Semi-Infinite Body with a Revolutionary Pit Under Tension*

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In this paper, based on the body force method, a sophisticated way to analyze the semi-infinite body with a revolutionary pit under uniaxial tension is presented, and the special fundamental solutions necessary for the analysis are given. The accurate numerical results are shown for the case of the semi-infinite bodies with a semispheroidal or drill-hole-shaped pit, and the stress distributions around the pit are discussed.

Key Words : Elasticity, Stress Concentration Factor, Revolutionary Pit, Semi-Infinite Body, Tension, Body Force Method

1. Introduction

It is important to analyze the problem of the stress concentration in semi-infinite bodies with a semi-ellipsoidal pit under tension, in relation to the evaluation of components with surface defects. Therefore, semi-infinite bodies or finite thickness plates with a semispheroidal pit under biaxial tension have been analyzed by many researchers^{(1)~(9)}. On the other hand, concerning the problems under uniaxial tension, only the semi-infinite body with a semispherical pit has been analyzed^{(10)~(12)}.

In this paper, based on the body force method⁽¹³⁾, a sophisticated way to analyze the semi-infinite body with a revolutionary pit under uniaxial tension is presented. The highly accurate results can be obtained even in the case of small number of division, because

the present method needs only the body force on the pit and does not need the division along the circumference of the pit. The numerical results are shown for the case of the semi-infinite bodies with a semispheroidal or drill-hole shaped pit under tension.

2. Method of Analysis

In this paper, the problem shown in Fig. 1 is treated. This problem can be considered to be composed of two problems.

'Problem under uniaxial tension shown in Fig. 1'
= 'Problem (A)' + 'Problem (B)' (1)

Loading conditions at infinity for the 'Problems (A) and (B)' are expressed as follows by using the coordinates defined in Fig. 2.

$$\begin{aligned} \text{'Problem (A)': } \sigma_r = \sigma_\theta = \frac{1}{2} \sigma_\infty, \\ \tau_{r\theta} = 0 \text{ at infinity} \end{aligned} \quad (2)$$

$$\begin{aligned} \text{'Problem (B)': } \sigma_r = -\sigma_\theta = \frac{1}{2} \sigma_\infty \cos 2\theta, \\ \tau_{r\theta} = -\frac{1}{2} \sigma_\infty \sin 2\theta \text{ at infinity} \end{aligned} \quad (3)$$

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2.1 Solution of 'Problem (A)'

2.1.1 Fundamental solution Since this is an axisymmetric problem, the stresses due to the r -directional ring force and z -directional ring force in a semi-infinite body are used as fundamental solutions shown in Fig. 3(a). That is, the boundary conditions on the surface of the semi-infinite body are satisfied automatically, and those on the pit are satisfied by distributing these ring forces.

As the expression of fundamental solutions were given by Murakami et al.⁽⁸⁾, they are not presented in this paper. But new devices are introduced in the numerical integration, which will be described later. This makes the present analysis highly accurate.

2.1.2 Density of body force The individual densities of the body force are defined in each pit. Denoting dP_r and dP_z respectively as the r - and z -directional body force acting on the infinitesimal area $\rho d\varphi \sqrt{(d\rho)^2 + (d\zeta)^2}$, the following definitions of the densities are used.

Fig. 1 Semi-infinite body with a revolutional pit under tension

Fig. 2 Explanation of coordinate

$$\left. \begin{aligned} \text{For a semispheroidal pit shown} \\ \text{in Fig. 4(a),} \\ \rho_r = \frac{dP_r}{\rho d\varphi d\zeta} \\ \rho_z = \frac{-dP_z}{\rho d\varphi d\rho} \end{aligned} \right\} \quad (4)$$

$$\left. \begin{aligned} \text{For a drill-hole-shaped pit shown} \\ \text{in Fig. 4(b), section A-C:} \\ \rho_r = \frac{dP_r}{\rho d\varphi d\zeta} \\ \rho_z = \frac{dP_z}{\rho d\varphi d\zeta} \frac{c}{\zeta} \\ \text{section C-B:} \\ \left\{ \begin{aligned} \rho_r &= \frac{dP_r}{\rho d\varphi d\zeta} \quad (\alpha \neq \pi) \\ \rho_r &= \frac{-dP_r}{\rho d\varphi d\rho} \frac{a}{\rho} \quad (\alpha = \pi) \end{aligned} \right. \\ \rho_z &= \frac{-dP_z}{\rho d\varphi d\rho} \end{aligned} \right\} \quad (5)$$

