BEM Analysis of a Cylindrical Three-Point Bend Specimen with a Chevron Crack for Fracture Toughness Test of Rock*

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The stress intensity factor along the periphery of a chevron crack in a cylindrical three point bend specimen (CB) is evaluated by BEM for the establishment of a rock fracture toughness test using CB specimens. So far, in the test, the stress intensity factor is assumed to be constant along the front edge of the crack and is evaluated by the compliance method. It is revealed by the present analysis that the stress intensity factor takes a minimum value at the center of the front edge so that the crack growth during the fracture toughness test makes the front edge of the crack concave. It is also shown that the stress intensity factor evaluated by the compliance method is larger than that of the concave crack with a homogeneous distribution of the stress intensity factor along the front edge. This implies that the fracture toughness test leads to overestimation.

Key Words: Elasticity, fracture, fracture toughness of rock, CB specimen, chevron crack, fracture toughness test, stress intensity factor

1. Introduction

Artificial subsurface cracks have received extensive attention in a variety of geotechnical fields, such as geothermal heat extraction, oil or gas well stimulation, in-situ tectonic stress measurement and storage of nuclear wastes^{(1)~(3)}. Basically, the behavior of subsurface cracks is governed by fracture mechanics, and thus the methodology for design, creation and control of artificial subsurface cracks must be based on fracture mechanics. In these respects, the fracture toughness of rock is one of the important material constants. Furthermore, the crucial role of the fracture toughness of rock in the explosion and fragmentation of rock and drilling of wells has been recognized in recent years⁽⁴⁾. Under these circumstances,

standardization of the fracture toughness test of rock is being attempted by ISRM (International Society of Rock Mechanics)^{(5)~(7)}.

Rock samples, especially those from deep in the earth's crust, are usually given as boring core samples, and the method suggested by ISRM uses two types of cylindrical specimens with chevron cracks, i. e., short rod specimens (SR) and three point bend specimens (CB). In order to determine fracture toughness, a distribution of the stress intensity factor along the front edge of the chevron crack must be evaluated beforehand. For SR specimens, the stress intensity factor has already been evaluated by FEM^{(8)~(10)} and BEM⁽¹¹⁾⁽¹²⁾ analyses. However, for CB specimens, such analyses have not yet been attempted and, instead, the stress intensity factor obtained experimentally by the so-called compliance method has been used in the fracture toughness test. As can be readily understood, the compliance method does not provide the information on the distribution of the stress intensity factor.

In the present paper, the distribution of the stress intensity factor along the front edge of the chevron crack of CB specimens is clarified in detail by BEM analyses and the stress intensity factor obtained by

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the compliance method is examined by comparison with that evaluated by the BEM analyses. Furthermore, the shape of a crack with a homogeneous distribution of the stress intensity factor is studied for the establishment of the fracture toughness test of rock using CB specimens.

2. Stress Intensity Factor of CB Specimens

BEM analysis of the CB specimen shown in Fig. 1 is performed using six-noded triangular and eight-noded rectangular isoparametric elements. Taking advantage of the symmetry of the specimen, we partition the surface of a quarter of the specimen into surface elements. On the middle plane, including one of the crack surfaces, the tractions and the displacements behave asymptotically as $1/\sqrt{r}$ and $\sqrt{r}$, respectively, as the crack edge is approached, where r is the distance from the crack edge. Here, on the crack surface, the usual singular elements with the displaced sidenodes are placed along the crack edge⁽¹³⁾.

Fig. 1 Geometry of the CB specimen

For the elements which are along the crack edge but are not on the crack surface, the shape functions that are obtained by multiplying the following function (Eq. (1)) to the usual shape function for the eight-noded isoparametric elements are used in order to express the singularity of the tractions near the crack edge⁽¹³⁾:

$$K(r) = \frac{1}{\sqrt{r}} \left\{ 1 - \frac{r}{L} (1 - \sqrt{L}) \right\}, \quad (1)$$

where L is the width of the elements measured in the direction normal to the crack edge. The stress intensity factor is evaluated by

$$K_I = \frac{\mu}{2(1-\nu)} \sqrt{2\pi/r} v, \quad (2)$$

where μ is the shear modulus, ν the Poisson's ratio and v the crack opening displacement. The following nondimensional stress intensity factor is introduced:

$$Y_c = \frac{K_I}{0.25(S/D)P/D^{1.5}}. \quad (3)$$

Here, D , P and S are given in Fig. 1. In the BEM analyses, S/D and a_0/D are set to be 3.33 and 0.205, respectively. This geometry is recommended by ISRM for the fracture toughness test. In Fig. 2, an example of partitioning, where the total numbers of nodes and elements are 534 and 184, respectively, is presented.

In the calculations, two cases, i.e., $L/D=0.025$ and 0.035 , are examined. The discrepancy between the resulting two sets of stress intensity factors is less

Fig. 2 Example of partitioning ($a/D=0.4$)

than 1%. The results presented next are obtained with $L/D=0.035$. The Poisson's ratio is set to be 0.12.

