Optimum Compliance Design of Stiffener Layout of Thin Plate*

(A Treatment as Pseudocontinuous Structure)

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An optimum design technique of a stiffener layout to achieve minimum compliance is developed. A design model of a thin plate with stiffeners is treated as an anisotropic pseudocontinuum and discretized into finite elements. The minimum compliance design problem subjected to constant volume in which the angles of the stiffener arrangement and its density distributions in the orthogonal directions are varied, is solved by a recursive quadratic programming technique. Optimal stiffener layouts of a rectangular plate under some typical loading and supporting conditions are obtained numerically.

Key Words : Design Engineering, Minimum Compliance Design, Optimum Layout of Stiffener, Pseudocontinuous Structure. Recursive Quadratic Programming, Finite Element Method.

1. Introduction

The reinforcement of plates and shell structures by stiffeners is a useful and practical means for obtaining stronger and more rigid structures than other-type ones, such as solid plates of varying thickness, which have the same volume as the stiffened structures. While the optimum stiffener design problem of a thin plate is very important practically, very few works to predict the optimum stiffener shape and layout have been done. Recently Cheng and Olhoff^{(1),(2)} studied the minimum compliance design of annular plates with circumferential stiffeners by introducing a new concept of integral stiffeners, that is, the plate is equipped with an infinite number of integral stiffeners with varying height and infinitesimal width. This new plate model can express the theoretical limitation of

unidirectionally stiffened plate structures, and they discussed the distribution of the stiffeners. But the possibility and limitation of bidirectional stiffeners to decrease the compliance and suitable direction of the stiffener arrangement have never been investigated.

In this paper, a new plate model equipped with bidirectional integral stiffeners will be proposed to obtain the optimum layout and distribution of stiffeners which can achieve the minimum compliance under any given loading and supporting conditions. The model to minimize the plate compliance which can be used for searching the theoretical limitations will be developed from Cheng and Olhoff's integrally stiffened plate. The new design model of thin plate with stiffeners is discretized into finite elements with anisotropic material. Then the minimum compliance design problem subjected to volume constraint will be formulated by taking the directions and rigidities of the bidirectional stiffeners in each element as design variables, and solved by using the recursive quadratic programming technique. Pshenichny's linearization method will be adopted here together with design sensitivity analysis. To demonstrate the applicability of the bidirectional integral stiffener model, optimum

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stiffener layouts and density distributions of rectangular plates under typical loading and supporting conditions will be determined numerically.

2. Minimum Compliance Design Formulation

2.1 Integral stiffener model

Consider a thin plate equipped on upper and lower surfaces with an infinite number of bidirectional stiffeners with continuously varying height and infinitesimal width, so-called integral stiffeners. At first, the bidirectional integral stiffeners are assumed to cross orthogonally to one another and to be arranged along an orthogonal curvilinear net when the minimum compliance design is achieved, because the orthogonal arrangement of the stiffeners is most effective in a biaxial moment field. Then the plate equipped with the integral stiffeners in the orthogonal directions can be treated as a pseudocontinuum of the orthotropic solid plate.

Let us begin to write the stress-strain relationship of this pseudocontinuum. The middle plane of the orthotropic solid plate will be taken as the xy -plane and the z -axis is taken perpendicular to the middle plane, as shown in Fig. 1(a). The local Cartesian coordinates (α, β) are defined in the tangent directions to the curved integral stiffener. An angle between the x -axis and the tangent to the α -axis is denoted by φ . Consider a small element cut out from the plate by two pairs of planes to the αz and βz planes, as shown in Fig. 1(b). The deflection of the middle plane being denoted by w , the stress components in the α and β directions will be derived in the following. The thin-plate theory gives the relationship between the stress components $(\sigma_{\alpha\alpha}^p, \sigma_{\beta\beta}^p, \sigma_{\alpha\beta}^p)$ in the real solid part of the plate ($2|z| \leq h_0$, where h_0 denotes the solid plate thickness) and the deflection w as follows:

$$\begin{aligned}\sigma_{\alpha\alpha}^p &= -\frac{Ez}{1-\nu^2}(w_{,\alpha\alpha} + \nu w_{,\beta\beta}), \\ \sigma_{\beta\beta}^p &= -\frac{Ez}{1-\nu^2}(\nu w_{,\alpha\alpha} + w_{,\beta\beta}), \quad \sigma_{\alpha\beta}^p = -\frac{Ez}{1-\nu^2}w_{,\alpha\beta}\end{aligned}\quad (1)$$

(a) Local coordinates definition

in which E and ν denote Young's modulus and Poisson's ratio of the plate, and subscripts " $\alpha\alpha$ ", " $\beta\beta$ " and " $\alpha\beta$ " imply the partial differentiation with respect to α and β . On the other hand, if the resistance of the orthogonally crossing integral stiffeners for the twisting deformation is neglected, we can use the well-known beam theory and write the stress components $(\sigma_{\alpha\alpha}^s, \sigma_{\beta\beta}^s, \sigma_{\alpha\beta}^s)$ in the integral stiffener part of the plate ($2|z| \leq h_0$) in terms of the deflection w as follows:

$$\sigma_{\alpha\alpha}^s = -Ezw_{,\alpha\alpha}, \quad \sigma_{\beta\beta}^s = -Ezw_{,\beta\beta}, \quad \sigma_{\alpha\beta}^s = 0 \quad (2)$$

Next let us consider the equilibrium of the bending moment acting on the section of the element in Fig. 1(b). Denoting the heights of the integral stiffener in the α and β directions by h_α and h_β , the normal stresses distributed over the lateral side of both parts of the solid plate and the stiffener in the α direction can be reduced to a couple, the magnitude of which per unit length is equal to the bending moment $M_{\alpha\alpha}$. In this way, we obtain

$$\Delta\beta M_{\alpha\alpha} = -2\Delta\beta \int_0^{\frac{h_\alpha}{2}} \sigma_{\alpha\alpha}^p z dz - 2\Delta C_\alpha \int_{\frac{h_0}{2}}^{\frac{h_\alpha}{2}} \sigma_{\alpha\alpha}^s z dz \quad (3)$$

where $\Delta\beta$ is the width in the α section and ΔC_α is the total stiffener width per $\Delta\beta$. By substituting from Eqs. (1) and (2) into Eq. (3) and noting that $\Delta\beta$ and ΔC_α are infinitesimal, the bending moment can be expressed by the deflection w as

$$M_{\alpha\alpha} = -D_0 \left[1 + (1-\nu^2) \left\{ \left(\frac{h_\alpha}{h_0} \right)^3 - 1 \right\} b_\alpha \right] w_{,\alpha\alpha} - D_0 \nu w_{,\beta\beta}, \quad (4)$$

in which $D_0 = Eh_0^3/12(1-\nu^2)$ and b_α is a ratio of stiffener density defined as $b_\alpha = \Delta C_\alpha/\Delta\beta$. In the same way, we can define the bending moment $M_{\beta\beta}$ in the β direction and the twisting moment $M_{\alpha\beta}$:

$$M_{\beta\beta} = -D_0 \nu w_{,\alpha\alpha} - D_0 \left[1 + (1-\nu^2) \left\{ \left(\frac{h_\beta}{h_0} \right)^3 - 1 \right\} b_\beta \right] w_{,\beta\beta}, \quad (5)$$

$$M_{\alpha\beta} = -D_0(1-\nu)w_{,\alpha\beta}. \quad (6)$$

In Eq. (5), $b_\beta = \Delta C_\beta/\Delta\alpha$ is a ratio of stiffener density in