(a) Fundamental solutions for 'Problem (A)'

(b) Fundamental solutions for 'Problem (B)'

Fig. 3 Fundamental solutions

2.2 Solution of 'Problem (B)'

2.2.1 Fundamental solution For this problem, the stress σ_{ij}^r due to the r -directional ring force F_r , the stress σ_{ij}^θ due to the θ -directional ring force F_θ , and the stress σ_{ij}^z due to the z -directional ring force F_z are used as fundamental solutions (Fig. 3(b)). The variations in r - and z -directional ring forces are represented by $\cos 2\varphi$, whereas the variation in the θ -directional ring force is represented by $\sin 2\varphi$. These stresses are obtained as follows by using the stress fields⁽¹⁴⁾ due to a point force acting in a semi-infinite body.

$$\begin{aligned} \sigma_r^r = B & \left[(1-2\nu)(\rho I_{3,0} + r I_{3,1} - 4\rho I_{3,2} - 2r I_{3,3} + 4\rho I_{3,4}) \right. \\ & - \frac{3}{r_m^2} (r^2 \rho I_{5,0} - r(r^2 + 2\rho^2) I_{5,1} + \rho^3 I_{5,2} \\ & + r(2r^2 + 3\rho^2) I_{5,3} - 2\rho(2r^2 + \rho^2) I_{5,4} + 2r\rho^2 I_{5,5}) \\ & + (1-2\nu)\{(3-4\nu)\rho J_{3,0} - (5-4\nu)r J_{3,1} \\ & - 4(1-2\nu)\rho J_{3,2} + 2(5-4\nu)r J_{3,3} - 4\rho J_{3,4}\} \\ & + \frac{3(3-4\nu)}{r_m^2} \{-r^2 \rho J_{5,0} + r(r^2 + 2\rho^2) J_{5,1} \\ & - \rho^3 J_{5,2} - r(2r^2 + 3\rho^2) J_{5,3} + 2\rho(2r^2 + \rho^2) J_{5,4} \\ & - 2r\rho^2 J_{5,5}\} + \frac{6\zeta}{r_m^2} \left[\zeta(\rho J_{5,0} - 3r J_{5,1} + 6r J_{5,3} - 4\rho J_{5,4}) \right. \\ & - (z + \zeta)\{(1-2\nu)\rho J_{5,0} - (3-2\nu)r J_{5,1} + 4\nu\rho J_{5,2} \\ & + 2(3-2\nu)r J_{5,3} - 4\rho J_{5,4}\} - \frac{5z}{r_m^2} \{-r^2 \rho J_{7,0} \\ & + r(r^2 + 2\rho^2) J_{7,1} - \rho^3 J_{7,2} - r(2r^2 + 3\rho^2) J_{7,3} \\ & + 2\rho(2r^2 + \rho^2) J_{7,4} - 2r\rho^2 J_{7,5}\} \left. \right] \\ & - \int_0^\pi \frac{4(1-\nu)(1-2\nu)}{R_2^2(R_2 + z + \zeta)^2} \left\{ (3r \cos \varphi - \rho - 2\rho \cos^2 \varphi) \right. \\ & - \frac{(3R_2 + z + \zeta)(r - \rho \cos \varphi)^2(r \cos \varphi - \rho)}{R_2^2(R_2 + z + \zeta)} \left. \right\} \\ & \times \cos 2\varphi d\varphi \Big] \cos 2\theta \end{aligned} \quad (6)$$