In Fig. 3, the nondimensional stress intensity factor is presented for various crack depths. It is clear that the nondimensional stress intensity factor is not constant along the crack edge. It takes a minimum value at the center of the front edge of the crack and reaches a maximum value at both ends of the front edge.

3. Discussions on the Fracture Toughness Test by CB Specimens

The fracture toughness test by CB specimens has used so far the stress intensity factor which is defined by

$$K_I = \left(\frac{P^2 E'}{2b} \cdot \frac{d\lambda}{da} \right)^{1/2}, \quad (4)$$

where $E' = E/(1-\nu^2)$, E is Young's modulus, b the width of the crack (Fig. 1) and λ the compliance. In Eq. (4), the first derivative of the compliance with respect to the crack depth is evaluated through three point bend tests of CB specimens made of metal, such as aluminium alloy, with various crack depths. The nondimensional stress intensity factor is given by

$$Y_c = \left(\frac{P^2 E'}{2b} \cdot \frac{d\lambda}{da} \right)^{1/2} / \left\{ 0.25 \left(\frac{S}{D} \right) P/D^{1/2} \right\}. \quad (5)$$

It is known that Y_c determined from Eq. (5) decreases with a , takes a minimum value at some crack depth a_c and then increases with $a^{(6)}$. Therefore, during the fracture toughness test, the load P takes a maximum value $P_{\max}$ at $a=a_c$, if crack growth resis-

tance is absent. On the basis of, this in the fracture toughness test by CB specimens, the fracture toughness K_c is evaluated through Eq. (4) by using $P_{\max}$ and $d\lambda/da$ at $a=a_c$. The relationship between λ and a is determined experimentally by using CB specimens of metal with various crack depths.

The compliance evaluated by the BEM analyses is shown in Fig. 4, together with that determined experimentally⁽¹⁴⁾, where the former is determined from the displacement at the intersection point between the crack edge and the loading line. From both of these two results of the compliance, Y_c can be determined through Eq. (5), and the results are shown in Fig. 5. The solid circles in Fig. 5 indicate the averaged values of Y_c along the crack front edge determined by the BEM analyses. The nondimensional stress intensity factors Y_c determined from the two

Fig. 3 Variation of the stress intensity factor along the front edge of the crack

Fig. 4 Variation of the compliance with respect to the crack depth

Fig. 5 Variation of the stress intensity factor with respect to the crack depth

Fig. 6 The shape of the crack with an almost homogeneous distribution of the stress intensity factor ($a/D = 0.4$)

compliances just stated agrees well with each other; however, these are considerably larger than that determined directly by the BEM analyses. The discrepancy is larger than 8%, as can be seen in Fig. 5.

As shown in Fig. 3, for a straight front edge, the stress intensity factor is not constant along the front edge and takes a maximum value at the ends of the front edge. This means that, during the fracture toughness test, the crack gradually changes its shape into a crack with a concave front edge due to crack growth if crack growth resistance is absent, i.e., if the crack growth takes place under the condition that $K_I = K_{IC}$. Based on these considerations, we have determined the shape of the front edge which makes the stress intensity factor almost constant along the front edge. The result for the case that the crack depth is $a/D = 0.4$ is shown in Fig. 6. The nondimensional stress intensity factor of the curved front edge shown in Fig. 6 is presented in Fig. 7. This value is less than that calculated from the compliance method and is larger than the averaged value of the stress intensity factor determined directly by the BEM analyses for the straight front edge. Therefore, if the stress intensity factor obtained by the compliance method is used for the fracture toughness test, fracture toughness will be overestimated. On the contrary, if the averaged stress intensity factor obtained from BEM analyses is used, fracture toughness will be underestimated.

4. Conclusions

For the establishment of the rock fracture toughness test, the stress intensity factor along the periphery of a chevron crack in a cylindrical three point

Fig. 7 The stress intensity factor for the crack shown in Fig. 6

bend specimen (CB) was evaluated by BEM and the results were compared with those obtained experimentally by the compliance method. The conclusions that are drawn from the present study can be summarized as follows:

(1) The stress intensity factor takes a minimum value at the center of the front edge of the crack and a maximum value at the ends of the front edge for any crack depth. The difference between the minimum and maximum values is nearly 15%.

(2) The compliance of a CB specimen evaluated experimentally agrees well with that evaluated by the BEM analyses, and, as a result, the stress intensity factor determined experimentally also agrees well with that determined from the compliance evaluated by the BEM analyses. However, this value of the stress intensity factor is considerably larger than the averaged value of the stress intensity factor along the front edge that are evaluated directly by the BEM analyses. The discrepancy is more than 8%.

(3) The shape of the crack which makes the stress intensity factor almost constant along the front edge is obtained.

The stress intensity factor of this crack is less than that determined by the compliance method. This means that, if the stress intensity factor determined experimentally by the compliance method is used in the rock fracture toughness test, the test results in an

overestimation.

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