Some Aspects of Structural Reliability Assurance for Random Excitation Processes*

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Certain service loads, when applied to practical machines and structures, could be regarded as approximately deterministic. However, most of them would be, to a degree, of a random nature with complicated temporal variation. Hence, in order to assure the desired level of reliability of critical structural components under random loading, it is of crucial importance to clarify, in particular, the effect of the randomness of the applied load on the life distributions or probabilities of failure of the components, in addition to the effects of other various uncertainty factors usually taken into account in the reliability assurance of structural components subjected to constant amplitude loadings. In this respect, in the present paper, the fundamental concept has been discussed and a state-of-the-art survey has been made to ascertain the structural reliability for stationary random loads from the viewpoint of whether the effect of the applied random load on a critical structural component is cumulative or not.

Key Words: Structural Reliability Assurance, Uncertainty Factor, Probability of Failure, Fatigue Damage, Random Excitation Process, Cumulative Effect, Noncumulative Effect, Semicumulative Effect

1. Introduction

Certain service loads applied to practical machines and structures could be regarded as approximately deterministic. However, most of them would be, more or less, of a random nature with complicated temporal variation like wind load, seismic load, wave load and so forth⁽¹⁾⁻⁽³⁾. Hence, in order to assure the desired level of reliability of critical structural components of machines and structures under these random loads, it is of crucial importance to clarify the effects of such statistical properties of the applied random load as mean, variance and correlation time on the life

distributions or probabilities of failure of the components, in addition to the effects of other various uncertainty factors usually taken into consideration in the structural reliability assurance for constant amplitude loadings.

The effect of random load on the critical structural component might differ according to the circumstances of whether or not damage induced by the random load is cumulative. Consequently, either the theoretical methods or approximation techniques to be introduced in the structural reliability assurance would be different according to the corresponding situations.

In this respect, in the present paper, the fundamental concept is discussed and a state-of-the-art survey is conducted to ascertain the structural reliability for stationary random loads from the viewpoint of whether or not the effect of the applied random load on the critical structural component is cumulative.

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2. Effects of Random Excitation Processes

To begin with, let us consider a simplified situation of structural reliability assurance for the uncertainty factors due only to the randomness of the applied load.

Let Z_t be the state at time t of the structural system subjected to random loading. Z_t could be a variety of different quantities according to the phenomenon concerned. For example, in the case of fatigue problems, Z_t would be the damage accumulated by time t , in the case of crack growth problems, the crack length at time t , in the case of yielding problems, the stress induced at time t , to name but a few. It should be noted that the state Z_t becomes indeterministic or stochastic when the applied external load is random⁽³⁾. Hence, in general, Z_t needs to be treated as a random vector.

Next, let D be the safe domain in the sense that if the state of the system lies in this domain D , then the system is deemed safe, otherwise it is regarded as unsafe or in the failed state. It is of interest to note that sometimes such a safe domain D could hardly be considered deterministic, as in the case of the critical value of cumulative fatigue damage⁽⁴⁾.

At this point, the random variable T is introduced which is defined as follows:

$$T \equiv \inf\{t > 0 | Z_t \notin D, Z_0 \in D\} \quad (1)$$

T thus defined means the time at which the state in D with initial value Z_0 at $t=0$ gets out of D , by transition, for the first time. When the number of cycles instead of time is concerned, the symbols T and t are replaced by N and n , respectively.

It is unquestionably of crucial importance to clarify the distribution of the random variable T in order to solve the problems associated with structural reliability assurance. Suppose that the distribution of T can be obtained in the following form

$$P\{T \leq t\} \equiv G(t; \Theta) \quad (2)$$

where the notation $P\{A\}$ represents the probability of the event A , and $\Theta = (\theta_1, \theta_2, \dots, \theta_n)$ with θ_i 's being design and/or maintenance parameters. Since $G(t; \Theta)$ gives, in case of known parameter values of Θ , the probability that the state is transferred to the unsafe domain by design or inspection life t , that is, the probability of failure, this probability needs to be kept not greater than a certain prescribed small value p_a which is termed hereinafter as the allowable probability of failure. In other words, it becomes of great importance to obtain the region of t or Θ which satisfies the relationship $G(t; \Theta) \leq p_a$. As is well known, if γ_a denotes the allowable reliability, then the following relationship holds between p_a and γ_a :

$$p_a + \gamma_a = 1 \quad (3)$$

From the viewpoint stated above, the key to the problem solution is whether or not the distribution of T can be derived or obtained rigorously for the phenomenon concerned, or, if this could not be done perfectly, whether or not an alternative approximation method could provide practically sufficient accuracy, since a fairly small value in the order of $10^{-3} \sim 10^{-5}$ is usually required for the allowable failure probability. However, many difficulties will arise in the process of the rigorous derivation of the distribution of T , so that an appropriate approximation method needs to be introduced as the occasion may demand.

Generally speaking, in order to clarify the distribution of T , it is necessary to take into consideration, in addition to the randomness of the applied load, the randomness associated with the material itself, lack of knowledge on the initial state Z_0 and the safe domain D , estimation errors of design and maintenance parameters, modelling uncertainties, and so forth⁽⁵⁾⁻⁽¹¹⁾. Unfortunately, however, taking all these uncertainties into account is far too complicated⁽¹²⁾, so, in the present paper, discussions will center mainly on those major uncertainty factors associated with the randomness of the applied load, the initial state Z_0 and the safe domain D .