(a) Semispheroidal pit

(b) Drillhole-shaped pit

Fig. 4 Shape of pit in numerical calculation

$$\begin{aligned} \sigma_\theta^r = B & \left[(1-2\nu)(-\rho I_{3,0} - r I_{3,1} + 4\rho I_{3,2} + 2r I_{3,3} \right. \\ & - 4\rho I_{3,4}) + \frac{3}{r_m^2} (-\rho^3 I_{5,0} + r\rho^2 I_{5,1} + 3\rho^3 I_{5,2} - 3r\rho^2 I_{5,3} \\ & - 2\rho^3 I_{5,4} + 2r\rho^2 I_{5,5}) + (1-2\nu)\{(5-4\nu)\rho J_{3,0} \\ & - (3-4\nu)r J_{3,1} - 4(3-2\nu)\rho J_{3,2} + 2(3-4\nu)r J_{3,3} \\ & + 4\rho J_{3,4}\} + \frac{3(3-4\nu)\rho^2}{r_m^2} (-\rho J_{5,0} + r J_{5,1} + 3\rho J_{5,2} \\ & - 3r J_{5,3} - 2\rho J_{5,4} + 2r J_{5,5}) + \frac{6\zeta}{r_m^2} \left[\zeta(3\rho J_{5,0} - r J_{5,1} \right. \\ & - 8\rho J_{5,2} + 2r J_{5,3} + 4\rho J_{5,4}) - (z + \zeta)\{(3-2\nu)\rho J_{5,0} \\ & - (1-2\nu)r J_{5,1} - 4(2-\nu)\rho J_{5,2} + 2(1-2\nu)r J_{5,3} \\ & + 4\rho J_{5,4}\} - \frac{5z\rho^2}{r_m^2} (-\rho J_{7,0} + r J_{7,1} + 3\rho J_{7,2} \\ & - 3r J_{7,3} - 2\rho J_{7,4} + 2r J_{7,5}) \left. \right] \\ & - \int_0^\pi \frac{4(1-\nu)(1-2\nu)}{R_2^2(R_2 + z + \zeta)^2} \left\{ (r \cos \varphi - 3\rho + 2\rho \cos^2 \varphi) \right. \\ & - \frac{\rho^2 \sin^2 \varphi (3R_2 + z + \zeta)(r \cos \varphi - \rho)}{R_2^2(R_2 + z + \zeta)} \left. \right\} \\ & \times \cos 2\varphi d\varphi \Big] \cos 2\theta \end{aligned} \quad (7)$$

$$\begin{aligned} \sigma_z^r = B & \left[(1-2\nu)(\rho I_{3,0} - r I_{3,1} - 2\rho I_{3,2} + 2r I_{3,3}) \right. \\ & - \frac{3(z - \zeta)^2}{r_m^2} (\rho I_{5,0} - r I_{5,1} - 2\rho I_{5,2} + 2r I_{5,3}) \\ & - (1-2\nu)(\rho J_{3,0} - r J_{3,1} - 2\rho J_{3,2} + 2r J_{3,3}) \\ & - \frac{3(3-4\nu)(z + \zeta)^2}{r_m^2} (\rho J_{5,0} - r J_{5,1} - 2\rho J_{5,2} + 2r J_{5,3}) \\ & + \frac{6\zeta}{r_m^2} \left\{ [\zeta + (1-2\nu)(z + \zeta)](\rho J_{5,0} - r J_{5,1} - 2\rho J_{5,2} \right. \\ & + 2r J_{5,3}) + \frac{5z(z + \zeta)^2}{r_m^2} (\rho J_{7,0} - r J_{7,1} - 2\rho J_{7,2} \\ & + 2r J_{7,3}) \left. \right\} \Big] \cos 2\theta \end{aligned} \quad (8)$$

$$\begin{aligned} \tau_{rz}^r = B & \left[(1-2\nu)(z - \zeta)(I_{3,1} - 2I_{3,3}) - \frac{3(z - \zeta)}{r_m^2} \{ r\rho I_{5,0} \right. \\ & - (r^2 + \rho^2) I_{5,1} - r\rho I_{5,2} + 2(r^2 + \rho^2) I_{5,3} - 2r\rho I_{5,4} \} \end{aligned}$$

