Notch Effects Under Ultrasonically Vibrating Stress*

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In this study, stress concentration factors and fatigue strength reduction factors are obtained for the case of a round bar of mild steel with a V-groove under ultrasonically vibration stress. The stress concentration factors are determined by FEM analysis on the eigenmodes. The fatigue strength reduction factors are obtained by combining the above analysis and the results of the ultrasonic fatigue tests. These factors are determined under ultrasonically vibrating stress. These results could make the ultrasonic fatigue test more popular than the ordinary fatigue test.

Key Words: Fatigue, Material Testing, Ultrasonics, Stress Concentration Factor, Fatigue Strength Reduction Factor, Ultrasonic Fatigue Test, FEM

1. Introduction

In the ultrasonic fatigue test, the testpiece is vibrated longitudinally. By utilizing vibration stress in the resonant state of the specimen, a fatigue test is conducted. Contrary to the ordinary fatigue test, the ultrasonic fatigue test can be conducted within a short period of time. (In the case of 17 kHz, the time required for 10^8 -times repeated loading is about 100 min.) The test also attracts attention as an effective means for studying high-speed fatigue, and studies have been made from various standpoints. They include, for instance, studies on changes in mechanical properties due to ultrasonic fatigue⁽¹⁾⁽²⁾, the fatigue limits^{(4)~(6)}, the effects of cooling water on corrosion⁽⁴⁾, the relation between ultrasonic fatigue and internal friction⁽³⁾⁽⁷⁾, and ultrasonic fatigue strength under mean stress⁽⁸⁾. The stress concentration factor and fatigue strength reduction factor under ultrasonically vibrating stress are considered indispensable in

designing ultrasonic application equipment but, so far, no studies have been made on these factors. This is considered attributable to the fact that in terms of the principles of the ultrasonic fatigue test, measurements of force and stress are difficult. In the present study, therefore, an eigenmode analysis of the ultrasonic vibrating system and an ultrasonic fatigue test applied to a round bar testpiece of mild steel with a circular V-groove were conducted. From the results of these tests, the stress concentration factor and fatigue strength reduction factor were obtained and the notch effect under ultrasonically vibrating stress was clarified.

2. Object and Method of Analysis

In general, the vibrating system for the ultrasonic fatigue test consists of a vibrator, cone, horn and testpiece. In the present study, however, to make matters simple, a system consisting of a testpiece and an adjoining horn is used as the object of analysis. An exponential horn for 17 kHz made of titanium alloy (6 Al 4 V) and a testpiece of a round bar with a circular V-groove are shown in Table 1 and Figs. 1 and 2. These vibrating systems are subjected to an eigen-

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mode analysis by the finite element method (FEM). Further, by assuming that the horn is a part of the driving system, displacement amplitudes at the nodal points are normalized by the axial displacement amplitude at the center of the larger end surface of the horn.

The finite element model of the vibrating system and the finer mesh in the region of the V-groove are

Table 1 Horn and Testpiece

	Horn	Testpiece
Material	Ti-Alloy (6Al4V)	S 10 C
Density	$4.42 \times 10^3 \text{ kg/m}^3$	$7.86 \times 10^3 \text{ kg/m}^3$
Young's Modulus	$1.127 \times 10^2 \text{ GPa}$	$2.073 \times 10^2 \text{ GPa}$
Shear Modulus	$4.41 \times 10 \text{ GPa}$	$8.23 \times 10 \text{ GPa}$

$$d_1 = 56 \text{ mm}, d_2 = 12 \text{ mm}, d_3 = 6 \text{ mm}, h = 10 \text{ mm} \\ l_0 = 165 \text{ mm}, l = 144 \text{ mm}, \gamma = 0.0093$$

Fig. 1 Ultrasonically vibrating System

Detailed drawing of A

NOTCH RADIUS ρ (mm)			
0.2	0.4	0.8	1.2

NOTCH DEPTH t (mm)				
0.2	0.4	0.6	0.8	1.0

Fig. 2 Shape of notched testpiece

shown in Figs. 3 (a)-(d). These elements are divided into triangular and square ring elements, and in the vicinity of the notch root surface, iso-parametric triangular ring elements are used to improve analytical accuracy.

