Key Words: Forming, Plasticity, Sheet Metal, Thickness Dependence, Modified M-K Theory, Gotoh Theory

1. Introduction

It is known that the in-plane stretch forming limit of a ductile metal depends on the sheet thickness⁽¹⁾⁽²⁾. The modified M-K theory⁽³⁾ which deals with the growth of inhomogeneity, such as surface roughness, is one of the theories in which the thickness dependence can be directly taken into consideration. In fact, the modified M-K theory gives a good agreement between experiment and theory, and also gives a prediction that the reduction of thickness results in a decreased forming limit, as reported previously⁽⁴⁾. Nevertheless, from the experimental viewpoint, this thickness dependence must be confirmed by performing the separate experiments to understand the contribution of each factor included in the theory. On the other hand, the theory proposed by Gotoh⁽⁵⁾ (Gotoh's theory) is one of the theories available to explain the thickness dependence of the forming limit by considering the thickness dependence of the factors involved in

the theory. Gotoh's theory assumes that the onset of localized necking comes from a bifurcation from a state of uniform deformation as in the Stören-Rice theory⁽⁶⁾ and that the forming limit is independent on the work-hardening exponent. However, since no explicit explanations for the thickness dependence have yet been made in his theory, it is worthwhile to discuss the thickness factors in Gotoh's theory.

The sheet thickness can be changed easily by rolling, but, in a cold-rolled sheet, it is necessary to relieve the residual stress and to consider the change of relative grain size, while in a hot-rolled sheet, it is necessary to consider the change of grain size due to recrystallization and the growth of texture. As a result, the grain size dependences of yield and flow stresses known as the Hall-Petch relation are unavoidably introduced. Furthermore, it is also reported that the surface roughness growth corresponding to deformation of ductile thin sheets has a grain size dependence. Therefore, it is not appropriate to evaluate the thickness dependence of the forming limit by using a sheet prepared by rolling. Consequently, the electropolishing technique was introduced in the present study as an excellent method to prepare the sheets

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of different thicknesses without any change of the grain structure.

The effects of sheet thickness on the in-plane forming limit were examined using aluminum sheets of different thicknesses prepared by the unique technique. The results were analyzed by the modified M-K theory which estimates the thickness effect directly. Also investigated was the thickness dependence of coefficients in the Gotoh's theory and the effect of work-hardening law equations on this theory. Note that the similar effect on the modified M-K theory has already been reported⁽⁴⁾.

2. Experiment and Analysis

2.1 Experimental procedure

The thickness of the commercial 1100 aluminum of nominal thickness 1.0 mm was thinned to 0.9, 0.7, and 0.5 mm by electropolishing, and standard JIS-6 tensile specimens of the respective thicknesses were machined. The tensile properties were measured using an Instron-type tensile test machine. It was found that the Hollomon law equation (n -th power law) was the best-fit work-hardening law equation to the tested material:

$$\bar{\sigma} = c \bar{\epsilon}^n \quad (1)$$

Although a slight variation between the stress-strain relations was observed, no thickness dependence of the stress-strain relation was observed in the sheet thickness range, similar to the results reported⁽⁸⁾. The average of the stress-strain relations was presented as,

$$\bar{\sigma} = 176 \bar{\epsilon}^{0.254} \quad (\text{MPa}).$$

Since the anisotropy coefficient, r , of the average value of three directions in the sheet was 1.03, the materials tested was assumed isotropic and to obey the Mises yield criterion. The in-plane stretch forming limit was measured by means of the Marciniak biaxial stretching test using a double-action hydraulic press machine equipped with the test device with the 80 mm blank radius and 36 mm punch radius. The stretch forming tests were performed at three constant strain ratios; i. e., $a = d\epsilon_2/d\epsilon_1 = 1.0, 0.6$, and 0.3 , where $d\epsilon_1 \geq d\epsilon_2 > 0 > d\epsilon_3$. Surface roughness represented by the maximum roughness height, R , was measured along the major strain direction by a roughness tester. The mean grain diameter along the sheet thickness direction was also measured by optical microscope and was about 17 μm .

2.2 Analytical procedure

The theoretical forming limit was calculated by the modified M-K theory⁽³⁾, in which the growth of imperfection such as surface roughness was taken into account. Since the details of the modified M-K theory have been reported in the previous paper⁽⁴⁾, only the

basic part of the theory is summarized here.