$$\begin{aligned}
& + (1-2\nu)(z-\zeta)(-J_{3,1}+2J_{3,3}) + \frac{3(3-4\nu)(z+\zeta)}{r_m^2} \\
& \{-r\rho J_{5,0} + (r^2+\rho^2)J_{5,1} + r\rho J_{5,2} - 2(r^2+\rho^2)J_{5,3} \\
& + 2r\rho J_{5,4}\} + \frac{6\zeta}{r_m^2} \left[z(z+\zeta)(J_{5,1}-2J_{5,3}) - (1-2\nu) \right. \\
& \{-r\rho J_{5,0} + (r^2+\rho^2)J_{5,1} + r\rho J_{5,2} - 2(r^2+\rho^2)J_{5,3} \\
& + 2r\rho J_{5,4}\} - \frac{5z(z+\zeta)}{r_m^2} \{-r\rho J_{7,0} + (r^2+\rho^2)J_{7,1} \\
& + r\rho J_{7,2} - 2(r^2+\rho^2)J_{7,3} + 2r\rho J_{7,4}\} \left. \right] \cos 2\theta \quad (9) \\
\tau_{\theta r}^F = & -2B \left[(1-2\nu)(-rI_{3,1}+2\rho I_{3,2}+rI_{3,3}-2\rho I_{3,4}) \right. \\
& + \frac{3\rho}{r_m^2} \{-r\rho I_{5,1} + (r^2+\rho^2)I_{5,2} - (r^2+\rho^2)I_{5,4} \\
& + r\rho I_{5,5}\} + (1-2\nu)\{rJ_{3,1}-2\rho J_{3,2}-rJ_{3,3}+2\rho J_{3,4}\} \\
& - \frac{3(3-4\nu)}{r_m^2} \{r\rho^2 J_{5,1} - \rho(r^2+\rho^2)I_{5,2} \\
& \rho(r^2+\rho^2)J_{5,4} - r\rho^2 J_{5,5}\} - \frac{6\zeta z}{r_m^2} \left[rJ_{5,1}-2\rho J_{5,2}-rJ_{5,3} \right. \\
& + 2\rho J_{5,4} - \frac{5\rho}{r_m^2} \{r\rho J_{7,1} - (r^2+\rho^2)J_{7,2} \\
& + (r^2+\rho^2)J_{7,4} - r\rho J_{7,5}\} \left. \right] - \int_0^\pi \frac{4(1-\nu)(1-2\nu)}{R_2(R_2+z+\zeta)^2} \\
& \left. \left\{ r \sin \varphi - 2\rho \sin \varphi \cos \varphi \right. \right.
\end{aligned}$$

$$\left. - \frac{(3R_2+z+\zeta)\rho \sin \varphi (r-\rho \cos \varphi)(\rho-r \cos \varphi)}{R_2^2(R_2+z+\zeta)} \right\} \sin 2\varphi d\varphi \Big] \sin 2\theta \quad (10)$$

$$\begin{aligned}
\tau_{\theta z}^F = & -2B \left[-(1-2\nu)(z-\zeta)(I_{3,1}-I_{3,3}) + \frac{3\rho}{r_m^2} (-\rho I_{5,1} \right. \\
& + rI_{5,2} + \rho I_{5,3} - rI_{5,4} + (1-2\nu)(z-\zeta)(J_{3,1}-J_{3,3}) \\
& - \frac{3(3-4\nu)(z+\zeta)\rho}{r_m^2} \{\rho(J_{5,1}-J_{5,3}) - r(J_{5,2}-J_{5,4})\} \\
& + \frac{6\zeta}{r_m^2} \left\{ -z(z+\zeta)(J_{5,1}-J_{5,3}) + (1-2\nu)\rho[J_{5,1} \right. \\
& - J_{5,3}) - r(J_{5,2}-J_{5,4}) \left. \right\} + \frac{5z(z+\zeta)\rho}{r_m^2} [\rho(J_{7,1}-J_{7,3}) \\
& - r(J_{7,2}-J_{7,4}) \left. \right] \Big] \sin 2\theta \quad (11)
\end{aligned}$$

$$B = \frac{F_r \rho}{4\pi(1-\nu)}, \quad r_m = \sqrt{2r\rho} \quad (12)$$

$$I_{n,m} = \frac{1}{r_m^3} \int_0^\pi \frac{\cos^m \varphi}{(e_1 - \cos \varphi)^{n/2}} d\varphi \quad (13)$$

$$J_{n,m} = \frac{1}{r_m^3} \int_0^\pi \frac{\cos^m \varphi}{(e_2 - \cos \varphi)^{n/2}} d\varphi \quad (14)$$

$$e_1 = 1 + \frac{(r-\rho)^2 + (z-\zeta)^2}{2r\rho} \quad (15)$$

$$e_2 = 1 + \frac{(r-\rho)^2 + (z+\zeta)^2}{2r\rho} \quad (16)$$

The expressions of σ_{ij}^F and σ_{ij}^F are omitted because of the limited space. Definite integrals in Eqs. (13) and (14) have already been given by Nisitani and Noda⁽¹⁵⁾.

