Inelastic Constitutive Equation on the Basis of Distribution Function of Yield Stress*

(Basic Equation and Parameter for Work Hardening and Softening)

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In this paper, we propose the distribution function of the yield stress to characterize the inelastic behavior of metals. This inelastic model consists of infinitely small cells which have perfectly elastoplastic properties. The distribution function of the yield stress means the content of infinitely small cells. The integral constitutive equation for this model is derived under the combined stress state. To give the effect of the work hardening and softening to the model, we propose the scalar parameter which is the integrated value of inner products of incremental plastic strain vectors along the plastic strain path. This parameter is elaborated by the experimental data of the tensile stress-strain curve and the uniaxial cyclic curves for SUS 304 stainless steel. Biaxial stress-strain relations are calculated with the constitutive equation, including this parameter. It is shown that the work hardening and softening phenomena can be simulated with this constitutive equation and this parameter.

Key Words: Plasticity, Constitutive Equation, Strain History, Combined Stress, Cyclic Plasticity, Stainless Steel

1. Introduction

Mathematical modeling of inelastic behavior of materials is one of the most difficult problems. The main reasons for the difficulties are usually as follows:

(1) Inelastic behavior is both a nonlinear and hysteretic phenomenon.

(2) The behavior depends upon the kind of materials.

(3) The behavior strongly depends upon the environmental condition, for example, temperature.

Recently the detailed inelastic analysis of structures has become necessary in many technical fields. Therefore, many inelastic constitutive theories considering both plastic and viscous properties have been proposed^{(1)~(6)}. Inoue and his co-workers⁽⁷⁾ have formed detailed comparisons of accuracy of several constitutive equations.

In this paper, we propose a new plastic mathemat-

ical model and a hardening and softening parameter applicable to the combined stress-strain response of a material. Our model is based on the distribution function of yield stress⁽⁸⁾. The function is the content of infinitely small cells which have perfectly elastic plastic properties. It is found that strain hardening of this model corresponds to a shift of the distribution function. To indicate strain softening caused by a curvature of the strain path, the parameter of hereditary integration is derived with respect to the shape of the plastic strain path. The material constants of the model are determined using the experimental data of SUS 304 stainless steel. The comparison of the calculated results with the experimental results is conducted.

2. Basic Constitutive Equation

2.1 One-dimensional model of distribution function of yield stress

The one-dimensional Fraction model is schematically shown in Fig. 1. This model consists of a number of units of the ideal perfect plastic materials, placed in parallel. The constitutive equation of the Fraction model is as follows,

* Received 20th June, 1988. Paper No. 87-0440 A

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$$\langle \sigma \rangle = \sum_{i=1}^N \phi_i \sigma_i(\varepsilon), \quad \sum_{i=1}^N \phi_i = 1 \quad (1)$$

where $\langle \sigma \rangle$, $\sigma_i(\varepsilon)$ and ϕ_i are the stress of the model, the stress of the i -th cell deformed by the strain ε and the volume content of the i -th cell, respectively. If the Fraction model is expanded to consist of an infinite number of cells, ϕ_i is substituted for the continuous function and Eq. (1) is transformed into the following equation,

$$\langle \sigma \rangle = \int_0^\infty \psi(\xi) \cdot \sigma(\varepsilon, \xi) \cdot d\xi, \quad \int_0^\infty \psi(\xi) \cdot d\xi = 1 \quad (2)$$

where ξ , $\psi(\xi)$ and $\sigma(\varepsilon, \xi)$ are the variables which correspond to the yield stress, the distribution function of yield stress and the stress of the cell of the yield stress of ξ deformed by the straining of ε , respectively. The physical ground of the distribution function $\psi(\xi)$ is that $\psi(\xi) \cdot d\xi$ corresponds to the volume content of an ideal plastic material in which yield stress is between ξ and $\xi + d\xi$.

