The Dynamic Behavior of an Elastic Circular Tube Due to Longitudinal Impact*

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Two-dimensional stress waves in an elastic circular tube are analyzed numerically when axisymmetric impacts are applied to the end face of the tubes. For the numerical analysis, a finite difference method based on integration along the bicharacteristics is employed. By obtaining the numerical results for various tube sizes, and for the impact loading condition, it is clearly demonstrated that, when the rise time of the applied impact velocity is sufficiently short, a large tensile stress arises on the boundary of the internal surface of the tube. It is also found that the maximum value of the caused tensile stress on the boundary of the internal surface becomes larger when the rise time becomes shorter, and when the ratio of the thickness of the tube to the outside diameter becomes larger than about 0.3.

Key Words: Numerical Analysis, Stress Wave, Circular Tube, Impulsive Load, Stress Concentration

1. Introduction

It often occurs that machine elements or structures are badly deformed or fractured when they receive an impact. In particular, the problem of failure prevention posed by apparatuses or structures made of brittle material under impact loads is serious.

Validity of the Hopkinson bar method has been examined^{(1),(2)} through the two-dimensional analysis on elastic waves in a circular cylinder under longitudinal impact. By examining the stress concentration⁽³⁾ and distribution of the maximum tensile stress⁽⁴⁾ arisen in a circular cylinder under various kinds of impact loads, the generation of the peculiar failure patterns⁽⁵⁾ under impact loading has been explained. Also, two-dimensional analyses on the elastic/viscoplastic wave motion in a circular cylinder have been carried out^{(6)~(8)}. It does not seem, however, that the

dynamic behavior of a circular cylinder containing a hole concentric with the axis, or a circular tube, under a longitudinal impact applied to the end surface has been investigated thoroughly.

In this study, the authors analyzed the two-dimensional wave motion in elastic cylinders containing a hole concentric with the axis and circular tubes, under a longitudinal impact featuring a ramp-shaped rising pattern, using the numerical integration along the bicharacteristics and investigated both stress variation and tensile stresses occurring in various types of circular bodies. As a result, the authors have obtained interesting results, including the fact that the tensile stress distribution in a circular tube varies greatly depending on the shape of the circular tube, and that the peak value of the tensile stress could be larger than that occurring in a circular cylinder.

2. Governing Equations

Figure 1 shows the elastic, circular tube (or a hollow column) having an outer diameter of $2R_2$, an inner diameter of $2R_1$, and a length of L_0 which was adopted by the authors as a model for the study of the propagation of two-dimensional stress waves. The

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impact having a ramp-shaped rising velocity $V_0(t)$ is assumed to be applied to the end surface of this circular tube. Cylindrical coordinates (r, θ, z) are taken, where the z axis coincides with the axis of the tube. From the symmetric property, it is sufficient for us to consider only the plane $\theta=0$. The governing equations to the axisymmetric problem like this are given as follows⁽¹⁾⁻⁽³⁾:

$$L[W] = A^r W_r + A^\theta W_\theta + A^z W_z - B = 0 \quad (1)$$

where the matrices A^r , A^θ and A^z are square matrices of the sixth order⁽³⁾ consisting of elements independent of unknown functions.

W and B are vectors as follows:

$$W^t = [u, v, p, q, s, n]$$

$$B^t = [2q/\eta, n/\eta, u/\eta, -u/\eta, 0, 0]$$

where

$$\tau = \frac{c_1}{r_0} t, \quad \eta = \frac{r}{r_0}, \quad \xi = \frac{z}{r_0}, \quad u = \frac{v_r}{c_1}, \quad v = \frac{v_z}{c_1}$$

$$p = \frac{\sigma_r + \sigma_\theta}{2\rho c_1^2}, \quad q = \frac{\sigma_r - \sigma_\theta}{2\rho c_1^2}, \quad s = \frac{\sigma_z}{\rho c_1^2}, \quad n = \frac{\tau_{rz}}{\rho c_1^2}.$$

σ_r , σ_θ , σ_z and τ_{rz} denote each stress component. v_r and v_z represent each particle velocity component, while c_1 is the velocity of propagation of the dilatational wave. ρ denotes the density, t the time, and r_0 the representative length.

3. Numerical Calculation

Equation (1) represents a system of semi-linear partial differential equations of the first order, and we can obtain its bicharacteristics and the differential relations along the bicharacteristic directions⁽¹⁾⁻⁽³⁾⁽⁶⁾⁽⁹⁾. In addition, by simultaneously solving the above-mentioned relations and the difference equations, which can be obtained by integrating Eq.(1) along the time axis, we can obtain respective increments δu , δv , δp , δq , δs and δn of respective components of vector W . At boundaries such as the end surface of the circular tube, the number of significant bicharacteristics decreases, namely, the number of difference equations decreases. This can be compensated, however, by introducing boundary conditions as necessary. Thus, combination of the required number of difference equations can be obtained for each boundary. The load function $V(\tau)$ is assumed to be as

Eq.(2) (refer to Fig. 2).

$$V(\tau) = \begin{cases} V_m \tau / \tau_r & (\tau \leq \tau_r) \\ V_m & (\tau_r \leq \tau) \end{cases} \quad (2)$$

where V_m denotes a constant impact velocity, which is dimensionless, after the rise-time τ_r . In addition, the ratio of the wall thickness of the tube to the outer radius is expressed as $H = (R_2 - R_1)/R_2$.