As seen from the above expressions, if σ_{ij}^F , σ_{ij}^F and σ_{ij}^F are used as the fundamental solutions, the stresses due to the above ring forces vary with θ in the form of $\cos 2\theta$ or $\sin 2\theta$ along the circumference. These

variations are similar to variations in the stresses to be cancelled on the imaginary pit. Therefore, if the stresses expressed in Eq. (3) are cancelled at an arbitrary point along the circumference by the body forces, the boundary conditions are automatically satisfied at all points along the same circumference.

Consequently the present method does not require the division along the circumference and the problem can be treated with high accuracy with a comparatively small number of unknowns.

2.2.2 Density of body force Denoting dP_r , dP_θ and dP_z respectively as the r -, θ - and z -directional body force acting on the infinitesimal area $\rho d\varphi \sqrt{(d\rho)^2 + (d\zeta)^2}$, the following definitions of the densities of body forces are used.

$$\left. \begin{aligned} & \text{For a semispheroidal pit,} \\ & \rho_r = \frac{dP_r}{\cos 2\varphi \rho d\varphi d\zeta} \\ & \rho_\theta = \frac{dP_\theta}{\sin 2\varphi \rho d\varphi d\zeta} \\ & \rho_z = \frac{-dP_z}{\cos 2\varphi \rho d\varphi d\zeta} \end{aligned} \right\} \quad (17)$$

For a drill-hole-shaped pit, section A-C :

$$\left. \begin{aligned} & \rho_r = \frac{dP_r}{\cos 2\varphi \rho d\varphi d\zeta} \\ & \rho_\theta = \frac{dP_\theta}{\sin 2\varphi \rho d\varphi d\zeta} \\ & \rho_z = \frac{dP_z}{\cos 2\varphi \rho d\varphi d\zeta} \frac{c}{\zeta} \end{aligned} \right\} \quad (18)$$

section C-B :

$$\left. \begin{aligned} & \rho_r = \frac{dP_r}{\cos 2\varphi \rho d\varphi d\zeta} \quad (\alpha \neq \pi) \\ & \rho_r = \frac{-dP_r}{\sin 2\varphi \rho d\varphi d\zeta} \frac{a}{\rho} \quad (\alpha = \pi) \\ & \rho_\theta = \frac{dP_\theta}{\sin 2\varphi \rho d\varphi d\zeta} \quad (\alpha \neq \pi) \\ & \rho_\theta = \frac{-dP_\theta}{\sin 2\varphi \rho d\varphi d\zeta} \frac{a}{\rho} \quad (\alpha = \pi) \\ & \rho_z = \frac{-dP_z}{\cos 2\varphi \rho d\varphi d\zeta} \end{aligned} \right\} \quad (18)$$

3. Numerical Results

Before analyzing the semi-infinite body with a pit under tension, the infinite body with a spheroidal cavity under tension perpendicular to the axis of its revolution was analyzed by means of the present method in order to check the accuracy of the present method. These results coincide with the Sadowsky and Sternberg's exact solution⁽¹⁶⁾ to 8 significant digits.

The numerical procedure can be put concretely as follows. The boundary of the pit is divided into N elements and the densities of the body forces are assumed to be step functions which take constant values in each element. The exact stresses of the

specified points corresponding to $N \rightarrow \infty$ are estimated by extrapolation⁽¹³⁾ from two values corresponding to two values of N . The pits in concrete numerical calculation are a semispheroidal and drill-hole shaped pit shown in Fig. 4.

The stress concentration factors of a semispheroidal and drill-hole-shaped pit under tension are given in Tables 1 and 2, respectively. The result⁽⁴⁾ by Fujita et al. are also put in Table 1, and the table shows that they coincide with the present results.

The stresses of point A and the maximum stress along A-C are plotted against the aspect ratio in Fig.

Table 1 Semi-infinite body with a semispheroidal pit under tension ($\nu=0.3$)

	a/b	Present result	Fujita et al. ⁽¹¹⁾
$\sigma_{xB}/\sigma_\infty$	0.2	2.7088 (2.7063)	2.047
	0.4	2.5157 (2.5118)	
	0.6	2.3312 (2.3290)	
	0.8	2.1748 (2.1738)	
	1.0	2.0460 (2.0454)	
$\sigma_{xA}/\sigma_\infty$	0.2	2.486 (2.8371)	1.986
	0.4	2.363 (2.5948)	
	0.6	2.228 (2.3751)	
	0.8	2.105 (2.1932)	
	1.0	1.994 (2.0454)	
$\sigma_{x \text{ max along A-B}}/\sigma_\infty$	0.2	2.896	2.062
	0.4	2.632	
	0.6	2.397	
	0.8	2.209	
	1.0	2.062	

() : Infinite body with a spheroidal cavity under tension⁽¹⁶⁾

Fig. 5 Stress concentration factor of point A

5 and 6, respectively. The figures show that all results smoothly extend to the results⁽¹⁷⁾ by Kawai and Endoh corresponding to $a/b \rightarrow 0$.