(b) Detailed structure of pseudocontinuum

Fig. 1 Pseudocontinuum model of an integrally stiffened plate

the β direction. By comparing these moment-deflection relationships with those of a solid plate of thickness h_0 , one finds that the new stiffened plate model can be treated as a pseudo-orthotropic plate with the following stress-strain relationship :

$$\begin{aligned}\sigma_{\alpha\alpha} &= \frac{E\{1+(1-\nu^2)\rho_\alpha\}}{1-\nu^2}\varepsilon_{\alpha\alpha} + \frac{E\nu}{1-\nu^2}\varepsilon_{\beta\beta}, \\ \sigma_{\beta\beta} &= \frac{E\nu}{1-\nu^2}\varepsilon_{\alpha\alpha} + \frac{E\{1+(1-\nu^2)\rho_\beta\}}{1-\nu^2}\varepsilon_{\beta\beta}, \\ \sigma_{\alpha\beta} &= \frac{E}{2(1+\nu)}\varepsilon_{\alpha\beta},\end{aligned}\quad (7)$$

in which ρ_α and ρ_β represent some ratios of stiffener rigidity to that of the plate defined as

$$\rho_\alpha = \left\{ \left(\frac{h_\alpha}{h_0} \right)^3 - 1 \right\} b_\alpha, \quad \rho_\beta = \left\{ \left(\frac{h_\beta}{h_0} \right)^3 - 1 \right\} b_\beta. \quad (8)$$

The above derivation allows us to treat the minimum compliance design problem of an integrally stiffened plate as a material design of an anisotropic plate having a constant thickness h_0 and varying flexural rigidity.

2.2 Minimum compliance design formulation

If we consider the minimum compliance design problem of the pseudo-orthotropic plate having the stress-strain relationship in Eq. (7), the objective function f with the angle φ of stiffener arrangement and the ratio of stiffener rigidities ρ_α, ρ_β as the design variables can be formulated as follows.

$$f(\varphi, \rho_\alpha, \rho_\beta) = \frac{1}{2} \iint_\Omega p w dx dy \rightarrow \min, \quad (9)$$

subjected to

$$\rho_\alpha \geq 0, \quad \rho_\beta \geq 0, \quad (10)$$

where the plate is applied a distributed load p . In Eq. (9), the integration has to be carried out over the plate surface.

Equation (9) is equivalent to minimize the strain energy of the loaded plate and implies to maximize the average flexural rigidity. Furthermore, the deflection w in Eq. (9) has to be a stationary point of the following total potential energy functional π :

$$\begin{aligned}\pi &= \frac{D_0}{2} \iint_\Omega \{ [1+(1-\nu^2)\rho_\alpha] w_{,\alpha\alpha}^2 + 2\nu w_{,\alpha\alpha} w_{,\beta\beta} \\ &\quad + [1+(1-\nu^2)\rho_\beta] w_{,\beta\beta}^2 + 2(1-\nu) w_{,\alpha\beta}^2 \} dx dy \\ &\quad - \iint_\Omega p w dx dy.\end{aligned}\quad (11)$$

For existence of bounded values of ρ_α, ρ_β , we must constrain the upper limit of the total volume as

$$\begin{aligned}g &= \iint_\Omega [h_0 + (h_\alpha - h_0)b_\alpha + (h_\beta - h_0)b_\beta \\ &\quad - \{\min(h_\alpha, h_\beta) - h_0\} b_\alpha b_\beta] da d\beta - V_0 \leq 0,\end{aligned}\quad (12)$$

in which V_0 is a prescribed total volume. Otherwise, the lower the compliance is required, the greater the ratios ρ_α, ρ_β of stiffener rigidity become. Since it is difficult to write the relationship between the stiffener heights h_α, h_β and the densities ρ_α, ρ_β when the minimum compliance is achieved, and since the stiffener

height will be constrained by the buckling failure in a discrete realistic stiffener design, we will here deal with some cases where the stiffener height or the density of width has a constant value.