Now, let us assume here an arbitrary critical structural component subjected to random loading and direct our attention to the effect induced in the component by the applied random load. As stated later, this effect would vary greatly according to the failure mode concerned and the quantity to be dealt with, that is, the phenomenon and the physical quantity under consideration. There have been so far a variety of research projects dealing with different uncertainty factors or introducing different approximation methods. Some are concerned with cumulative effects such as fatigue damage where the quantity of interest accumulates its value gradually by the long-term application of the random load, and some with noncumulative effects such as the crushing phenomena of concrete or ceramic materials.

In view of the above, the stochastic excitation processes have been classified into three categories, as shown in Fig. 1, based not on their statistical properties but on their inducing effects. In the following, the state-of-the-art survey of the studies in each category will be summarized and discussed briefly.

3. Topics Associated with the Cumulative Effect

In this section, a brief discussion of typical examples of linear cumulative phenomena is followed by a discussion of the nonlinear phenomena.

For those phenomena where the effect induced by the random excitation is merely cumulative, the first

◎ Effect induced by applied random load

(a) Cumulative

(a-1) Linear accumulation

(a-2) Nonlinear accumulation

(b) Semicumulative

(c) Noncumulative

(c-1) Linear response

(c-2) Nonlinear response

Fig. 1 Classification of random excitation processes according to their inducing effect

passage problem is comparatively easy to deal with in case of the deterministic domain D , since transition of the state to a certain new situation is always the initial occurrence. In this case, however, it is often necessary to consider the uncertainties associated with domain D and the initial state Z_0 . A typical example of the linear cumulative phenomenon is the linear cumulative damage law (which is also known as the modified Miner's rule) in the treatment of systemic fatigue damage.

Let S_m be the random stress amplitude of the m -th cycle of loading, s_0 , the normalization stress level and b , the material constant. Then, the damage state Z_n after n cycles of stressing can be given as

$$Z_n = \sum_{m=1}^n \left(\frac{S_m}{s_0} \right)^b \quad (4)$$

Suppose that safe domain D can be expressed as

$$D = \{z \in R | 0 \leq z \leq Z_c\} \quad (5)$$

where R is the entire set of real numbers. The stationary random process S_m^b has, in general, a certain amount of correlation between two different m 's, say, m and $m+k$, and its correlation time n_c , which implies the number of cycles over which correlation extends between values of the process S_m^b , can be defined as follows⁽¹³⁾:

$$n_c = \frac{2 \int_{-\infty}^{\infty} \{E[S_m^b S_{m+k}^b] - E[S_m^b]E[S_{m+k}^b]\} dk}{E[S_m^b]^2 - (E[S_m^b])^2} \quad (6)$$

In the case that the change in damage is only of interest during the number of cycles n which is large enough in comparison to the correlation time n_c , Z_n follows, with the aid of the central limit theorem, a normal distribution⁽¹⁴⁾⁽¹⁵⁾ with the mean

$$n\lambda \equiv \frac{n}{s_0^b} \int_0^\infty s^b dR(s) \quad (7)$$

and the variance

$$n_c n \nu^2 \equiv \frac{n_c n}{s_0^{2b}} \left[\int_0^\infty s^{2b} dR(s) - \left\{ \int_0^\infty s^b dR(s) \right\}^2 \right] \quad (8)$$

in which $R(s)$ is the distribution function of the random process S_m , and λ and ν are the mean and the variance of the process $(S_m/s_0)^b$.

At this point, suppose that S_m is constant in the

amount of s_0 regardless of m . Then, Z_n in Eq. (4) becomes the number of cycles n under constant stress amplitude s_0 , and hence Z_c proves to represent the fatigue life under constant stress amplitude s_0 , since Z_c is the critical value of Z_n . According to experimental results, however, Z_c has a large scatter and hence, should be treated as a random variable. As is often the case, the distribution of Z_c may be assumed to be of a two-parameter Weibull type⁽¹⁶⁾⁽¹⁷⁾ or of a log-normal type⁽¹⁸⁾⁽¹⁹⁾.

Now, let $F(z_c)$ be the distribution function of Z_c , and the distribution of the number of the first passage cycles N can be given as follows:

$$P[N \leq n] \equiv G(n) = P[Z_n > Z_c] \\ = \int_{-\infty}^{\infty} F(z_c) \frac{1}{\sqrt{2\pi n_c n \nu}} \exp \left[-\frac{(z_c - n\lambda)^2}{2n_c n \nu^2} \right] dz_c \quad (9)$$

In the case that $n/n_c \gg 1$, the probability density function of Z_n has a very sharp peak around the mean $n\lambda$. Consequently, with the aid of the Taylor's series expansion of $F(z_c)$ around $z_c = n\lambda$, Eq. (9) reduces to

$$G(n) = F(n\lambda) + \frac{n_c n \nu^2}{2} F''(n\lambda) + O \left(\left(\frac{n_c}{n} \right)^2 \right) \quad (10)$$

The second and higher terms of the right-hand side of the above equation are small enough compared with the first term⁽¹⁴⁾⁽¹⁵⁾. That is, in this case, the effect of the randomness of the applied load (which can be measured by the second term) could be treated as a complementary term.

Thus, the kind of probability model applicable to $F(z_c)$ and the way in which the uncertainties associated with parameter estimation can be taken into account become very important, since the parameter values are still unknown even if a proper probability model is adopted. The latter aspect has been discussed so far in some detail⁽¹¹⁾⁽¹⁴⁾⁽¹⁵⁾⁽²⁰⁾⁽²¹⁾. First, an attempt was made by A. M. Freudenthal to account for the effect of not-so-large sample size by introducing the notion of the so-called scatter factor⁽²⁰⁾. Unfortunately, however, the physical meaning of his scatter factor was not absolutely clear, and hence a proper revision

has been provided by Shinozuka et al.⁽¹¹⁾ and Ishikawa et al.⁽²¹⁾. At the present stage, the parameter values in Eq. (10) are replaced by their maximum likelihood estimators so as to construct the estimator of the failure probability for a given number of cycles of loading, and its distribution properties are discussed in detail⁽¹⁴⁾⁽¹⁵⁾⁽²¹⁾.