Let t be the sheet thickness and t_0 be its initial value, then using subscripts F and G attached to the parameters corresponding to the outside and inside of the groove, respectively, the inhomogeneity factor, f , is obtained by using Eq. (2) from the ratio of the minimum to maximum thickness of the sheet at the instability condition.

$$f = \frac{t_{0G}}{t_{0F}} = \frac{\sigma_{1F}}{\sigma_{1G}} \exp(\epsilon_{3F} - \epsilon_{3G}). \quad (2)$$

The compatibility and equilibrium equations are:

$$d\epsilon_{2F} = d\epsilon_{2G}, \quad (3)$$

and

$$t_F \sigma_{1F} = t_G \sigma_{1G}, \quad (4)$$

respectively. Therefore, applying the above equations to the post-diffuse necking condition, the forming limit strain is obtained from a set of differential equations derived from Eqs. (3) and (4).

In the Gotoh's theory⁽⁵⁾, on the other hand, the constitutive equation can describe a vertex-formation on the yield locus and the growth of the pointed vertex with plastic deformation. In addition, it is assumed in his theory that the inception of ductile fracture in solids is the result of the bifurcation from a state of uniform deformation by Stören and Rice theory (S-R theory)⁽⁶⁾. One of the advantages of Gotoh's theory is that the half angle of the pointed vertex, θ_0 , does not depend on the work-hardening exponent, n . Therefore, the forming limit strain is different, depending on the θ_0 value, even if the mechanical properties such as flow curve, yield stress, uniform elongation, etc. are the same. Using the Mises yield criterion, the ϵ_1 value of the forming limit is given by:

$$\epsilon_1 = \frac{n}{2(2+a)} \left\{ 4 + \frac{3(1-gn)a^2}{gn(1+a+a^2)} \right\}, \quad (5)$$

where,

$$g = \cos \theta_0 / \{n(1 + \cos \theta_0)\}. \quad (6)$$

θ_0 is given by the function in its simplest form:

$$\theta_0 = \pi/2 - \rho \bar{\epsilon}^2, \quad (7)$$

where, ρ is a positive constant and determined from the relation between Eqs. (6) and (7) using a g value which satisfies the consistency of the experimental value of ϵ_1 with the value by Eq. (5) in equibiaxial tension ($a=1$). Therefore, the theory and experiment agree precisely at $a=1$ of the forming limit diagram. In other words, when $g=1$, Gotoh's theory coincides with the S-R theory.

3. Results and Discussion

3.1 Thickness dependence of surface roughness growth

The surface roughness growth shown in Fig. 1 is expressed by the relation between surface roughness and effective strain. It has already been pointed out⁽¹²⁾

that the surface roughness growth could not be expressed only by a function of effective strain. In the present investigation too, the strain ratio dependence of the relation between surface roughness and effective strain was not observed. The same tendency has been reported also in some other papers⁽³⁾⁽¹³⁾. The surface roughness, R , itself can be expressed by the linear function of $\bar{\epsilon}$ (Fig. 1) :

$$R = \alpha \cdot \bar{\epsilon} + R_0, \quad (8)$$

where, α is an incremental factor of the surface roughness and R_0 is the initial surface roughness. As shown in Fig. 1, the α value is slightly varied with the initial sheet thickness but the R_0 value is about $1.9 \mu\text{m}$ regardless of the initial sheet thickness.

The experimental α value was 35.8, 36.8, 36.0, and $38.4 \mu\text{m}$ for $t_0 = 1.0, 0.9, 0.7$, and 0.5 mm , respectively. These values of α seem to increase slightly with the decrease in sheet thickness, but the tendency is not so clear that one can assume α as constant. Therefore, the analysis was done using $\alpha = 36.8 \mu\text{m}$ which is an average of the above four values. Regarding this point, since the mean grain diameter of the sheets is $17 \mu\text{m}$, the least number of grains contained in the thickness direction is more than 30 even for the thinnest sheet of $t_0 = 0.5 \text{ mm}$. Therefore, it is thought that the difference in grain constraints for slip deformation accompanying plastic deformation cannot be recognized. This is the reason why the effect of sheet

Fig. 1 Relation between surface roughness and effective strain

thickness on the α value is not observed. Consequently, the virtual difference among the prepared sheets is only the initial sheet thickness, so that the net thickness dependence on the forming limit can be examined with great accuracy. When the average value of $\alpha = 36.8 \mu\text{m}$ is used, the relation between inhomogeneity factor and the strain ratio, which is calculated from Eq. (2), indicates that the inhomogeneity factor decreases with a decrease in initial sheet thickness.