3. Finite Element Model and Its Optimization

3.1 Discretization into finite elements

Let us discretize the new plate model into isoparametric elements with 8 nodes for seeking the optimum stiffener layout. If we introduce normalized curvilinear coordinates (ξ, η) defined on the middle plane of the plate and adopt a linear function $R_i(\xi, \eta)$ to interpolate the angle φ to arrange the stiffener and the ratios ρ_α, ρ_β of stiffener rigidity at any point, then,

$$\varphi = \sum_i R_i \varphi_i, \quad \rho_\alpha = \sum_i R_i \rho_{\alpha i}, \quad \rho_\beta = \sum_i R_i \rho_{\beta i}, \quad (13)$$

in which $\varphi_i, \rho_{\alpha i}$ and $\rho_{\beta i}$ are the values at a corner node i .

If we denote the nodal displacement vector at node i as $\{\delta\}_i = (w_i, \theta_{xi}, \theta_{yi})^T$, in which θ_{xi} and θ_{yi} are the rotations about the x - and y - axes, and if a quadratic interpolation function $N_i(\xi, \eta)$ is adopted, we can write the displacement components u, v and w in the direction of the global x, y and z -axes as

$$\begin{aligned}u &= -\sum_i N_i \zeta \frac{h_0}{2} \theta_{yi}, \quad v = \sum_i N_i \zeta \frac{h_0}{2} \theta_{xi}, \\ w &= \sum_i N_i w_i,\end{aligned}\quad (14)$$

where ζ is the normalized coordinate normal to the $\xi\eta$ plane^{(3),(4)}. Then, if the total nodal displacement vector in the element is denoted by $\{\delta\} = (\delta_1, \dots, \delta_8)^T$, the global coordinates of the strain vector $\{\varepsilon\} = (\varepsilon_x, \varepsilon_y, \gamma_{xy})^T$ in the element are given in the usual manner.

$$\{\varepsilon\} = [B]\{\delta\}, \quad (15)$$

where $[B]$ is the matrix defined in terms of the displacement derivatives with respect to the global Cartesian coordinates. We are now interested in the strain components, which are denoted by $\{\bar{\varepsilon}\} = (\varepsilon_{\alpha\alpha}, \varepsilon_{\beta\beta}, \varepsilon_{\alpha\beta})^T$, in the α, β local coordinates, and they are obtained from Eq. (15) by the necessary coordinate transformation.

$$\{\bar{\varepsilon}\} = [T][B]\{\delta\}, \quad (16)$$

where

$$[T] = \begin{bmatrix} c^2 & s^2 & cs \\ s^2 & c^2 & -cs \\ -cs & cs & c^2 - s^2 \end{bmatrix}, \quad \begin{aligned} c &= \cos \varphi, \\ s &= \sin \varphi, \end{aligned} \quad (17)$$

and the corresponding stress components $\{\bar{\sigma}\} = (\sigma_{\alpha\alpha}, \sigma_{\beta\beta}, \sigma_{\alpha\beta})^T$ are also given by

$$\{\bar{\sigma}\} = [D][T][B]\{\delta\}, \quad (18)$$

in which $[D]$ is the stress-strain matrix derived from Eq. (7). Finally, we can evaluate the element stiffness matrix $[K]^e$ of the pseudocontinuum in the global coordinates as follows :

$$[K]^e = \int_{-1}^1 \int_{-1}^1 \int_{-1}^1 [B]^T [T] [D] [T] [B] f d\xi d\eta d\zeta,$$

(19)

in which $|J|$ represents the determinant of the Jacobian matrix.

3.2 Optimization and sensitivity analysis

After the discretization into finite elements, the values φ_i , ρ_{ai} and $\rho_{\beta i}$ at every corner node i in each element are taken as the design variables, and the objective function in Eq. (9) must be minimized taking account of the constraints of Eqs. (10) and (12).

