Specimen Size Requirement for J Characterization Under Mixed Mode Loading*

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An elastic/plastic finite element analysis was performed to provide insight into the minimum specimen size limitations for J characterization in large-scale yielding under a mixed-mode condition. The power-law hardening constitutive equation was employed and a plane strain condition was assumed. The criterion used to judge the degree of dominance was the extent of agreement of the stress field near the deformed crack tip with the results for the small-scale yielding. It was found that the ratio of the minimum ligament L to J/σ_o for the specimen under a mixed-mode condition necessary to ensure a J -based characterization is required to satisfy the next condition.

$L \geq 80J/\sigma_o$ for Mode I tensile specimen,

$L \geq (80 \sim 50)J/\sigma_o$ for Mode I tensile + Mode II Specimen and

$L \geq 50J/\sigma_o$ for Mode II specimen.

Key Words: Fracture, J -integral, Mixed-mode Condition, Minimum Specimen Size Limitation, Large Scale Yielding

1. Introduction

The investigation of elastic/plastic fracture is required for safety assessment of structures made of high-toughness material. Structure members are usually subjected to complex loadings and the tip of the existing crack is deformed under mixed-mode loadings. Rice's J -integral⁽¹⁾ has been widely employed as a criterion for the onset of crack growth under contained and fully plastic conditions of Mode I loading. For applying the J -integral as a measure of stress severity near a crack tip, the relevant dimensions of the specimen should satisfy some minimal size requirements, which can be stated as

$$L > MJ/\sigma_o \quad (1)$$

where σ_o is the yield stress and the value of M depends

on the type of loading. McMeeking and Parks⁽²⁾ have performed finite-strain/finite-element analyses of center-cracked tensile panel and single-edge crack bend specimens, and suggested that to ensure a J -characterizing singular field of the type derived by Hutchinson⁽³⁾ and Rice and Rosengren⁽⁴⁾ (the HRR field), the uncracked ligament length L should be

$$L \geq 25J/\sigma_o \quad \text{for bend-type specimen, and}$$

$$L \geq 200J/\sigma_o \quad \text{for center-cracked specimen.}$$

(2)

The above value of 200, however, may be reduced to 80 for ensuring J -controlled crack-tip characteristics⁽⁵⁾. Shih⁽⁶⁾ has also obtained the size requirements of cracked bodies subjected to combined tension and bending loading. Although the size requirements have been investigated for the Mode I loading, mixed-mode loading problems have received little attention⁽⁷⁾.

In this paper, the stress and strain distribution near a crack tip under mixed-mode (Mode I + Mode II) loading is analyzed by a large deformation finite element method based upon the updated Lagrangean method and the minimum size requirement necessary to ensure the J -controlled crack-tip characteristics is discussed.

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2. Numerical Procedure

The large deformation finite element program⁽⁸⁾ developed by the authors is used. The material behavior is modeled by the the following power hardening rule

$$\frac{\bar{\tau}_s}{\tau_o} = \begin{cases} \gamma/\gamma_o & \sigma_m \leq \sigma_o \\ (\gamma/\gamma_o)^N & \sigma_m > \sigma_o \end{cases} \quad (3)$$

where $\bar{\tau}_s$ denotes the effective shear stress ($\bar{\tau}_s = \sigma_m/\sqrt{3}$), σ_m the effective tensile stress, τ_o the yield stress in

shear ($\tau_o = \sigma_o/\sqrt{3}$), γ_o the yield strain in shear and γ the total (elastic plus plastic) equivalent shear strain. The ratio of σ_o to Young's modulus is specified by $\sigma_o/E = 1/300$, the strain hardening coefficient $N=0.1$, and the Poisson's ratio $\nu=0.3$. The incremental form of Eq. (3) according to the J_2 flow theory of plasticity is employed and a plane strain condition is assumed.