As an example of this illustration, let us consider here the case that $F(z_c)$ follows a two-parameter Weibull distribution and examine the outline of this treatment. Suppose that a two-parameter Weibull distribution is given as follows:

$$F_{\theta}(z_c) = \begin{cases} 1 - \exp\{-e^{\theta_1} z_c^{\theta_2}\} & (\text{for } z_c \geq 0) \\ 0 & (\text{otherwise}) \end{cases} \quad (11)$$

Strictly speaking, for the estimator $\hat{G}(n)$ of $G(n)$, the distribution of the variate

$$\hat{G}(n) = F_{\theta}(n\lambda) + \frac{n_c n \nu^2}{2} F''_{\theta}(n\lambda) + O\left(\left(\frac{n_c}{n}\right)^2\right) \quad (12)$$

should be discussed where the parameter θ is replaced by its maximum likelihood estimator $\hat{\theta}$. However, it is difficult to obtain the distribution of $\hat{G}(n)$ concretely. Hence, the discussion of the distribution

$$\hat{G}_a(n) = F_{\theta}((n\lambda)_{\text{eff}}) \quad (13)$$

will be made the best of here in view of the fact that the second term of the right-hand side of Eq. (10), and hence that of Eq. (12), is a complementary term. The quantity $(n\lambda)_{\text{eff}}$ in Eq. (13) is defined as

$$(n\lambda)_{\text{eff}} \equiv n\lambda + F_{\theta}^{-1}\left\{\frac{n_c n \nu^2}{2} F''_{\theta}(n\lambda)\right\} \quad (14)$$

where θ is no longer treated as a random variable. For the two-parameter Weibull distribution defined in Eq. (11), the following approximation can be applied asymptotically for a sample of large size:

$$\begin{aligned} P[\hat{G}_a(n) \leq p_a] \\ \cong \Phi\left[\frac{\ln(-\ln(1-p_a)) - \theta_1 - \theta_2 \ln(n\lambda)_{\text{eff}}}{\sqrt{[1 + 6(\theta_1 + \theta_2 \ln(n\lambda)_{\text{eff}} + \gamma_E - 1)^2 / \pi^2] / k}}\right] \end{aligned} \quad (15)$$

where $\Phi(\cdot)$ represents the standardized normal distribution function, γ_E the Euler's constant and k the sample size. In this discussion, since the effect of finite sample size reflects on the probability of the event $\{\hat{G}_a(n) \leq p_a\}$, it goes without saying that the proposition that the failure probability assumes a value not greater than p_a becomes more reliable with increasing sample size k .

Next, as a typical example of nonlinear cumulative phenomena, let us examine the crack length in the fatigue crack growth process. Let Z be the nondimensional crack length after n cycles of stressing normalized by an appropriate length, and D be the safe domain defined, in the same way as before, as

$$D = \{z \in \mathbf{R} | 0 \leq z \leq Z_c\} \quad (16)$$

Suppose that the crack growth follows the Paris-

Erdogan's law, with stress amplitude replaced by the random stress amplitude S_n , such that

$$\frac{dZ}{dn} = \varepsilon S_n^{2(\lambda+1)} Z^{\lambda+1} \quad (\lambda \geq 0) \quad (17)$$

where the nonnegative constant λ of the form

$$\lambda = m/2 - 1 \quad (18)$$

is introduced in place of the power exponent m in the law, and ε is the smallness parameter which takes on a sufficiently small positive constant value.

Although Eq. (17) should be treated as a stochastic differential equation, most of the related studies so far solve the equation in a deterministic manner by replacing the effect of the random applied load with a certain equivalent constant stress amplitude. There are quite few papers which deal directly with diffusion effect of the stochastic process $S_n^{2(\lambda+1)}$. Recently, the authors successfully solved the stochastic differential equation to obtain, in a closed form, the distribution of the number of cycles N necessary for a fatigue crack to grow from its initial size z_0 to its critical size z_c at the time of failure, with consideration that the problem can be treated with the aid of the Markov approximation method⁽²²⁾⁽²³⁾, when focussed on a sufficiently large number of cycles in comparison to the correlation time of $S_n^{2(\lambda+1)(5)(6)(24)}$.

In general, in dealing with this kind of problem, careful attention must be paid to the possibility that the crack length may tend to infinity after a limited number of cycles of stress. Thus, the death point $\{\infty\}$, which corresponds to the state of infinite crack length, needs to be involved in the state space. It is necessary to consider that the crack remains at the death point forever once it reaches such a state.

The importance of introducing the concept of the death point is made apparent by the following reasoning. The Paris-Erdogan deterministic law of crack growth under constant stress amplitude S_a can be given in a normalized form, as in Eq. (17), as

$$\frac{dz}{dn} = \varepsilon S_a^{2(\lambda+1)} z^{\lambda+1} \quad (19)$$

where z represents the normalized deterministic crack length after n cycles of stress. Solving this differential equation with the initial condition $z=z_0$ at $n=0$, we obtain

$$z_0^{-\lambda} - z^{-\lambda} = \lambda \varepsilon S_a^{2(\lambda+1)} n \quad (20)$$

As is clearly seen from Eq. (20), the crack length z tends monotonically to infinity with increasing n . For values of n not less than $n_1 = z_0^{-\lambda} (\lambda \varepsilon S_a^{2(\lambda+1)})^{-1}$, z no longer assumes a real number. This discussion could be regarded as the case of a sample function of the stochastic fatigue crack growth process when S_a is replaced by the random stress amplitude S_n and z by a random variable of crack length Z . However, there are some other sample functions, as shown in Fig. 2, of

which the values still remain less than infinity even if n becomes greater than the aforementioned value of n_1 . This is really the case when dealing with stochastic processes, hence, insufficiency arises as long as we consider the state space as one-dimensional Euclidian space.