We adopted Pshenichny's linearization method⁽⁵⁾ as the practical optimization technique. This recursive quadratic programming technique is used repeatedly for improving the design point, at which the objective function is expanded to the second order approximation and the constraints are linearized. Since this method needs design sensitivities of the objective and constraint functions, these sensitivities will be formulated in the following.

3.2.1 Design sensitivity of objective function

Dems and Mróz have derived the first variation of arbitrary stress, strain and displacement functionals corresponding to the variation of material parameters within a specified domain by using the solution for primary and adjoint systems⁽⁶⁾. We can use their formulation to the displacement functional of Eq. (9). If we denote the general design vector as $x_i = (\varphi_i, \rho_{ai}, \rho_{\beta i})$ and the stress-strain relationship as $\sigma = D\epsilon$, then the first variation of Eq. (9) can be written as

$$\delta f = -\frac{1}{2} \int_{\Omega} \epsilon^T \frac{\partial D}{\partial x_i} \epsilon \delta x_i d\Omega. \quad (20)$$

In our case, since the strain vector ϵ and the D correspond to $\{\bar{\epsilon}\}$ and $[T]^T [D] [T]$, respectively, the practical form of the first variation δf will be

$$\begin{aligned} \delta f = & -\frac{1}{2} \iint_{\Omega} \left[\{\bar{\epsilon}\}^T \left(\frac{\partial [T]}{\partial \varphi_i} [D] [T] \right. \right. \\ & + [T]^T [D] \frac{\partial [T]}{\partial \varphi_i} \left. \right) \{\bar{\epsilon}\} \delta \varphi_i \\ & - \{\bar{\epsilon}\}^T [T]^T \frac{\partial [D]}{\partial \rho_{ai}} [T] \{\bar{\epsilon}\} \delta \rho_{ai} \\ & \left. - \{\bar{\epsilon}\}^T [T]^T \frac{\partial [D]}{\partial \rho_{\beta i}} [T] \{\bar{\epsilon}\} \delta \rho_{\beta i} \right] dxdy, \end{aligned} \quad (21)$$

because the matrix $[T]$ depends on only φ_i and the matrix $[D]$ only ρ_{ai} and $\rho_{\beta i}$. From Eqs. (7) and (17), the components of $\partial [T] / \partial \varphi_i$, $\partial [D] / \partial \rho_{ai}$ and $\partial [D] / \partial \rho_{\beta i}$ can be derived as

$$\begin{aligned} \frac{\partial [D]}{\partial \rho_{ai}} &= ER_i \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \frac{\partial [D]}{\partial \rho_{\beta i}} = ER_i \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\ \frac{\partial [T]}{\partial \varphi_i} &= \frac{\partial [T]}{\partial \varphi} R_i. \end{aligned} \quad (22)$$

3.2.2 Design sensitivity of volume constraint

Next, the sensitivity of the volume constraint Eq. (12) will be derived for both cases when the stiffener heights h_a and h_b are prescribed, and when the densities of width b_a and b_b are prescribed. When h_a and

h_b are prescribed, the variables ρ_a and ρ_b will be changed by the variation of b_a and b_b . Then the evaluation of the volume constraint will be discretized by the transformation to the normalized coordinates ξ, η as

$$\begin{aligned} g = & h_0 \sum_e \int_{-1}^1 \int_{-1}^1 \left\{ 1 + \frac{h_0^3}{h^2 + hh_0 + h_0^3} \right. \\ & \left. \left(\rho_a + \rho_b - \frac{h_0^3}{h^3 - h_0^3} \rho_a \rho_b \right) \right\} |J| d\xi d\eta - V_0 \leq 0. \end{aligned} \quad (23)$$

in which $\sum$ means the summation of all elements and the partial volume of each element will be integrated numerically. Differentiating Eq. (23) with respect to the design variables, we obtain the design sensitivities of the volume constraint.