Figure 1 shows the finite element mesh used for the present investigation. A slit, which simulates a crack, is introduced on the negative segment of the X axis and the initial radius of the slit is represented by $b_o/2$. The ratio of crack length a to specimen width W is assumed to be $1/2$. Boundary conditions applied for the pure Mode I and Mode II deformations are illustrated in Figs. 2(a) and (b), respectively. For the Mode I deformation, the uniaxial stress σ_{appl} is prescribed along the upper and lower specimen boundaries, which corresponds to the problem of a center-notched plate subjected to a uniaxial tension. For the Mode II deformation, the displacement u_{appl} along the X direction is prescribed. For mixed-mode conditions, both σ_{appl} and u_{appl} are applied simultaneously. The mixed-mode ratio is defined by ψ which is the angle between the uniaxial tensile loading direction and the crack plane in the inclined crack problem (see Fig 3) and is expressed as, for a small-scale yielding condition,

$$\psi = \tan^{-1}(K_I/K_{II}) \quad (4)$$

(a) The whole configuration of the finite element mesh. A total of 2327 nodes and 4504 constant strain triangular elements are used

(b) A minute subdivision of the finite element mesh near the crack tip

Fig. 1 The finite element mesh

Fig. 2 Schematic representation of specimen geometries and boundary conditions considered. Mixed-mode loadings are given by the superposition of two loading conditions.

- (a) Uniaxial tensile stress for Mode I loading
(b) Shear displacement for Mode II loading

where K_I and K_{II} are the stress intensity factors of Mode I and II, respectively. The relationship between the mixed-mode ratio and the ratio of applied tensile stress σ_{appl} to shear displacement u_{appl} has been computed preliminarily using the small strain finite element method with the mesh shown in Fig. 4, and the results are shown in Fig. 5. In the present paper, four cases of mixed-mode ratios are considered: (1) $\psi = 90^\circ$, (2) $\psi = 60^\circ$, (3) $\psi = 30^\circ$, (4) $\psi = 0^\circ$. These ratios correspond to (1) $K_I/K_{II} = \infty$, (2) $K_I/K_{II} =$

Fig. 3 Angle between crack surface and uniaxial tensile loading direction

Fig. 4 Finite element breakdown used for computing mixed mode ratio ψ

1.732, (3) $K_I/K_{II} = 0.577$, (4) $K_I/K_{II} = 0$, respectively.

3. Results and Discussion

3.1 The shape of plastic region

Figures 6(a)~(d) show the configuration of the plastic region at various loading stages; the level of loading is indicated by $L/(J_r/\sigma_o)$ where J_r is the value of the J -integral computed by the line integral along the contour far from the crack tip. X_o and Y_o are the rectangular Cartesian coordinates in the undeformed state. In a small deformation range, a self-similar expansion of the plastic region takes place as the load increases, and its configuration is similar to that obtained in a previous paper⁽⁹⁾ with the assumption of small-scale yielding. However, as the specimen undergoes large deformation, the shape of the plastic zone is gradually affected by the specimen boundaries.

3.2 Stress field

Figures 7(a)~(d) show the variation of the normalized circumferential stress σ_θ/σ_o along the radial line on which σ_θ takes its maximum value. The distance R_o of the material point from the crack tip in the undeformed configuration is normalized by J_r/σ_o and the dashed line is the small-scale yielding result of Aoki et al⁽⁹⁾. The stress field near the crack tip can be divided into four characteristic fields⁽⁵⁾⁽⁹⁾. (i) the field dominated by the stress intensity factor, A_K (the K field), (ii) the field dominated by the HRR singularity, A_H (the HRR field), (iii) the field dominated by the deformation of the crack tip, A_B (the blunted crack-tip field), and (iv) the region nearest the crack tip affected by microvoids, A_D (the damaged region). These fields are indicated by hatching in the figures with the K field outside the HRR field omitted. Under well-contained plasticity or small-scale yielding conditions, the calculated fields are in good agreement with the small-scale yielding results, i.e., the HRR

Fig. 5 Mixed-mode ratio ψ versus the ratio of tensile stress σ_{appl} to shear displacement u_{appl}

field is achieved. With increasing plastic deformation, the stresses gradually fall below the small-scale yielding curve from remote regions. The disappearance of the HRR field occurs at lower J_r values with an increasing Mode I component. It is noted that even when the stress distribution deviates from the HRR field, there exists a J -value range where the stress field in the immediate vicinity of the crack tip (A_D and A_B region) is still in close agreement with that under the small-scale yielding condition.