Figures 7 to 11 show the distribution of σ_x along the pit edge A-C. We can consider that the stresses near point A are almost independent of the pit shape near point B if $a/b \leq 0.5$. Also, σ_x values along edge B-C-A of a drill-hole shaped pit are displayed in Figs. 12 to 14, when α is constant. These figures show that the stress fields near edge B-C are similar if the values of α are the same.

Figures 15 and 16 show the stress distributions from point A and point B, respectively, in the case of a semispheroidal pit. In Fig. 16, their lines become one line independent of the shape of an ellipse. This is explained by the concept of 'Linear Notch Mechanics'⁽¹⁸⁾.

Figures 17 to 19 show the stress distributions

Table 2 Semi-infinite body with a drill-hole-shaped pit under tension ($\nu=0.3$)

	a/b	α		
		$\pi/2$	$2/3 \pi$	π
$\sigma_{xA}/\sigma_\infty$	0.2	2.523	2.526	2.529
	0.4	2.412	2.440	2.427
	0.6	2.200	2.303	2.386
	0.8	1.953	2.151	2.293
	1.0	—	2.001	2.213
$\sigma_{x \text{ max along A-C}}/\sigma_\infty$	0.2	2.98	2.99	3.01
	0.4	2.68	2.75	2.83
	0.6	2.32	2.48	2.65
	0.8	1.97	2.24	2.49
	1.0	—	2.03	2.36

Fig. 6 Maximum stress along pit edge A-C

Fig. 7 Stress distribution along pit edge A-C ($a/b=0.2$) Fig. 10 Stress distribution along pit edge A-C ($a/b=0.8$) Fig. 8 Stress distribution along pit edge A-C ($a/b=0.4$) Fig. 11 Stress distribution along pit edge A-C ($a/b=1.0$) Fig. 9 Stress distribution along pit edge A-C ($a/b=0.6$) Fig. 12 σ_x along edge B-C-A of a drill-hole shaped-pit ($\alpha=n/2$)

Fig. 13 σ_x along edge B-C-A of a drill-hole-shaped pit ($\alpha=2\pi/3$)

Fig. 14 σ_x along edge B-C-A of a drill-hole shaped pit ($\alpha=\pi$)

Fig. 15 Stress distribution from point A in case of a semi-spheroidal pit

Fig. 16 Stress distribution from point B in case of a semi-spheroidal pit

(a) $\alpha=\pi/2$

(b) $\alpha=2\pi/3$

(c) $\alpha=\pi$

Fig. 17 Stress distribution from point A in case of a drill-hole-shaped pit

Fig. 18 Stress distribution from point B in case of a drill-hole-shaped pit

Fig. 19 Stress distribution from point C in case of a drill-hole-shaped pit

from point A, point B and point C, respectively, in the case of a drill-hole shaped pit. Especially in Fig. 18, they are plotted on a logarithmic scale to compare with the singularity of the conical notch by Bazant and Keer⁽¹⁹⁾. We find that the stress distributions near point B approach Bazant and Keer's singularity gradually and decrease rapidly.

Finally, in order to consider the free-surface effect of the semi-infinite body, σ_x along the pit edge A-C divided by σ_{xA} at point A are plotted against z/a

in Fig. 20. We may think that the effect vanishes at a depth of about $a/2$. Moreover, Kawai and Endoh's results corresponding to the limit case are also drawn in this figure. It shows a similar tendency with the present result for $a/b=0.2$.

4. Conclusion.

Based on the body force method, the sophisticated way to analyze the semi-infinite body with a revolutionary pit under tension was presented. The

Fig. 20 Relative stress distribution along pit edge A-C

accurate numerical results were shown for the case of the semi-infinite bodies with a semispheroidal or drill-hole-shaped pit, and the stress distributions around the pit were discussed.

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