$$\begin{aligned} \frac{\partial g}{\partial \rho_{ai}} &= \frac{h_0^3}{h^2 + hh_0 + h_0^3} \\ & \sum_e \int_{-1}^1 \int_{-1}^1 R_i \left(1 - \frac{h_0^3}{h^3 - h_0^3} \sum_j R_j \rho_{\beta j} \right) |J| d\xi d\eta, \\ \frac{\partial g}{\partial \rho_{\beta i}} &= \frac{h_0^3}{h^2 + hh_0 + h_0^3} \\ & \sum_e \int_{-1}^1 \int_{-1}^1 R_i \left(1 - \frac{h_0^3}{h^3 - h_0^3} \sum_j R_j \rho_{aj} \right) |J| d\xi d\eta. \end{aligned} \quad (24)$$

On the other hand, when b_a and b_b are prescribed, the variables ρ_a and ρ_b will be changed by the variation of h_a and h_b . Then the evaluation of g will be discretized as

$$\begin{aligned} g = & h_0 \sum_e \int_{-1}^1 \int_{-1}^1 \left\{ (1-b)^2 + \left(\sqrt[3]{\frac{\rho_a}{b}} + 1 + \sqrt[3]{\frac{\rho_b}{b}} + 1 \right) b \right. \\ & \left. - 3 \sqrt[3]{\frac{\min(\rho_a, \rho_b)}{b}} + 1 b^2 \right\} \times |J| d\xi d\eta. \end{aligned} \quad (25)$$

and the corresponding sensitivities of the volume constraint will be given in the region where $\rho_a \geq \rho_b$ as

$$\begin{aligned} \frac{\partial g}{\partial \rho_{ai}} &= \frac{h_0}{3} \sum_e \int_{-1}^1 \int_{-1}^1 R_i \left(\frac{1}{b} \sum_j R_j \rho_{aj} + 1 \right)^{-\frac{2}{3}} |J| d\xi d\eta, \\ \frac{\partial g}{\partial \rho_{\beta i}} &= \frac{h_0(1-b)}{3} \\ & \sum_e \int_{-1}^1 \int_{-1}^1 R_i \left(\frac{1}{b} \sum_j R_j \rho_{\beta j} + 1 \right)^{-\frac{2}{3}} |J| d\xi d\eta, \end{aligned} \quad (26)$$

and in the region where $\rho_a < \rho_b$ as

$$\begin{aligned} \frac{\partial g}{\partial \rho_{ai}} &= \frac{h_0(1-b)}{3} \sum_e \int_{-1}^1 \int_{-1}^1 R_i \left(\frac{1}{b} \sum_j R_j \rho_{aj} + 1 \right)^{-\frac{2}{3}} |J| d\xi d\eta, \\ \frac{\partial g}{\partial \rho_{\beta i}} &= \frac{h_0}{3} \sum_e \int_{-1}^1 \int_{-1}^1 R_i \left(\frac{1}{b} \sum_j R_j \rho_{\beta j} + 1 \right)^{-\frac{2}{3}} |J| d\xi d\eta. \end{aligned} \quad (27)$$

The derivative $\partial g / \partial \varphi_i$ is equal to zero in both regions.

4. Numerical Example

4.1 Simply supported plate under uniform load

As the first design example, let us consider a simply supported square plate of width $2a$ and thickness h_0 under a uniformly distributed load p , as shown in Fig. 2. Because of symmetry, a quarter region is discretized into 4 elements and the values φ_i , ρ_{ai} and $\rho_{\beta i}$ in each corner node are taken as the design vari-

ables. We prescribe the ratio $h_0/a=0.01$, stiffener heights $h_a=h_b=2h_0$ and volume limitation $V_0=1.19a^2h_0$, and adopt the initial values of $\varphi_i=0$ and $\rho_{ai}=\rho_{bi}=0.7$.