2.2 Extension to the combined stress state

To extend the one-dimensional model into a multidimensional one, we discuss on the basis of five-dimensional stress and strain spaces^{(8),(9)}. The components of these spaces are given by,

$$\begin{aligned} \bar{\varepsilon}_1 &= e_{11}, & \bar{\varepsilon}_2 &= (e_{11} + 2e_{22})/3, & \bar{\varepsilon}_3 &= 2e_{12}/\sqrt{3}, \\ \bar{\varepsilon}_4 &= 2e_{23}/\sqrt{3}, & \bar{\varepsilon}_5 &= 2e_{31}/\sqrt{3} \\ \bar{\sigma}_1 &= 3S_{11}/2, & \bar{\sigma}_2 &= 3 \cdot (S_{11} + 2S_{22})/2, & \bar{\sigma}_3 &= \sqrt{3}S_{12}, \\ \bar{\sigma}_4 &= \sqrt{3}S_{23}, & \bar{\sigma}_5 &= \sqrt{3}S_{31} \end{aligned} \quad (3)$$

where $\bar{\varepsilon}_i$ and $\bar{\sigma}_i$ are the i -th components of the strain and stress vectors. The notations of e_{ij} and S_{ij} show the ij -components of the deviator strain and stress tensors. Equation (2) is extended for the combined stress state and rewritten as follows:

$$\langle \bar{\sigma}_i \rangle = \int_0^\infty \psi(\xi) \cdot \bar{\sigma}_i(\varepsilon, \xi) \cdot d\xi \quad (4)$$

where $\bar{\sigma}_i(\varepsilon, \xi)$ is the stress vector of the ideal elastic perfect plastic material with the yield stress ξ , which is deformed by the strain ε .

Figure 2 shows the geometrical relation between the strain increment and the stress increment in the

Fig. 1 One-dimensional parallel structure model

plastic state of the elastic perfect plastic material. The relation between the stress incremental vector $d\bar{\sigma}$ and the strain incremental vector $d\bar{\varepsilon}$ becomes as follows,

$$\begin{aligned} d\bar{\sigma}(\xi) &= E'(d\bar{\varepsilon} - d\bar{\varepsilon}^p(\xi)) \\ E' &= 3E/2(1+\nu) \\ d\bar{\varepsilon}^p(\xi) &= \begin{cases} 0 & \dots |\bar{\sigma}(\xi)| < \xi \\ (d\bar{\varepsilon} \cdot \bar{\sigma}(\xi)) \cdot \bar{\sigma}(\xi) / \xi^2 & \dots |\bar{\sigma}(\xi)| = \xi \end{cases} \quad (5) \end{aligned}$$

where E and ν are Young's modulus and Poisson's ratio. The notation $d\bar{\varepsilon}^p(\xi)$ is the plastic incremental vector of the ξ -element (the element in which yield stress is equal to ξ).

For the calculation of the stress-strain relation with this constitutive equation, it is necessary to determine the distribution function, $\psi(\xi)$ by the experimental result. Then it is necessary to obtain the increment of $d\bar{\sigma}(\xi)$ corresponding to the given strain increment, $d\bar{\varepsilon}$, by Eq. (5), and to substitute $d\bar{\sigma}(\xi)$ into Eq. (4).

3. Work Hardening and Work Softening

3.1 Work hardening

In order to calculate the stress-strain curves using Eqs. (3), (4) and (5), it is necessary to obtain the function $\psi(\xi)$. If $\psi(\xi)$ is a given function, the flow stress is saturated to a certain value when the all elements are yielded. Figure 3 shows the principle of the work hardening for this constitutive equation. The distribution function, A , includes lower yield stress elements than the function B . Therefore, the flow stress of the function A is less than the function B . If we assume that the distribution function $\psi(\xi)$ shifts from the function A to B due to a certain hardening parameter, it is possible to introduce the idea of hardening into the constitutive equation. For example, the isotropic hardening model represents only one element which is shifted by plastic strain as shown in Fig. 4, and $\psi(\xi)$ of this model is formulated according to the following equations:

$$\psi(\xi) = \delta(f(\varepsilon^p) - \xi)$$

Fig. 2 Stress increment $d\bar{\sigma}$ by $d\bar{\varepsilon}$ in plastic state of element ξ_c

$$\sigma = f(\varepsilon^p), \quad (6)$$

where the function f expresses the uniaxial stress-strain relation. The function δ is the delta-function.