3.2 Thickness dependence of modified M-K theory

Figure 2 shows the comparisons of experimental forming limits (plotted points) with theoretical values (solid line) predicted by the modified M-K theory in which the inhomogeneity factor calculated from Eq. (2) for $\alpha = 36.8 \mu\text{m}$ is used. Although there is a slight scatter, the experimental forming limits decrease with a decrease in the initial sheet thickness, as predicted by the modified M-K theory, and good agreement between the theory and experiments is observed. Since the factors in the theory, such as surface roughness growth, n -value, etc., with the exception of the initial thickness, can be assumed to be the same, as mentioned previously, the initial sheet thickness is the only variable parameter affecting the forming limit. Therefore, the difference of the initial sheet thickness is the origin of variation of the forming limits.

The fracture profiles of the $\epsilon_1 - \epsilon_2$ plane are nearly chisel-point type formed by continuous shear deformation on the two cross planes. The measured thickness at the fractured portion by necking is thinner than the value calculated by the modified M-K theory under the condition that the necking portion reaches the plane-strain condition and no more deformation occurs in the uniform thickness portion. These results agree with the assumption of the modified M-K theory as in the previous results⁽¹⁴⁾.

3.3 Thickness dependence of Gotoh's theory

Figure 3 shows the comparison of experimental

Fig. 2 Comparison of the experimental forming limits with the results of the modified M-K theory

forming limits (plotted points) with theoretical values by Gotoh's theory (solid line) using Eqs. (5)–(7). Differences between the theory and the experiment are not observed for the initial thickness of 1.0 and 0.9 mm. However, the experimental values at $a=0.3$ and 0.6 become larger than the theoretical values in accordance with the decrease in the initial thickness. Comparing the theoretical forming limits with the diffuse necking strain⁽⁹⁾ shown in Fig. 2 by a single dotted line, the theoretical value in the plane-strain side is less in the case of $t_0=0.5$ mm.

In Fig. 4, the growth of the pointed vertex corresponding to the experimental forming limits is compared with the relation between a half angle of the pointed vertex, θ_0 , and the effective strain, $\bar{\epsilon}$. The solid line in Fig. 4 is a theoretical one obtained by Eq. (7) and the plotted points are the experimental values. It is observed that θ_0 values decrease with increasing $\bar{\epsilon}$, but its values are only in the range from 70 to 90 degrees. This fact suggests that the development of the pointed vertex is relatively small. The ρ value of Eq. (7) shown by the solid line in Fig. 4 is 19.5, 21.7, 30.6, and 46.6 degrees for $t_0=1.0, 0.9, 0.7$, and 0.5 mm, respectively. Its thickness dependence was examined to obtain the most suitable functional expression by the least square method for various functions, and it was found that

$$\rho \approx 19.3/t_0^{1.27} \text{ (degree)}$$

is the most acceptable relation between ρ and t_0 . This relation shows the tendency of the thinner initial thickness to produce a better development of the pointed vertex.

The difference in θ_0 between the theoretical values calculated by Eq. (7) and the experimental values corresponds directly to the difference in the forming limits between the theory and experiment. Therefore, a good agreement is achieved only when θ_0 can be expressed by a proper function of effective strain, $\bar{\epsilon}$. Such functions other than that indicated in Eq. (7)

are,

$$\theta_0 = \psi_0 - \rho_0 \bar{\epsilon}^2, \quad (9)$$

$$\psi_0 = 0.5n/(1-0.5n), \quad (10)$$

as proposed previously⁽⁵⁾. Another one is an even function of $\bar{\epsilon}$ and its simplest form is given by:

$$\theta_0 = \pi/2 - \rho_1(\bar{\epsilon}^2 + \bar{\epsilon}^4). \quad (11)$$

ρ_0 and ρ_1 in Eqs. (9) and (11) are positive constants. In the relation indicated in Eqs. (9) and (10), since the initial value of θ_0 , that is equivalent to ψ_0 , is about 82 degrees (i.e., $n=0.254$) and less than 90 degrees, θ_0 becomes small in the plane-strain side and the difference is enlarged compared to the experimental values shown in Fig. 4. The dotted lines in Fig. 4 are the results of Eq. (11) and the dotted lines in Fig. 3 indicate the corresponding forming limits. θ_0 given by Eq. (11) is always larger than that given by Eq. (7) except on the equibiaxial tension side. The experimental forming limits in $a=0.3$ and 0.6 almost agree with the theoretical ones for $t_0=0.7$ and 0.5 mm but are less than the theoretical ones for $t_0=1.0$ and 0.9 mm. Hence, a more appropriate function should be introduced, and a parabolic function containing more than two parameters is one of such functions. Example of such functions predicted from Eqs. (7) and (11) is,

$$\theta_0 = \pi/2(\rho_2 \bar{\epsilon}^2 + \rho_3 \bar{\epsilon}^4), \quad (12)$$

or

$$\theta_0 = \pi/2 - q_1 \bar{\epsilon}^{q_0}. \quad (13)$$

However, a method to decide the positive constants ρ_2 , ρ_3 , q_1 , and q_0 should be established.