For more detailed discussion, let $Z(n)$ be the nondimensional crack length after n cycles of stress, let

$$P[Z(n)=\infty]=P_{\infty}(n) \quad (21)$$

be the probability that $Z(n)$ lies in the death point $\{\infty\}$, and let

$$P[Z(n)\leq z|Z(0)=z_0]=W(z, n|z_0) \quad (22)$$

be the probability that the crack of initial size z_0 grows to a finite value less than or equal to z after n cycles of stress. Thus, it is evident that the following relationship holds:

$$\lim_{z \rightarrow \infty} W(z, n|z_0) + P_{\infty}(n) = 1 \quad (23)$$

Suppose that the function $W(z, n|z_0)$ is differentiable with respect to z and its derivative is $w(z, n|z_0)$.

When the correlation time n_c is smaller than the number of cycles of interest, the equation to describe the temporal variation of the crack length distribution $w(z, n|z_0)$, which is called a generalized Fokker-Planck equation, can be derived by use of the perturbation method⁽²²⁾⁻⁽²⁴⁾ as

$$\begin{aligned} \frac{\partial w}{\partial \tau} = & -\frac{\partial}{\partial z} \left[\left\{ z^{\lambda+1} + \frac{\mu}{2}(\lambda+1)z^{2\lambda+1} \right\} w \right] \\ & + \frac{\mu}{2} \frac{\partial^2}{\partial z^2} \{ z^{2(\lambda+1)} w \} \end{aligned} \quad (24)$$

where τ and μ are defined in the following forms:

$$\tau = \varepsilon E[S_n^{2(\lambda+1)}]n \quad (25)$$

$$\begin{aligned} \mu = & \frac{2\varepsilon}{E[S_n^{2(\lambda+1)}]} \int_0^\infty \{ E[S_n^{2(\lambda+1)} S_{n+n'}^{2(\lambda+1)}] \\ & - E[S_n^{2(\lambda+1)}] E[S_{n+n'}^{2(\lambda+1)}] \} dn' \end{aligned} \quad (26)$$

in which $E[\cdot]$ denotes an operator to take expectation.

Fig. 2 Schematic representation of the importance of the notion of death point $\{\infty\}$

After some mathematical manipulation, the solution of the generalized Fokker-Planck equation that satisfies

$$\lim_{\tau \rightarrow 0} w(z, \tau|z_0) = \delta(z - z_0) \quad (27)$$

can be obtained as follows:

$$\begin{aligned} w(z, \tau|z_0) = & \frac{1}{z^{\lambda+1} \sqrt{2\pi\mu\tau}} \\ & \times \exp \left[-\frac{(z_0^{-\lambda} - z^{-\lambda} - \lambda\tau)^2}{2\lambda^2\mu\tau} \right] \end{aligned} \quad (28)$$

By use of this solution, the distribution of N , namely, $G(n|z_0, z_c)$ is given as follows:

$$\begin{aligned} G(n|z_0, z_c) = & 1 \\ & - \Phi \left[\frac{z_0^{-\lambda} - z_c^{-\lambda} - \lambda\varepsilon E[S_n^{2(\lambda+1)}]n}{\lambda\varepsilon \sqrt{\{E[S_n^{4(\lambda+1)}] - (E[S_n^{2(\lambda+1)}])^2\} n_c n}} \right] \end{aligned} \quad (29)$$

where the correlation time n_c in the random excitation process $S_n^{2(\lambda+1)}$ is defined by Eq. (6) with b being replaced by $2(\lambda+1)$. In this case, $P_{\infty}(n)$ defined by Eq. (21) is given as follows:

$$\begin{aligned} P_{\infty}(n) = & 1 \\ & - \Phi \left[\frac{z_0^{-\lambda} - \lambda\varepsilon E[S_n^{2(\lambda+1)}]n}{\lambda\varepsilon \sqrt{\{E[S_n^{4(\lambda+1)}] - (E[S_n^{2(\lambda+1)}])^2\} n_c n}} \right] \end{aligned} \quad (30)$$

where both z_0 and z_c are supposed to be deterministic.

In this kind of phenomenon where the accumulation rate increases gradually, the effect of the uncertainty of z_0 becomes greater than that of z_c . It is interesting to note that, even if z_0 is not deterministic and follows a distribution $\Psi(z_0)$, the distribution $G(n|z_c)$, for instance, can be obtained in the following form by straightforwardly considering the result of Eq. (29) to be the conditional probability⁽²⁵⁾:

$$G(n|z_c) = \int_0^\infty G(n|z_0, z_c) d\Psi(z_0) \quad (31)$$

By expanding this concept, cases such as random crack propagation resistance⁽⁵⁾⁽¹⁰⁾⁽²⁶⁾, finite width of the specimen⁽⁸⁾⁽²⁷⁾, inspections during service⁽²⁸⁾⁽²⁹⁾ and elliptical crack growth⁽³⁰⁾⁻⁽³²⁾ have recently been studied.