The eigenmode analysis in the present study was conducted using the Integrated Structure Analysis System II (ISAS II) at the Calculation Center of Tokyo University.

3. Notch Shape and Resonant Frequency for the System

Figure 4 shows the relations among the notch depth, notch radius and resonant frequency obtained by the eigenmode analysis of the resonant frequency. It indicates that as the notch depth increases and as the notch radius increases, the resonant frequency of the system tends to decrease. In this study, however, the resonant frequency only changes within 0.1 kHz.

(a) Horn and testpiece

(b) Notched testpiece

(c) Detailed drawing of A

(d) Detailed drawing of B

Fig. 3 Finite element model of the ultrasonically vibrating system

4. Stress Concentration Factor Under Ultrasonically Vibrating Stress

4.1 Calculation method for the stress concentration factor

When a static tensile load is applied to the circular V-grooved round bar, the nominal stress of the notch section, namely, the mean stress, has a value obtained by dividing the load by the area of the cross section. In the case of ultrasonic vibration, however, measurements of the load are difficult and, thus, it is impossible to obtain the mean stress of the notch section, unlike under ordinary static load conditions. It is necessary, therefore, to obtain the mean stress of the notch section by calculation using the results of the eigenmode analysis as follows.

Figure 5 denotes the axial stress at the center of gravity of the triangular ring element on the notch section by σ_i^* , and the radial sectional area of the element by S_i . The mean stress σ_{mean}^* of the notch section can then be obtained by the following equation:

$$\sigma_{\text{mean}}^* = \frac{\sum \sigma_i^* S_i}{\sum S_i} = \frac{\sum \sigma_i^* (r_i^2 - r_{i-1}^2)}{r^2} \quad (1)$$

The relations among σ_{mean}^* , notch depth and

Fig. 4 The relation among the resonant frequency, f_r , notch depth, t , and notch radius, ρ

Fig. 5 Finite element mesh and distribution of the axial stress on the cross section of the notch region

notch radius are shown in Fig. 6. Also, the maximum stress σ_{max}^* of the notch root obtained from the analysis is shown in Fig. 7 for various notch depths and radii.

Then the stress concentration factors, α_m , under ultrasonically vibrating stress for each shape of the circular V-groove can be obtained using the following equation:

$$\alpha_m = \frac{\sigma_{\text{max}}^*}{\sigma_{\text{mean}}^*} \quad (2)$$

Namely, for a total of 20 shapes with notch depths, t , of 0.2, 0.4, 0.6, 0.8 and 1.0 mm and notch radii, ρ , of 0.2, 0.4, 0.8 and 1.0 mm, the values of α_m under ultrasonically vibrating stress are obtained.

σ_{mean}^* : mean stress on the cross section of the notch bottom
 ρ : notch radius
 t : notch depth

Fig. 6 The relation among σ_{mean}^* , ρ and t

σ_{max}^* : maximum stress at the notch bottom

Fig. 7 The relation among σ_{max}^* , ρ and t

Fig. 8 The relation between the stress concentration factor, α , and notch radius, ρ

4.2 Stress concentration factor under ultrasonically vibrating stress

Values α_m under ultrasonically vibrating stress obtained from Eq. (2) are shown in Figs. 8 (a) to (e). Furthermore, the value α in the static case obtained by the FEM is also plotted in the figures. The condition for α was that the large end surface of the horn was fixed, and a static tensile load of 100 N was applied to the free end of the testpiece. It is clear that α_m under ultrasonically vibrating stress agrees with α under a static load.

Figure 9 shows the relation of d/D and ρ/D to α_m under ultrasonically vibrating stress. These relationships show good agreement in accordance with the chart by Peterson⁽⁹⁾.