3.3 Size requirement for the J -integral characterization

The departure of the crack-tip stress from the HRR field is gradual rather than abrupt; there is a certain amount of arbitrariness in determining when the stress and strain fields are no longer of the HRR type. For the Mode I case, McMeeking and Parks⁽²⁾ suggested Eq. (2) as the size limitation based on their comparison of crack-tip fields with those derived from small-scale yielding analyses. This limitation corresponds to the condition that the stress deviates about 30% from HRR singular field (see Fig. 7(a)). The applicability limit of the J -integral under mixed-

Fig. 6 The shape of the plastic region

Fig. 7 Variation of circumferential stress σ_θ on the radial line on which σ_θ takes its maximum value. Distance axis is normalized by J/σ_o . Dashed curve is solution for small-scale yielding⁽⁹⁾

mode loading, estimated from this criterion, is shown by the solid line in Fig. 8. The minimum ligament size to ensure the HRR field decreases with an increase in the Mode II component and takes the value of $115J/\sigma_o$ in the pure Mode II case, which is about 60% of that in the pure Mode I case.

In the actual fracture of materials, the fracture process such as the nucleation of voids and their coalescence takes place within a small region near the crack tip and the process of void nucleation, growth and coalescence is largely controlled by the local tensile and hydrostatic stress history. Thus, the applicability range of the J -integral for the Mode I case may not necessarily coincide with the loading range to ensure the HRR field. Figure 9 shows the variation of the normalized circumferential stress σ_θ/σ_o for $\psi = 30^\circ$ at fixed material points of $R_o/b_o = 30, 94, 215$ and 318 along the radial line on which σ_o takes its maximum value. Since the distance axis is normalized by J_r/σ_o ,

Fig. 8 Dependence of J -characterization size parameter M on the mixed mode ratio ψ . Solid line represents the size requirement to ensure the dominance of the HRR singular fields. Dashed line gives the minimum ligament length to assure J characterization of ductile crack initiation

Fig. 9 Stress history of material points. Distance axis is normalized by J_r/σ_0 . Dashed curve is solution for small-scale yielding⁽⁹⁾

values for fixed material points are plotted to move toward the origin with an increase in J_r value.

At $L/(J_r/\sigma_0)=97$, the stress values indicated by the symbols $\bullet$, $\blacksquare$, $\blacktriangle$ and $\blacklozenge$ have already fallen below the small-scale yielding result in region A_H , but the stress at the material point $R_0/b_0=30$ continues to be in close agreement with it. This result means that even though the HRR singular field has vanished, the stress history at the immediate vicinity of the crack tip remains essentially similar to that of the small-scale yielding. Since the distance axis is normalized by the J -integral, this stress history can be characterized by the J -integral. Based on these considerations, it is concluded that the applicability range of the J -integral as a characterizing parameter of ductile crack initiation may be judged from the criterion whether or not the stress on the boundary between A_D and A_B coincides with that in the small-scale yielding at the same J -value. The size requirement estimated by this criterion is shown as the dashed line in Fig. 8 and is expressed as

$$\begin{aligned} L &\geq 80J/\sigma_0 && \text{for Mode I tensile specimen} \\ L &\geq (80 \sim 50)J/\sigma_0 && \text{for Mode I tensile + Mode II specimen} \\ L &\geq 50J/\sigma_0 && \text{for Mode II specimen} \end{aligned} \quad (5)$$

The above result for the Mode I tensile specimen coincides with the previous results by the authors⁽⁵⁾. It is noted that the size requirement for ductile fracture initiation under J -dominant conditions is satisfied with about half the length of the ligament as compared with that to ensure the dominance of the HRR singularity, and does not so much depend on the mixed-mode ratio.

Figure 10 shows the value of the J -integral, computed as a contour integral, versus distance of the contour from the crack tip. The J -integral is almost

Fig. 10 Plot of J -integral, computed as a contour integral, versus distance of contour from the crack tip.

path-independent in regions A_H and A_B , and the path dependence of J is only detectable within the A_D region. This result also suggests that the near-tip field is characterized by the J -integral even when the HRR singular field vanishes.

4. Conclusions

The applicability range of the J -integral under mixed-mode loading conditions was investigated by using a finite element method suitably formulated to admit large geometry change. It was found that even when the stress distribution deviates from the HRR singular field, there exists a J -value range where the stress field in the immediate vicinity of the crack tip is in close agreement with that of small-scale yielding. This suggests that for such a J -value range, the fracture process near the crack tip is the same as in the small-scale yielding and that the fracture initiation can be characterized by the J -integral. Based on the present computation, we would propose Eq. (5) as a size limitation to ensure the J characterization under mixed-mode loadings.

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