4. Topics Associated with the Semicumulative Effect

Following on from the discussions in the preceding section, this section is concerned with fairly complex processes rather than simple accumulation, where both accumulation and decay alternate repeatedly. This may be regarded as a process with a semicumulative effect, one example of which would be the case of the cumulative damage process in consideration of recovery. However, it is rare to successfully model this phenomenon reasonably well. Even if the modelling is successful, it is almost impossible to obtain the distribution of T , namely, $G(t; \Theta)$, due to the intricacies of the mathematical manipulation. Even for a particularly simple case, it is no easy task to obtain yet lower order moments of T .

With the aid of the importance sampling method⁽³³⁾, some studies have recently been performed on the reliability analysis based on the two-criteria approach⁽³⁴⁾⁽³⁵⁾ where both brittle failure due to crack propagation and ductile failure due to overload are assumed to take place. This analysis can be regarded as the model where the semicumulative damage is divided into two parts, that is, cumulative damage and noncumulative. In this kind of complicated situation, simulation techniques undoubtedly play an important role.

5. Topics Associated with the Noncumulative Effect

In this section, discussions center on the phenomenon that the effect of random loading is not cumulative. To begin with, let us consider the case of linear response of the system to random excitations. In the steel-reinforced concrete structure, for instance, assuming that the structure will fail or reach the unsafe domain when at least one element of the steel reinforcements yields or crushing occurs in the concrete, the response of the structure before reaching the failed state may be considered approximately linear. In fact, the response of each element differs, yet the safe domain of each element seems to be almost equal. However, in the case that all the vibration modes of the structure are known, it may be considered that the response of each element $\mathbf{Z}_i$ is the same, but the safe domain differs according to each element.

Let D_i be the safe domain of the i -th element and D be that of the structural system defined as

$$D = \bigcap_i D_i \quad (32)$$

In this case, it is difficult to obtain rigorously the distribution of T , $G(t; \Theta)$, and hence, by use of the mean outcrossing rate $\nu(t; \Theta)$, the distribution $G(t; \Theta)$ can usually be approximated as follows⁽³⁶⁾:

$$G(t; \Theta) \cong 1 - \exp\left\{-\int_0^t \nu(t'; \Theta) dt'\right\} \quad (33)$$

When D does not depend upon t , the mean outcrossing rate $\nu(t'; \Theta)$ can be represented by expanding Rice's method, as follows⁽³⁷⁾:

$$\nu(t'; \Theta) = \int_{\partial D} dA_m \int_0^\infty \dot{z}_{nt'} f_{z_{nt'}, \dot{z}_{nt'}}(z_{nt'}, \dot{z}_{nt'}) d\dot{z}_{nt'} \quad (34)$$

where $f_{z_{nt'}, \dot{z}_{nt'}}(z_{nt'}, \dot{z}_{nt'})$ is the joint probability density function between probability vector $\mathbf{Z}_{nt'}$ and random variable $\dot{Z}_{nt'} \equiv \mathbf{n}^T \dot{\mathbf{Z}}_{nt'}$, and ∂D represents the boundary of the m -dimensional domain D , and in the definition equation of $\dot{Z}_{nt'}$, $\mathbf{n}$ is the external unit normal vector and dA_m is the surface element on ∂D . However, in the case that $m > 1$, multifold integration in Eq. (34)

cannot easily be performed except in special cases. Hence, the upper or the lower bound of $\nu(t'; \Theta)$ needs to be evaluated in those cases where $\nu(t'; \Theta)$ cannot be reduced to a simple form. When $\mathbf{Z}_t$ is the stationary Gaussian process and ∂D is the hyperplane, multifold integration in Eq. (34) can be performed to yield the following results. In other words, let $\mathbf{R}_m$ be the m -dimensional Euclidian space, and D be defined by

$$D = \{\mathbf{z} \in \mathbf{R}_m | \mathbf{n}^T \mathbf{z} \leq r, r > 0\} \quad (35)$$

thus it follows from Eq. (34) that⁽³³⁾:

$$\nu(t'; E[\dot{Z}_i \dot{Z}_j]) = \frac{1}{2\pi} \left\{ \sum_{i,j=1}^m n_i n_j E[\dot{Z}_i \dot{Z}_j] \right\}^{\frac{1}{2}} e^{-\frac{r^2}{2}} \quad (i, j = 1, 2, \dots, m) \quad (36)$$

Based on this result, the upper bound of $\nu(t'; \Theta)$ is discussed in the case where D is the convex hyperpolyhedron. In cases where ∂D is the hyperspherical surface, the appropriate upper bound of $\nu(t'; \Theta)$ for the stationary Gaussian process $\mathbf{Z}_t$ is also obtained⁽³⁷⁾.

That is, for D defined by

$$D = \{\mathbf{z} \in \mathbf{R}_m | \mathbf{z}^T \mathbf{z} \leq r^2\} \quad (37)$$

it follows that

$$\nu(t'; E[\dot{Z}_i \dot{Z}_j], E[Z_i Z_j]) \leq \frac{r^{m-1}}{\sqrt{2\pi}} e^{-\frac{r^2}{2}} \frac{(\sum_{i=1}^m k_{ii}/m)^{\frac{1}{2}}}{2^{\frac{m}{2}-1} \Gamma(\frac{m}{2})} \quad (38)$$

where k_{ij} is defined by

$$k_{ij} = E[\dot{Z}_i \dot{Z}_j] - \sum_{k=1}^m E[Z_k \dot{Z}_i] E[Z_k \dot{Z}_j] \quad (39)$$

In a specific case where

$$k_{ij} = E[\dot{Z}_i^2] \delta_{ij} \quad (40)$$

is satisfied, the equality in Eq. (38) holds. This result is the same as that obtained by Veneziano et al.⁽³⁸⁾.