3.4 Effects of work-hardening law equation on Gotoh's theory

In the modified M-K theory, the forming limits are dependent on the work-hardening law equations employed, as reported previously⁽⁴⁾. The forming limit by the Gotoh's theory is examined in this study to

Fig. 3 Comparison of the experimental forming limits with the results the Gotoh theory

Fig. 4 Relation between a half angle of pointed vertex and the effective strain

determine the existence of the similar dependence on the work-hardening law equations. In practice, the Hollomon law, in which the logarithmic work-hardening exponent, n , is constant, is compared with the forming limits calculated by the Voce law (saturation law) in which the logarithmic work-hardening exponent is the function of strain. The stress-strain relation of the materials tested in this study is represented by the Voce law as:

$$\bar{\sigma} = 153\{1 - 0.546/\exp(4.26\bar{\epsilon})\} \quad (\text{MPa}).$$

The variations of the n -value are calculated by replacing the n -value in Eqs. (5) and (6) of the Gotoh's theory with

$$n = d \ln \bar{\sigma} / d \ln \bar{\epsilon}. \quad (14)$$

Here, the angle of the pointed vertex was assumed to be presented by the relation of Eq. (7). The results are shown in Fig. 5 by solid lines. For $t_0 = 1.0$ mm, the calculated value is greater than the experimental value in the range $\alpha = 0.3$ and 0.6 , but these differences decrease with a decrease in sheet thickness and both values coincides with each other at $t_0 = 0.5$ mm. This tendency and the profile of forming limits are contrary to the results of Hollomon law. This seems to be caused by the difference of the relation between the logarithmic work-hardening exponent and the effective strain. The corresponding θ_0 values are in the range between 85 and 90 degrees. These values are less than that of Hollomon law where θ_0 values are in the range between 70 and 90 degrees.

In the authors' previous report, the experimental forming limits of brass and copper were investigated, and it was revealed that the best-fit work-hardening law equation for the both materials was not Hollomon law. Hence, the effects of the work-hardening law equation on the forming limits by the modified M-K theory were analyzed⁽⁴⁾. The same forming limits of brass and copper are re-examined here by the Gotoh's theory and the obtained tendencies are compared with the experimental results. The best-fit work-hardening

Fig. 5 Effect of the work-hardening law equation used in the Gotoh theory on the forming limits

dening law equation for brass is Voce law which is given by,

$$\bar{\sigma} = 782\{1 - 0.870/\exp(1.90\bar{\epsilon})\} \quad (\text{MPa}).$$

And the corresponding Hollomon law is given by,

$$\bar{\sigma} = 752\bar{\epsilon}^{0.535} \quad (\text{MPa}),$$

For copper, the best-fit work-hardening law equation is Swift law (i.e., modified power law). This law and the corresponding Hollomon law are given by,

$$\bar{\sigma} = 470(\bar{\epsilon} + 0.029)^{0.382} \quad (\text{MPa}),$$

and

$$\bar{\sigma} = 463\bar{\epsilon}^{0.339} \quad (\text{MPa}),$$

respectively. In applying Hollomon law in the case of brass, $g = 1.093$ is obtained when the experimental value $\epsilon_1 = 0.420$ at $\alpha = 1$ is substituted into Eq. (5), but this g value cannot satisfy Eq. (6). Therefore, calculations are made using the values of $g = 1$ at $\alpha = 1$, and the calculated forming limits are shown in Fig. 6. The solid lines indicate the results calculated by applying the best-fit work-hardening law equation; the broken lines are those obtained by Hollomon law, and the plotted points represent the experimental values. Differences between the experimental and the theoretical values by the best-fit work-hardening law equations are almost the same as the differences for aluminum tested in the present study.

Figure 7 shows the relation between θ_0 and $\bar{\epsilon}$. The difference between the experiment (plotted points)

Fig. 6 Re-examination of the effect of the work-hardening law equation used in the Gotoh theory on the forming limits for brass and copper

and the theory (solid line) expressed by Eq. (7) is larger than the present results as shown in Fig. 4. However, the calculated θ_0 values are in the range between 70 and 90 degrees, which are the same as for the present materials. Therefore, the dependence of θ_0 on the work-hardening law equations is not observed even if the best-fit work-hardening law equation is used, and the fact that the θ_0 values are in the range between 70 and 90 degrees agrees with the results reported by Gotoh⁽⁵⁾. On the other hand, when the Hollomon law is used, the forming limits are enlarged both in the plane-strain side and in the equibiaxial tension side, and the pointed vertex is well developed, because θ_0 is in the range between 0 and 40 degrees, which corresponds to the case of brass. But noticeable differences are not observed for copper. Compared with the modified M-K theory, the effect of the work-hardening law equation used for the calculation of the forming limits is small for brass and copper; this tendency differs from that indicated by the results for aluminum in the present study.