After 113 repetitions of Pshenichny's linearization method, the ratio f to f_0 , which is the value of the corresponding uniform plate with volume V_0 , decreased from 1.27 to 0.811, that is, the flexural rigidity of the stiffened plate increased 56.7%. The optimum layout of the integral stiffener is shown in Fig. 3(a), which is drawn by solving the following ordinary differential equation derived from $dy/dx=\tan\varphi$ for the optimum value of φ_i

Fig. 2 Design model of simply supported plate under uniform load

(a) Layout drawn numerically in the quarter region

(b) Layout prediction in the full region

Fig. 3 Optimum layout illustrations of integral stiffener

$$\frac{d\xi}{d\eta} = - \frac{\sum_i \frac{\partial R_i}{\partial \xi} (y_i - x_i \tan \varphi)}{\sum_i \frac{\partial R_i}{\partial \eta} (y_i - x_i \tan \varphi)} \quad (28)$$

Many contours are calculated for arbitrary initial values φ_i , but the intervals between them don't relate to the stiffener density. From this figure, we can notice that the optimum layout consists of the stiffeners emanating from the center of the plate in the radial direction and of concentric circumferential stiffeners. The optimum stiffener layout in the full region drawn freehandedly by taking the characteristics of Fig. 3(a) and the condition of symmetry into account is shown in Fig. 3(b). Furthermore, the distributions of the rigidity ratios ρ_a and ρ_b are shown in Fig. 4 for a quarter region. From these figures, the integral stiffeners in the radial direction distribute densely at the center and the edge corner of the plate, but the circumferential stiffener density achieves its maximum at the center of the quarter region.

4.2. Square plate supported at 4 corner points

Next we consider a square plate of width $2a$ and thickness h_0 ($h_0/a=0.018$) supported at 4 edge corners and applied a concentrated load P at the center, as shown in Fig. 5. A quarter region is discretized into 9 elements and the total volume and the stiffener den

(a) Radial stiffener distribution

(b) Circumferential stiffener distribution

Fig. 4 Distributions of stiffener rigidity

sities are prescribed as $V_0 = 3.72a^2h_0$ and $b_\alpha = b_\beta = 0.0412$. Initial values of the stiffener heights at all nodes are assumed as $h_\alpha = h_\beta = 4h_0$.

The optimum values of the design variables are obtained after 275 iterative steps of Pshenichny's linearization method and the ratio f/f_0 changed from 0.663 to 0.567, that is, the integrally stiffened plate at the initial step has 1.5 times the rigidity of the corresponding uniform plate with the same volume, and the

Fig. 5 Design model of square plate supported at 4 corner points

(a) Layout drawn numerically in the quarter region

(b) Layout prediction in the full region

Fig. 6 Optimum layout illustrations of integral stiffener

rigidity of the optimum solution is 76.4% greater than that of the corresponding uniform plate.

Figure 6(a) shows the optimum stiffener layout drawn by the same manner of description as for Fig. 3. The optimum layout consists of an orthogonally crossing net which is similar to hyperbolas with asymptotes of the x , y -axes and the diagonals of the plate. The optimum layout in the full region is also illustrated in Fig. 6(b). The distributions of the rigidity ratios ρ_α , ρ_β are shown in Fig. 7 for a quarter region. The integral stiffener arranged along the hyperbola with the asymptotes of the x and y -axes is higher than that arranged along the hyperbola with the asymptotes of the diagonals.

5. Conclusions

A new pseudocontinuous model of a plate with orthogonal stiffeners is proposed to obtain an optimum layout and distribution of stiffeners for minimum compliance. The stress-strain relationship of the continuum is derived and the minimum compliance problem subjected to volume constraint is formulated as a mathematical programming technique by using the finite element method. Unique layouts of the stiffener are obtained numerically for two examples

(a) Hyperbolic stiffener with asymptotes of diagonals of the plate

(b) Hyperbolic stiffener with asymptotes of the x , y axes

Fig. 7 Distributions of stiffener rigidity

of a square plate under some supporting and loading conditions.

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