Even if the system of equations which describes and governs the behaviors of the system is linear, the aforementioned treatment becomes impossible unless the applied load is stationary. In the case of so-called parametric excitation, where the coefficients of the state variables are excited by the applied load, the treatment is virtually the same as in the case of the nonlinear system; it is also difficult to obtain the distribution of the first passage time. Hence, it is customary to deal only with lower order moments⁽³⁹⁾.

Next, let us consider the case of nonlinear response. In this case, roughly speaking, there are two different methods to obtain the distribution of the first passage time of the response. One method obtains the approximate distribution of the first passage time T through Eq. (33) with the mean outcrossing rate $\nu(t; \Theta)$ gained by Rice's formula Eq. (34), after obtaining the joint probability distribution between the response $\mathbf{Z}_{t'}$ and its derivative $\dot{\mathbf{Z}}_{t'}$ with the solution of the Fokker-Planck equation based upon the Markov approximation method⁽⁴²⁾⁽⁴³⁾. It is impossible at any

stage to obtain a rigorous solution due to the complexity of the problem, hence the introduction of an approximation method. As for the Markov approximation method, reference to the paper by Lin⁽⁴⁰⁾ is highly recommended. As for the response distribution method, papers by Caughey⁽⁴⁴⁾⁽⁴⁵⁾ and by Wen⁽⁴⁶⁾ should be referred to. In general, the density function $f_{z_t, \dot{z}_t}(\mathbf{z}_t, \dot{\mathbf{z}}_t)$ in Eq. (34) is not Gaussian and the computation of $\nu(t; \Theta)$ is not easy.

The other method is to obtain the distribution of the first passage time T with the aid of the solution of the initial boundary value problem of the Fokker-Planck equation⁽⁴⁷⁾⁽⁴⁸⁾. This method may not be practical, since many difficulties arise in solving boundary value problems with accuracy. Roberts reviewed the status of recent studies on the methods of obtaining the first passage probability of the Markov diffusion process⁽⁴⁹⁾.

Since these two methods have the drawback of requiring a great deal of computation, Suzuki and Minai recently proposed a new method used to avoid such difficulties in obtaining the outcrossing rate and/or solving the boundary value problem⁽⁴⁹⁾. In other words, their method deals with both response analysis and reliability analysis at the same time by regarding the current safety measure as one of the states of the system in a wide sense. For instance, with the state quantity $Z(t)$ being the deformation, and by representing the maximum ductility ratio

$$\eta(t) = \max_{0 \leq \tau \leq t} |Z(\tau)| \quad (41)$$

in differential form as

$$\dot{\eta} = |\dot{Z}|U(Z\dot{Z})U(|Z| - \eta) \quad (42)$$

η can be regarded as one of the components of the state vector. In Eq. (42), $U(\cdot)$ represents the unit step function. However, the Fokker-Planck equation with multicomponents is usually very difficult to deal with, inevitably involving a great deal of computation.

In view of this difficult situation, simulation techniques may be applicable to obtain directly the information on the distribution of the first passage time T ⁽⁵⁰⁾. In this case, to reduce computer memory occupation, it will be more convenient to solve the system equation in the process of generating the random sample functions to be used as an input. As for the generation of the sample functions, fast Fourier transform (FFT) techniques have been widely utilized so far⁽⁵¹⁾. However, from the aforementioned viewpoint, particularly in the case of a multicomponent system, FFT techniques are inconvenient and time-consuming due to their large computer memory requirement. To conquer these difficulties, the use of an AR-MA model is highly recommended. With the aid of an AR-MA model, this method generates sample functions of

desired statistical properties recursively from a series of independent random numbers in the following fashion:

$$\mathbf{y}_t + \sum_{i=1}^q A_i \mathbf{y}_{t-i} = \sum_{i=0}^q B_i \mathbf{x}_{t-i} \quad (43)$$

where $\mathbf{x}_t$ is an n -variate Gaussian white random time series. The effectiveness of this method has recently been confirmed even in multicomponent systems if the AR-MA coefficient matrix is determined by the two-stage procedure, which means that $C_{yx}(\tau)$ is first calculated through $C_{yy}(\tau)$ by use of a certain approximation method, and AR and MA coefficient matrices are determined at the same time by use of both $C_{yy}(\tau)$ and $C_{yx}(\tau)$ ⁽⁵²⁾.

6. Conclusions

In order to assure the reliability of important structural components, it is of crucial importance to obtain the distribution of the random variable $G(t; \Theta)$ defined by Eq. (2) in Section 2 or to estimate, with good accuracy, the distribution of the sample random variable $\hat{G}(t; \Theta)$ associated with $G(t; \Theta)$, taking into consideration the effects of not only the randomness of the applied load but also various other uncertainty factors associated with the random crack propagation resistance, the interspecimen variation of parameters, lack of information on parameters, and uncertainties of modelling and parameter estimation due to finite sample size. As stated in Sections 3 through 5, it is comparatively easy to obtain the distribution of the first passage time T in simple cumulative processes, since the problem of obtaining the distribution of T can be reduced to that of obtaining the distribution of the state of the system $\mathbf{Z}_t$ at time t . However, in the noncumulative processes, detailed discussion can hardly be developed as the first passage problem itself can never be solved except in a few special cases. Hence, some kind of approximation method is indispensable. Furthermore, for semicumulative processes, it appears very difficult to obtain an effective solution even by approximation.

In conclusion, the present paper sorts the problems of structural reliability assurance under random loading into three categories from the viewpoint of whether the induced effect by the random excitation process is cumulative or not. For each category, the fundamental concept as well as typical examples are presented and discussed concisely, and a comprehensive state-of-the-art survey is also provided to give an integrated view of the problems.