α_m : stress concentration factor under ultrasonically vibrating stress
 d : diameter of the notch bottom
 D : diameter of the plain testpiece ($D=6$ mm)

Fig. 9 The relation between α_m and d/D

5. Fatigue Strength Reduction Factor Under Ultrasonically Vibrating Stress

5.1 Calculation method for fatigue strength reduction factor

In general, the fatigue strength reduction factor under push-pull alternating stresses of conventional repeating speed is obtained by ordinary fatigue tests. In the case of ultrasonic vibration, however, no fatigue limit of notched specimens has yet been obtained by ultrasonic fatigue tests because of the difficulty of direct stress measurement, though it is possible to measure the displacement amplitude of the free-end surface of the testpiece. Therefore, assuming that the allowable displacement amplitude corresponds to the fatigue limit of the testpiece and that the displacement amplitude of the free end surface of the testpiece obtained by eigenmode analysis corresponds to the mean stress of notch section, the fatigue limit, σ , could be obtained as follows.

$$B^*/\sigma_{\text{mean}}^* = B_w/\sigma_w \quad \therefore \sigma_w = (B_w/B^*) \cdot \sigma_{\text{mean}}^* \quad (3)$$

where

B^* : Analyzed axial-direction displacement amplitude of the free-end surface of the testpiece under analysis.

σ_{mean}^* : Mean stress of the notch section by analysis

B_w : Allowable displacement amplitude in the ultrasonic fatigue test

σ_w : Fatigue limit of the notched testpiece

By substituting the fatigue limit σ_w obtained by the above equation into the following Eq. (4), the fatigue strength reduction factor β_m under ultrasonically vibrating stress is obtained.

$$\beta_m = \frac{\sigma_{w0}}{\sigma_w} \quad (4)$$

where

σ_{w0} : Fatigue limit for plain specimen whose diameter is equal to the notch-root diameter.

5.2 Ultrasonic fatigue test

5.2.1 Test equipment and test method Figures 10 and 11 show the ultrasonic fatigue test equipment which consists of the external synchronization type CR oscillator (9 to 55 kHz), wide-band power amplifier (2 kW), controller, input A.C. voltmeter, Nickel-magnetostrictive type transducer (17 kHz) and horn (titanium alloy; 17 kHz). The strain amplitude measuring equipment consists of the eddy current type displacement pickup, preamplifier, output digital voltmeter, synchroscope, and frequency counter. For the testpiece cooling, a freezer, tank and pump (labo-pump) are used. The V. C. I. (volatile corrosion inhibitor) aqueous solution kept at 5°C is used as cooling water⁽⁴⁾⁽⁸⁾. Through the use of the above-mentioned equipment, the ultrasonic fatigue test is conducted to obtain the allowable displacement amplitude. The material of the testpiece is S10C and its chemical composition is shown in Table 2. The plain testpiece having a diameter of 6 mm also undergoes a fatigue test.

5.2.2 Results of the fatigue tests Figure 12 shows the S-N curve of a plain testpiece. The stress amplitude on the ordinate has been obtained from the value of the displacement amplitude at the free-end surface using the vertical vibration theory of the rod.

Figure 13 shows the relation between the displacement amplitude of the free-end surface and the num-

Table 2 Chemical composition of the testpiece

S10C							(wt%)	
C	Si	Mn	P	S	Cu	T-Al	Ni+Cr	
0.10	0.02	0.46	0.019	0.016	0.01	0.046	0.03

Fig. 10 Equipment for ultrasonic fatigue tests

Fig. 11 Ultrasonic testing machine (17 kHz)

ber of cycles of stress. These A - N curves correspond to the ordinary S - N curves for the notch depth of 0.6 mm, from which the values of the allowable displacement amplitude for various notch shapes are obtained. Figure 14 shows these values of allowable displacement amplitudes with various notch depths and notch radii.

5.3 Fatigue strength reduction factor under ultrasonically Vibrating Stress

Now, by substituting the value of B_w , σ_{mean}^* and B^* into Eq.(3), the fatigue limit, σ_w , is obtained. Figure 15 shows the relation among σ_w , ρ and t . Fatigue strength reduction factor under ultrasonically vibrating stress is obtained by Eq.(4). Figure 16 shows the relation between ρ/D and β_m .