It is difficult to estimate the shape of the distribution function $\psi(\xi)$ from the experimental data. We assume the rectangular shape as the function $\psi(\xi)$, which is shown in Fig. 5. When all elements are yielded, the flow stress $\langle \sigma \rangle$ coincides with the center of the rectangular ξ_c . The flow stress in the plastic state is almost equal to ξ_c , and $\psi(\xi)$ for the uniaxial stress-strain relation can be approximated by the following equations:

$$\begin{aligned} \xi_c &= \sigma_y + H \cdot h^p, \\ \psi(\xi) &= \{u(\xi) - u(\xi - 2 \cdot \xi_c)\} / 2 \cdot \xi_c, \\ dh &= |d\varepsilon^p|, \\ u(x) &= \begin{cases} 1 & x > 0 \\ 1/2 & x = 0 \\ 0 & x < 0 \end{cases}, \end{aligned} \quad (7)$$

Table 1 Material constants of tensile stress strain relation

Temp.	σ_y (MPa)	H (MPa)	β
R. T.	190	48.7	0.69
200°C	113	34.9	0.80
400°C	93.3	36.5	0.78
600°C	72.1	37.1	0.76

Fig. 3 Relation between flow stress and the shape of the distribution function

Fig. 4 Distribution function of yield stress corresponding to isotropic hardening

where σ_y , H and β are material constants used to express the tensile stress-strain curves and these values are shown in Table 1⁽⁹⁾. The notation h is the parameter indicating the hardening degree, which is the accumulation of the Mises-type plastic strain in Eq. (7). Figure 6 shows the tensile and compressive stress-strain curves of the experiments and the calculations. The stress-strain relation calculated by Eq. (6) is almost equal to that of the isotropic hardening model because the parameter h represents the accumulation of plastic strain. In a real material, the softening phenomenon (Bauschinger's effect) occurs at the turning point of the loading.

3.2 Work hardening and softening parameter considering strain history

Considering two different plastic strain paths, as shown in Figs. 7 (a) and (b), the hardening value by the straight path (Fig. 7(b)) is greater than that of the path with a corner (Fig. 7(a)).

In order to consider a softening parameter, such as Bauschinger's effect, we propose the hereditary integration, $k(\rho)$, and the work hardening and softening parameter, h , as follows, instead of Eq. (7),

$$\begin{aligned} h &= \int_0^\rho x(s) \cdot ds \\ ds &= |d\varepsilon^p|, \end{aligned} \quad (8)$$

Fig. 5 Shape of the distribution function used in the calculations

Fig. 6 Bauschinger's curves of experiments and calculations

where s and ρ are the coordinate along the plastic strain path and the accumulated value of the plastic strain, respectively. The function $\kappa(\rho)$ is a scalar function and is proposed to describe the history of the degree of the curvature of the plastic strain path. If a plastic strain path is straight, such as is shown in Fig. 7(b), the function $\kappa(\rho)$ assumes unity, and the value of h in Eq. (8) coincides with the accumulation of the Mises-type plastic strain. If its path is bent or has some degree of curvature, the function $\kappa(\rho)$ assumes less than unity. This scalar function should assume the same value in symmetric of plastic strain paths.

Valanis^{(5),(10)} and Watanabe⁽¹¹⁾ proposed and extended Endochronic Theory in order to express a hereditary property by using the integral formulation, respectively. This expression is quite different from the expression of Eq. (8).