References

- (1) Ishikawa, H., Reliability Analysis of Fatigue Lives of Machines and Structures Under Service

- Loads, J. JSMS, (in Japanese), Vol. 24, No. 264 (1975), p. 914.
- (2) Ishikawa, H., A State-of-the-Art Survey of Reliability-Based Design of Structures, J. JSSC, (in Japanese), Vol. 18, No. 192 (1982), p. 3.
 - (3) Ishikawa, H. and Tsurui, A., Statistical Properties of Random Load, J. JSMS, (in Japanese), Vol. 31, No. 346(1982), p. 736.
 - (4) Tsurui, A., Ishikawa, H. and Kimura, H., On the Estimation Reliability of Failure Probability, J. JSMS, (in Japanese), Vol. 31, No. 347 (1982), p. 804.
 - (5) Ishikawa, H., Tsurui, A., A Stochastic Fatigue Crack Growth Model in Consideration of Random Propagation Resistance, Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 50, No. 454, A (1984), p. 1309.
 - (6) Tsurui, A. and Ishikawa, H., On the Reliability Degradation of Structural Component in Fatigue Crack Growth Process, Trans. Jpn. Soc. Mech. Eng., (in Japanese) Vol. 51, No. 461, A (1985), p. 17.
 - (7) Ishikawa, H., Tsurui, A., Utsumi, A. and Kimura, H., Determination of the Optimum Sample Size in the Reliability-Based Fatigue-Proof Design, Proc. of ICOSSAR'85, Vol. 2(1985), p. 415, IASSAR.
 - (8) Tsurui, A., Ishikawa, H., Utsumi, A. and Sako, A., A Stochastic Fatigue Crack Growth Model in Consideration of Size Effect, J. JSMS, (in Japanese), Vol. 35, No. 393(1986), p. 578.
 - (9) Ishikawa, H., Tsurui, A., Utsumi, A. and Tanaka, H., Sensitivity Study on a Stochastic Fatigue Crack Growth Model under Stationary Loading, Proc. of ICM-5, Vol. 1(1987), p. 699, Pergamon Press.
 - (10) Ishikawa, H., Tsurui, A. and Kimura, H., Stochastic Fatigue Crack Growth Model and Its Wide Applicability in Reliability-Based Design, Statistical Research on Fatigue and Fracture (CJMR: Current Japanese Materials Research), Vol. 2 (1987), p. 45, Elsevier.
 - (11) Shinozuka, M., Ishikawa, H. and Ishikawa, H., Practical Implementation of Fatigue-Proof Design in Structural Components with the Aid of Reliability-Based Scatter Factors, Recent Studies on Structural Safety and Reliability (CJMR), Vol. 5(1989), p. 25, Elsevier.
 - (12) Tanaka, H., Tsurui, A. and Ishikawa, H., Cross Effect of Uncertainty Factors Associated with Fatigue Crack Growth Process, J. JSMS, (in Japanese), Vol. 35, No. 399(1986), p. 1385.
 - (13) Stratonovich, R. L., Topics in the Theory of Random Noise, Vol. 1(1963), p. 22, Gordon and Breach Sci. Publ., Inc.
 - (14) Tsurui, A., Probability of Fatigue Failure as a Statistic, Int. J. Fatigue, Vol. 3(1981), p. 125.
 - (15) Ishikawa, H., Tsurui, A., Tanimoto, T. and Kimura, H., Reliability-Based Fatigue-Proof Design of Composite Materials, Proc. of ICM-4, Vol. 1(1983), p. 533.
 - (16) Sogliero, G. and Srinivasan, A. V., Fatigue Life Estimates of Mistuned Blades via a Stochastic Approach, AIAA J., Vol. 18(1980), p. 318.
 - (17) Shimokawa, T. and Tanaka, S., A Statistical Consideration of Miner's Rule, Int. J. Fatigue, Vol. 2(1980), p. 165.
 - (18) Nakagawa, T. and Nakabayashi, H., Prediction of Miner Damage Distribution by Bayesian Method, J. JSMS., (in Japanese), Vol. 28(1979), p. 9.
 - (19) Jono, M., Izuchi, T. and Kikukawa, M., Statistical Study on Fatigue Lives of Carbon Steel, S35C, under Repeated Two-Step Loadings, J. JSMS., (in Japanese), Vol. 30(1981), p. 8.
 - (20) Freudenthal, A. M., The Scatter Factor in the Reliability Assessment of Aircraft Structures, J. Aircraft, Vol. 14, No. 2(1977), p. 202.
 - (21) Ishikawa, H., Tanimoto, T., Kimura, H. and Shinozuka, M., Estimation of the Reliability of Machines and Structures with Life Distribution of a Weibull Type, J. JSMS, (in Japanese), Vol. 30, No. 328(1981), p. 39.
 - (22) Tsurui, A. and Ishikawa, H., Theoretical Study on the Distribution of Fatigue Crack Propagation Life under Stationary Loading, Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 51, No. 461(1985), p. 31.
 - (23) Tsurui, A. and Ishikawa, H., Application of the Fokker-Planck Equation to a Stochastic Fatigue Crack Growth Model, Structural Safety, Vol. 4, No. 4(1986), p. 15.
 - (24) Tsurui, A., Ishikawa, H. and Utsumi, A., Theoretical Study on the Distribution of Fatigue Crack Propagation Life under Stationary Random Loading, Proc. of Fatigue '84, Vol. 2(1984), p. 949.
 - (25) Tsurui, A. and Ishikawa, H., The Effect of Initial Flaws on the Reliability Degradation of Structural Components Based on a Stochastic Fatigue Crack Growth Model, Proc. of ICOSSAR'85, Vol. 3(1985), p. III-495, IASSAR.
 - (26) Tanaka, H. and Tsurui, A., Reliability Degradation of Structural Components in the Process of Fatigue Crack Propagation under Stationary Random Loading, Eng. Frac. Mech, Vol. 27, No. 5 (1987), p. 501.
 - (27) Tsurui, A., Tanaka, H. and Tanaka, T., Probabilistic Analysis of Fatigue Crack Propagation in Finite Size Specimens, Prob. Eng. Mech., to appear.
 - (28) Tsurui, A. and Sako, A., Reliability Analysis of Fatigue Crack Growth Process under Stationary Random Loading, Current Studies on Structural Safety and Reliability (CJMR), Vol. 5, p. 153, Elsevier.
 - (29) Tsurui, A., Sako, A. and Tanaka, T., The Effects of Repeated Inspections on the Structural Reliability Degradation, J. JSMS, (in Japanese), to appear.
 - (30) Tsurui, A. and Tanaka, H., Probabilistic Analysis for the Propagation of Surface Cracks of Semi-