Figure 17 shows the relation between α_m and β_m under ultrasonically vibrating stress. It shows a nearly linear relation between α_m and β_m .

The relation between the fatigue strength reduction factor and d/D is shown in Figs. 18 (a) to (d).

Fig. 12 S - N curve of a plain testpiece (diameter is 6 mm)

A : displacement amplitude that is proportional to stress amplitude

Fig. 13 A - N curves of notched testpiece

Here β_0 is the ratio of the allowable displacement amplitude of the notched testpiece to the allowable displacement amplitude of the plain specimen, obtained by the result of the ultrasonic fatigue test.

Fig. 14 The relation among B_w , ρ and t

Fig. 15 The relation among σ_w , ρ and t

β_m : fatigue strength reduction factor under ultrasonically vibrating stress

ρ : notch radius

D : 6 mm

Fig. 16 The relation between β_m and ρ/D

α_m : stress concentration factor under ultrasonically vibrating stress

β_m : fatigue strength reduction factor under ultrasonically vibrating stress

Fig. 17 The relation between α_m and β_m

(a) in the case of $\rho = 0.2$ mm

(c) in the case of $\rho = 0.8$ mm

(b) in the case of $\rho = 0.4$ mm

(d) in the case of $\rho = 1.2$ mm

●: the ratio of allowable displacement amplitudes, β_0 , obtained by Eq. (5)

○: β_k under conventional repeating speed

△: $\beta_{ex} = \beta_0 \times d^2/D^2$

×: β_m under ultrasonically vibrating stress

Fig. 18 The relation between the fatigue strength reduction factor, β , and d/D

Namely,

$$\beta_0 = B_b / B_a \quad (5)$$

where

B_a : Allowable displacement amplitude of notched testpiece

B_b : Allowable displacement amplitude of plain testpiece

Further, β_k in Figs. 18 (a) to (d) has been calculated from the following equation (6)⁽¹⁰⁾ which applies to the case of push-pull alternating stresses at a conventional repeated speed for the circular V-grooved round bar recommended by the Japan Society of Mechanical Engineers :

$$\beta_k = 1 + \xi_1 \xi_2 \xi_3 \xi_4 \xi_5 \quad (6)$$

Figure 18 indicates that there is a similar tendency between β_m under ultrasonically vibrating stress and β_k under push-pull alternating stresses at ordinary repeated speed, but β_m generally tends to become smaller than β_k .

Figure 19 shows the relation between ρ/d and β . It indicates that as ρ/d becomes smaller, β_m under ultrasonically vibrating stress gradually becomes larger.

6. Discussion

6.1 Method of experimentally obtaining the fatigue strength reduction factor in ultrasonic fatigue tests

The value β_0 shown in Figs. 18 (a) to (d) is the ratio between the allowable displacement amplitude of the plain testpiece and that of the notched testpiece obtained in the ultrasonic fatigue test. In all of the graphs β_0 is larger than β_m under ultrasonically vibrating stress, and this difference becomes large as the notch depth increases. The reason for this seems to be that no effects of the area of the cross section are

considered in β_0 . Namely, since sudden changes in stress occur at the notch, the plain testpiece and notched testpiece give different nominal stress values, even if the displacement amplitudes at the free-end surfaces of the test pieces are the same. Therefore, when the V-groove is replaced with a square groove with the same area of the notch section as shown in Fig. 20, we can take the effect of nominal stress into consideration. Namely, when the fatigue strength reduction factor taking an area of the notch section into consideration is denoted by β_{ex} , it can be defined by the following equation :

$$\beta_{ex} = \beta_0 \cdot \frac{d^2}{D^2} \quad (7)$$

The β_{ex} value thus obtained shows good agreement with β_m as shown in Fig. 18 (a) to (d) and Fig. 19. Namely, in the case of the circular V-groove, it is possible to estimate the fatigue strength reduction factor β_{ex} under ultrasonically vibrating stress. Further, the six kinds of notches shown by "x" marks in Table 3, out of the 20 kinds of notches, are round-grooved ones. In the case of the materials of the round-grooved notches, the values of β_{ex} and β_0 also show good agreement and in the case of the circular round-grooves, it is similarly possible to estimate the fatigue strength reduction factor under ultrasonically vibrating stress.