3.5 Modified M-K theory and Gotoh's theory

For the prediction of the forming limits deduced by the Gotoh's theory using best-fit work-hardening law equation, it is necessary to determine a more appropriate equation that expresses the relation between θ_0 and $\bar{\epsilon}$. Typical expressions are parabolic functions written as Eqs. (12) and (13); each contains two coefficients. Only Eq. (13) is analyzed here because of the relative ease in determining the coefficients. Figure 8 shows the log-log relation between $(\pi/2 - \theta_0)$ and $\bar{\epsilon}$, in which the plotted points and the broken lines represent the results of experiment and the least squares method, respectively. Once a reasonable fit was achieved for all materials, the results seem to be well arranged in a log-log relation. Solid lines in Fig. 8 are the results of the least square fitting method which was applied for the θ_0 values corresponding to the forming limit of modified M-K

Fig. 7 Relation between a half angle of a pointed vertex and the effective strain for brass and copper

theory.

When the relation given by the solid line Fig. 8 is used, the theoretical values of the Gotoh's theory coincide with that of the modified M-K theory and the difference explained in section 3.3 can be eliminated. Since the safety zone is the upper part above each line in Fig. 8, it seems to be an appropriate method for θ_0 values to be estimated from the results of the modified M-K theory and expressed by the relation indicated in Eq. (13). The sheet-thickness dependence of the coefficients q_0 and q_1 corresponding to the solid lines for aluminum in Fig. 8 is examined by several functions, and the best results obtained by least square method are

$$q_0 = 1.99/t_0^{0.625}, \quad q_1 = 19.3/t_0^{2.14} \text{ (degree)}.$$

These results provide a typical example indicating the sheet-thickness dependence of the parameters included in the Gotoh's theory. In the case of $t_0 = 1.0$ and 0.9 mm, $q_0 = 2$ is obtained, so that the theoretical values using Eq. (7) coincide with the experiments.

The modified M-K theory has one advantage in that the forming limit under a constant strain ratio can be estimated using only the results of a uniaxial tensile test. The Gotoh's theory also has an advantage in that it is easy to estimate the forming limit not only under a constant strain ratio, but also under any variable strain path. Both the modified M-K theory, as reported previously, and the Gotoh's theory, discussed in section 3.4, show the dependence of the work-hardening law equation used. One of the most approved methods used to predict the forming limit under a variable strain path can be obtained by combin-

Fig. 8 Comparison of a half angle of a pointed vertex and the effective strain for the representation of a logarithmic linear function $\theta_0 = \pi/2 - q_1 \bar{\epsilon}^{q_0}$

ing the advantages of both theories. Actually, the mechanical properties, including the best-fit work-hardening law equation required for the theory are first to be measured by the uniaxial tensile test. Then, the parameters for the Gotoh's theory (e.g., coefficients of q_0 and q_1 in Eq. (13)) are to be calculated from the forming limits estimated by a modified M-K theory. Finally, the forming limits needed can be calculated by the Gotoh's theory.

4. Conclusions

(1) In the forming limit theory, it is found that such factors as the surface roughness and other mechanical properties have little thickness dependence. This is attribute to the relatively small grain size compared to the sheet thickness. The surface roughness itself develops linearly as the effective strain increases.

(2) The reduction of the sheet thickness results in the decrease of the forming limit. This result agrees with the modified M-K theory, when the inhomogeneity factor is used and only the variation of the initial sheet thickness is taken into account.

(3) The effect of the work-hardening law equation is rather ambiguous in the Gotoh's theory compared to that in the modified M-K theory, but a clear difference is observed among half angles of the pointed vertex, θ_0 . The θ_0 values are in the range between 70 and 90 degrees for all the materials examined, when the best-fit work-hardening law equation is used.

(4) In order to express the thickness dependence associated with the Gotoh's theory, it is necessary to reconsider the relation between θ_0 and $\bar{\epsilon}$ so that the thickness dependence of growth of a pointed vertex can thoroughly be represented. The relation, $\theta_0 = \pi/2 - q_1 \bar{\epsilon}^{q_0}$, which satisfies the experiment is proposed and discussed as the method to calculate the coefficients q_0 and q_1 from the forming limits derived from the modified M-K theory.

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