We proposed this hereditary function $\kappa(\rho)$ as the integration of the inner product of tangential vectors of a strain path,

$$\kappa(\rho) = \frac{1}{l(1-\alpha)} \int_0^l \{e(\rho) \cdot e(\rho-\eta) - \alpha\} d\eta.$$

Hence

$$\rho > l, \quad \alpha < 1, \quad (9)$$

where the notations $e(\rho)$ and $e(\rho-\eta)$ are the tangential unit vectors of the plastic strain path at the present time and the past time, respectively. The notation η is the strain length, as shown in Fig. 8. The notation l is the length of the plastic strain, in which the curvature of the strain path accounts for the value of $\kappa(\rho)$. If any curves exist at the η point, the inner product of $e(\rho) \cdot e(\rho-\eta)$ becomes less than unity and also $\kappa(\rho) < 1.0$.

In the case that the strain path is bent at an angle

Fig. 7 Relation between the strain paths and hardening-softening value

Fig. 8 Coordinates along the plastic strain path

of 180° at the point of ρ^* , the value of $\kappa(\rho)$ is given by,

$$\kappa(\rho) = \begin{cases} 1 & \dots \rho \leq \rho^* \\ \frac{2(\rho - \rho^*) - l(1+\alpha)}{l(1-\alpha)} & \dots \rho^* < \rho < \rho^* + l \\ 1 & \dots \rho \geq \rho^* + l \end{cases} \quad (10)$$

Figure 9 shows the example of the shape of $\kappa(\rho)$ for various strain paths in the case of $d=0$. Once the strain path is bent at 180° , the value of function $\kappa(\rho)$ becomes less than zero. Consequently, the hardening parameter h decreases. Thus, the softening occurs. This process is schematically shown in Fig. 10.

3.3 Determination of the parameters with experimental results

It is important in the mechanical modeling of materials to determine the parameters or material constants of the model formulation. In the case of the mathematical description of the hardening and softening, as in Eqs. (8) and (9), it is necessary to deter-

Fig. 9 Shape of $\kappa(\rho)$ for the various strain paths

Fig. 10 Increment or decrement of hardening value of h in Bauschinger's curves

mine the hereditary length l , which expresses the softening phenomenon. Generally, when the material is hardened by the more prestrain, the softening phenomenon, such as Bauschinger's effect, becomes more remarkable. Therefore, it is reasonable to determine the length l as a function of the hardening parameter of h .

There is a uniaxial cyclic stress-strain relation in which the hardening and softening occur at the same time. Considering the stable cycle of this relation, the hardening and softening are in an equilibrium state in one cycle, and we assume this state by the following,

$$\Delta h = \int_{\rho-L}^{\rho} \kappa(s) ds \quad (11)$$

where L corresponds to the plastic strain width (2 times the plastic strain amplitude) of stable uniaxial cyclic stress-strain loop. The notation, Δh , is the increment of h in a half cycle. Substituting Eq. (9) into Eq. (11), the value of L , which satisfies $\Delta h=0$, is given by,

$$L = \frac{l}{1-\alpha} \quad (12)$$

The process used to obtain above relation is shown in the Appendix. If it is assumed that Eq. (12) is satisfied in a stable cycle, we can determine the function l with experimental results.

A similar quantitative approach was performed by Ohno⁽¹⁾. He proposed the non-hardening locus on a strain space to describe a softening phenomenon of cyclic loading. The length l corresponds to a diameter of their nonhardening locus.

The parameter, α , in Eqs. (9) and (12) is to describe the softening of the cyclic deformation, but it is difficult to estimate this from the experimental results. Therefore, we adopt $\alpha=0$ in the calculations.