6.2 Frequency effect upon fatigue limits of notched and plain testpieces

The value β_m under ultrasonically vibrating stress obtained in this study shows agreement with β_k at

$$\bullet : \beta_{ex} = \beta_0 \times d^2 / D^2$$

$\times$: β_m under ultrasonically vibrating stress

Fig. 19 The relation between β and ρ/d

Table 3 V-grooves and round-grooves

○ : V-groove

× : round-groove (without notch angle)

d : diameter of notch bottom

ρ : notch radius

ρ/d	4.0	4.4	4.8	5.2	5.6
0.2	○	○	○	○	○
0.4	○	○	○	○	×
0.8	○	○	○	×	×
1.2	○	○	×	×	×

Fig. 20 Round bar with a circular V-groove and with a square groove

normal repeated speed which has been obtained from Eq. (6) stipulated by the Japan Society of Mechanical Engineers. This seems to indicate that the frequency effect on the fatigue limits are substantially equal for the notched and plain testpieces. However, β_m generally shows a slightly lower value. This may be due to the fact that the rate of increase in the fatigue limit due to the frequency effect in the notched testpiece is slightly larger than in the plain one.

7. Conclusions

To clarify the notch effect under ultrasonically vibrating stress, an eigenmode analysis of the ultrasonic vibration system containing a round bar of mild steel with a circular V-groove and an ultrasonic fatigue test of the same testpiece were conducted. As a result, the values of the stress concentration factor and fatigue strength reduction factor under ultrasonically vibrating stress were obtained in the case of round bar of mild steel with the circular V-groove and the following two points have been clarified:

(1) The value of the shape coefficient α_m under ultrasonically vibrating stress is nearly equal to the value under the static load.

(2) The value of fatigue strength reduction factor β_m under ultrasonically vibrating stress substantially agrees with β_s at a conventional repeated speed that is recommended by the Japan Society of Mechanical Engineers. Further, the value of β_m sub-

stantially agrees with the value of β_{ex} which has been obtained by Eq. (7).

In the future, the authors intend to carry out eigenmode analyses and ultrasonic fatigue tests on various kinds of materials and notch shapes to further clarify the notch effect under ultrasonically vibrating stress.

References

- (1) Koganei, H., Tanaka, T. and Sakurai, T., *Material* (in Japanese), Vol. 24, No. 263 (1975-8), p. 45.
- (2) Awatani, J., *Trans. Jpn. Soc. Mech. Eng.*, (in Japanese), Vol. 40, No. 337 (1974-9), p. 2453.
- (3) Mason, W. P. and MacDonald, D. E., *J. Acoust. Soc. Am.*, 51 (1972), p. 894.
- (4) Kikukawa, M., Oji, S. and Ogura, K., *Trans. Jpn. Soc. Mech. Eng.*, (in Japanese), Vol. 32, No. 235 (1966-3), p. 363.
- (5) Udoguchi, T., Kato, N. and Yamane, M., *Jour., Jpn. Soc. Mech. Eng.*, (in Japanese), Vol. 68, No. 552 (1965), p. 17.
- (6) Yamane, M., *Trans. Jpn. Soc. Mech. Eng.*, (in Japanese), Vol. 28, No. 191 (1962), p. 893.
- (7) Ono, T. and Kaga, Y., *Prepr. of Jpn. Soc. Mech. Eng.*, (in Japanese), No. 720-1 (1972-4), p. 97.
- (8) Terao, K., Kuno, T. and Mizuno, M., *Bull. JSME*, Vol. 24, No. 196 (1981-10), p. 1708.
- (9) Peterson, R. E., *Stress Concentration Factors*, (1974), p. 50, A Wiley-Interscience Publication.
- (10) The Japan Society of Mechanical Engineers, *Kinzoku Zairyo-Tsukare Tsuyosa no Sekkei Shiryo I*, (in Japanese), (1961), p. 114., JSME.