- Elliptic Type, J. JSMS, (in Japanese), Vol. 37, No. 420(1988).
- (31) Tanaka, H. and Tsurui, A., Random Propagation of Semi-Elliptical Surface Crack as a Bivariate Stochastic Process, Eng. Frac. Mech., Vol. 33, No. 5(1989), p. 787.
 - (32) Tanaka, H., Stochastic Properties of Semi-Elliptical Surface Crack Propagation Based upon the Newman-Raju's K-Expression, Eng. Frac. Mech., Vol. 34, No. 1(1989), p. 189.
 - (33) Schuëller, G. I. and Stix, R., A Critical Appraisal of Methods to Determine Failure Probabilities, Structural Safety, Vol. 4(1987), p. 293, Elsevier.
 - (34) Tsurui, A., Nienstedt, J., Schuëller, G. I. and Tanaka, H., Time Variant Structural Reliability Analysis Using Diffusive Crack Growth Models, Eng. Frac. Mech. Vol. 34, No. 1(1989), p. 153.
 - (35) Nienstedt, J., Tsurui, A., Tanaka, H. and Schuëller, G. I., Time Variant Structural Reliability Analysis Using Bivariate Diffusive Crack Growth Models, Int. J. Fatigue, to appear.
 - (36) Rice, S. O., Mathematical Analysis of Random Noise, Bell System Tech. J., Vol. 23(1944), p. 282.
 - (37) Shinozuka, M., Kako, T. and Tsurui, A., Random Vibration Analysis in Finite Element Formulation, Random Vibration-Status and Recent Developments, (1986), p. 415, Elsevier.
 - (38) Veneziano, D., Grigoriu, M. and Cornell, C. A., Vector-Process Models for System Reliability, J. Eng. Div, ASCE, Vol. 103, No. EM3(1977), p. 441.
 - (39) Bergman, L. A. and Spencer, B. F. Jr., First Passage Time for Linear Systems with Stochastic Coefficients, Prob. Eng. Mech., Vol. 2(1987), p. 46.
 - (40) Lin, Y. K., Some Observations on the Stochastic Averaging, Prob. Eng. Mech., Vol. 1, No. 1(1986), p. 23.
 - (41) Roberts, J. B., Response of an Oscillator with Non-Linear Damping and a Softening Spring to Non-White Random Excitation, Prob. Eng. Mech, Vol. 1, No. 1(1986), p. 40.
 - (42) Lin, Y. K., First-Excursion Failure of Randomly Excited Structures, AIAA J., Vol. 8, No. 4(1970), p. 720.
 - (43) Crandall, S. H., Chandiramani, K. L. and Cook, R. G., Some First-Passage Problems in Random Vibration, J. Appl. Mech., Trans. ASME, Vol. 33, No. 3(1966), p. 532.
 - (44) Caughey, T. K., Nonlinear Theory of Random Vibrations, Adv. Appl. Mech., Vol. 11 (1971), p. 209, Academic Press.
 - (45) Caughey, T. K., On the Response of Non-Linear Oscillators to Stochastic Excitation, Prob. Eng. Mech., Vol. 1, No. 1(1986), p. 2.
 - (46) Wen, Y-K., Stochastic Response and Damage Analysis of Inelastic Structures, Prob. Eng. Mech., Vol. 1, No. 1(1986), p. 49.
 - (47) Spanos, P-T. D., Survival Probability of Non-Linear Oscillators Subjected to Broad-Band Random Disturbances, Int. J. Non-Linear Mech., Vol. 17, No. 516(1982), p. 303.
 - (48) Roberts, J. B., First-Passage Probabilities for Randomly Excited Systems: Diffusion Methods, Prob. Eng. Mech., Vol. 1, No. 2(1986), p. 67.
 - (49) Suzuki, Y. and Minai, R., Application of Stochastic Differential Equations to Seismic Reliability Analysis of Hysteretic Structures, Prob. Eng. Mech., Vol. 3, No. 1(1988), p. 43.
 - (50) For example, Shinozuka, M. and Wen, Y-K., Monte Carlo Solution of Nonlinear Vibrations, AIAA J., Vol. 10(1972), p. 37.
 - (51) For example, Shinozuka, M., Monte Carlo Solution of Structural Dynamics, Computers and Structures, Vol. 2(1972), p. 855.
 - (52) Samaras, E., Shinozuka, M. and Tsurui, A., ARMA Representation of Random Processes, J. Eng. Mech., ASCE, Vol. 111(1985), p. 449.