In order to decide a function of l , we assume that the relation between the maximum cyclic stress $\sigma_{\max}$ and the plastic strain width $\Delta \epsilon^p$ of the saturated stable uniaxial cyclic loop is similar to the tensile stress-strain relation, $\sigma_f = \sigma_y + H \cdot h^\beta$,

$$\sigma_{\max} = \sigma_y + H^* \cdot (\Delta \epsilon^p)^{\beta^*} \quad (13)$$

where H^* and β^* are new material constants which are determined by the experimental data of more than two uniaxial cyclic elastic-plastic tests. The values of H^* and β^* are obtained for room temperature and 600°C with the experimental results⁽¹²⁾. These values are shown in Table 2. We assume the relation $\sigma_f = \sigma_{\max}$, and obtain the relation between h and L as follows:

$$H \cdot h^\beta = H^* \cdot L^{\beta^*}, \quad L = l/(1-\alpha) \quad (14)$$

Equation (14) expresses the interaction between a softening parameter l and a hardening parameter h .

In order to calculate a stress-strain relation in the

combined stress state using our theory, we should first determine the five material constants, σ_y , H , β , H^* and β^* . The decision process is shown in Fig. 11(a). Secondly, we obtain the stress response by a strain increment using Eqs. (4), (5), (7), (8), (9), and (14) as shown in Fig. 11(b).

4. Calculation of Plastic Deformation

4.1 Calculation of uniaxial cyclic stress-strain relation

Figure 12 shows both the experimental and calculated results of tension-compression cyclic stress-strain curves at room temperature and 600°C. The marks indicate the experimental results of the first and ninth cycle. The solid curves show the calculated results. Differences are observed on the hysteresis curves, but the rates of stress amplitude of the calculation agree with the experiment. Figure 13 shows the relation between the hardening parameter h of Eq. (8) and the accumulation of plastic strain ρ for the same calculation as Fig. 12. The difference between room temperature and 600°C is caused by the parameters, H^* and β^* .

Table 2 Material constants of H^* and β^*

Temp.	$\Delta \epsilon$	$\Delta \epsilon^p$	$\sigma_{\max}$	H^*	β^*
R. T.	1%	0.70%	296 MPa	135 MPa	0.67
	2%	1.61%	376 MPa		
600°C	1%	0.70%	230 MPa	185 MPa	0.44
	2%	1.60%	300 MPa

Fig. 11 Process of determination of material constants and calculation

4.2 Calculation of complex combined deformation

Figure 14 shows the relation between the equivalent stress and accumulation of plastic strain for loading along the strain path with a corner, as shown in Fig. 14. In the calculating condition, the strain paths are bent to various angles after tensile prestraining of 5%. Note that, in the case of the angle of 90°, subsequent flow stress decreases slightly. Indeed, this phenomenon is observed in the same condition for any material. It is called the "delayed phenomenon" ⁽⁸⁾.

Figure 15 shows the stress and plastic strain trajectories for the biaxial nonproportional strain cycle, which shape is a $1.2\% \times 1.2\%$ square. The plastic strain trajectory is almost square with slightly rounded corners. In this loading condition, the fluctuation of $\chi(\rho)$ is shown in Fig. 16. When a hereditary length, l , becomes two times the $\Delta\epsilon^p$, hardening and softening are balanced. Equivalent stress is greater than that of uniaxial cyclic deformation by the 1.2% strain width. The same phenomenon was observed in the experimental results of nonproportional cyclic

tests performed by Ohashi et al. ⁽¹³⁾ and Tanaka et al. ⁽¹⁴⁾. This hardening and softening expression cannot deal with special hardening effects such as "latent hardening", which occurs under complex combined loading conditions ⁽¹⁵⁾.

Fig. 14 Calculated results of equivalent stress and plastic strain relation by the strain path with corner

Fig. 12 Calculated and experimental results of cyclic stress-strain curves

Fig. 15 Calculated result of stress trajectory and plastic strain trajectory by square total strain path

Fig. 13 Relation between h and ρ while cyclic deformation

Fig. 16 Change of h and l by the nonproportional cyclic loading

4.3 Consideration of cyclic softening by a parameter α

When a metallic material is cyclically loaded after being hardened with a large amount of prestrain, cyclic softening occurs. However, in the case of $\alpha=0$, this theory cannot simulate the cyclic softening phenomenon, because many relations, $L=l/n$ (here, n is any natural number) satisfy Eq. (11) instead of Eq. (12). If a cyclic loading with a small strain amplitude is operated after being hardened sufficiently, the value of L is increased above the plastic strain width and the hysteresis loop becomes stable at the nearest value of L which satisfies $L=l/n$.

Figure 17 shows the cyclic stress-strain curves for $\Delta\epsilon=0.8\%$ subsequent to 10% tensile prestrain. Cyclic softening does not occur in the case of $\alpha=0$, but does occur in the case of $\alpha=0.2$.

5. Conclusions

In order to characterize the inelastic behavior of metals, the inelastic mechanical model on the basis of the distribution function of the yield stress is proposed and an integral constitutive equation for this model is derived. To bring work hardening and softening into the constitutive equation, a hereditary integration expression for the plastic strain path is formulated. The material constants of the equation are determined with the experimental data of the tensile stress-strain curve and the uniaxial cyclic curves for SUS 304 stainless steel. Biaxial stress-strain relations are calculated using the constitutive equation. It is found that the work hardening and softening phenomena can be simulated by this constitutive equation.

Appendix

Figure 18 shows the plastic strain path and the variables in Eq. (9) for a strain path bent at $\rho=\rho^*$ with 180° . The point ρ corresponds to the plastic state

Fig. 17 Cyclic stress-strain curves subsequent to large tensile prestraining

at the present, and its value represents the accumulated plastic strain. The variable η is the plastic strain length from ρ to the past along the plastic strain path. The inner product of directional vectors, $e(\rho) \cdot e(\rho-\eta)$ is unity where $0 < \eta < \rho - \rho^*$, -1 where $\rho - \rho^* < \eta < l$, and unity again where $\eta > l$. Therefore, the value of $\chi(\rho)$ is given by,

$$\chi(\rho) = \begin{cases} \frac{\int_0^l (1-\alpha) d\eta}{l(1-\alpha)} \cdots \rho \leq \rho^* \\ \frac{\int_0^{\rho-\rho^*} (1-\alpha) d\eta + \int_{\rho-\rho^*}^l (-1-\alpha) d\eta}{l(1-\alpha)} \cdots \rho^* < \rho < \rho^* + l \\ \frac{\int_0^l (1-\alpha) d\eta}{l(1-\alpha)} \cdots \rho \geq \rho^* + l \end{cases} \quad (\text{App. 1})$$

Rearranging the above equation, Eq. (10) is given.

The resultant shape of Eq. (10) is shown in Fig. 19. In the case of uniaxial cyclic loading condition, the function shown in Fig. 19 is repeated. The area of $\chi(\rho) < 0$ corresponds to the value of softening, and that of $\chi(\rho) > 0$ to hardening. A value of L in Eq. (11) is equivalent to a plastic strain width which makes the area of $\chi(\rho) > 0$ equal to $\chi(\rho) < 0$ in cyclic loading. The areas of $\chi(\rho) < 0$ and $\chi(\rho) > 0$ are given, respectively,

Fig. 18 Plastic strain path bent at 180°

Fig. 19 Shape of $\chi(\rho)$ for the strain path bent at 180°

by,

$$S^{(-)} = \frac{l(1+a)^2}{4(1-a)} \quad \dots \text{the area of } x(\rho) < 0$$

$$S^{(+)} = \frac{l(1-a)}{4} + y \quad \dots \text{the area of } x(\rho) > 0$$

(App. 2)

where $y = L - l$. The relation of Eq.(11) corresponds to $S^{(-)} = S^{(+)}$ and by rearranging the above equation, Eq. (12) is given.

If plastic strain width, $\Delta \varepsilon^p$, is greater than L (the function of L depends on the hardening value h), the value of h increases to a certain value, satisfying Eq. (12).

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