The boundary conditions are as follows:

$$\left. \begin{array}{ll} \text{Impact end:} & v = V(\tau), \quad n = 0 \\ \text{Free end:} & s = 0, \quad n = 0 \\ \text{Inner surface:} & p + q = 0, \quad n = 0 \\ \text{Outer surface:} & p + q = 0, \quad n = 0. \end{array} \right\} \quad (3)$$

As the initial condition, it was assumed that the stress components and the particle velocity components were all zero at $\tau=0$. With regard to the stability of the numerical solution, the energy method was adopted, in which the relative energy error $[(E_{inp} - E_{int})/E_{inp}]$ is examined, using both input energy E_{inp} coming from the impact end and energy E_{int} possessed in the circular tube.

4. Numerical Results and Discussion

The numerical calculation was carried out on an example as follows: length of the circular tube (or a hollow column) $L_0 = 0.06$ m; representative length $r_0 = 0.01$ m; ratio of the wall thickness to the outer radius, $H = 0.17$ to 1.0 ; and nondimensional rise-time in the load function, $\tau_r = 1, 3$ and 6 . Since the material was steel, the material constants were chosen as follows: Young's modulus $E = 205.8$ GPa, density $\rho = 7.86 \times 10^3$ kg/m³ and Poisson's ratio $\nu = 0.25$.

4.1 Stress variation in circular tube

Figure 3 shows the variation with time of the stresses $(p+q)$, $(p-q)$ and s on the circumferential surface at the middle of the wall at the both impact end ($\xi=0$) and the middle of the tube length ($\xi=3$) when $H=0.17$ and $\tau_r=1$. From Fig. 3, it can be seen that stress $(p+q)$ varies periodically between the tensile stress range and the compressive stress range.

Figure 4 shows the distributions of both stresses $(p-q)$ and s on the inner surface of a circular tube when $H=0.50$ and $\tau_r=1$. From Fig. 4, it can be seen that on the inner surface of the tube the tensile stresses could be really remarkable.

Fig. 1 Circular tube and coordinates

Fig. 2 Load function

Figure 5 shows the stress distributions along the inner surface of a circular tube (or a hollow column) when $H=0.17$ and 0.91 and $\tau_r=1$ and 6 . From Fig. 5, we can see that the smaller τ_r , the greater the variation in the stress distribution. From Fig. 5, it can also be seen that, when $\tau_r=1$, the greatest variation in stress ($p-q$) occurs in the case of $H=0.91$, which is of a circular tube (or a hollow column) having a large wall thickness.

4.2 Position of failure

When an impact load is applied to a circular tube (or a hollow column) made of brittle material which is weak in resisting a large tensile stress, it is considered that the position of the failure is closely related to the position of the highest tensile stress⁽⁴⁾. In addition,

even if the material is not brittle, it is considered that the position of failure caused by the repeated impact loads closely corresponds to the position where the tensile stress is generated. In this connection, the authors investigated the value of the maximum tensile stress $\sigma_{\max}/V_m$ generated at various points in the circular tube (or a hollow column). The above-mentioned $\sigma_{\max}$ denotes the maximum value of the three principal stresses σ_I, σ_{II} and σ_{III} ; i.e. $\sigma_{I,II,III} = \{(s+p+q)/2 \pm [(s-p-q)^2/4 + n^2]^{1/2}, p-q\}$. Figures 6 to 9 show the distribution of the maximum tensile

Fig. 3 Stress variation on circumferential surface at the middle of the wall

Fig. 4 Stress distributions on the inner surface

Fig. 5 Stress distributions on the inner surface

Fig. 6 Distribution of the maximum tensile stress

stress generated in the wall of the circular tube. The values of τ_r , τ and H are indicated in each figure. In Fig. 6(a), the value of $\sigma_{\max}$ becomes larger in the middle of the wall and the distribution is of a convex shape. This is mainly due to the stress component $(p+q)$ in the radial direction (refer to Fig. 3). In Fig. 6(b), $\sigma_{\max}$ takes the maximum value in the vicinity of the free end ($\xi=6$). This is mainly due to the stress component s in the axial direction (refer to Figs. 3 and 5). This phenomenon agrees with that shown in Fig. 11 in the ref. (4). By comparing Figs. 6(b), 7 and 8, we can see that as the wall thickness increases relatively (in other words, as H increases, or as the inner diameter decreases), the values of $\sigma_{\max}$ on the inner surface of the circular tube, which correspond to the values at $\eta=1.0$ in Fig. 7 and at $\eta=0.1$ in Fig. 8, increase. This increase in the $\sigma_{\max}$ is mainly due to the increase in the circumferential stress component $(p-q)$ (refer to Figs. 4 and 5). By comparing Figs. 7 and 9, we can see that the value of $\sigma_{\max}$ becomes smaller as τ_r becomes longer.

Figure 10 shows the results of the maximum tensile stress σ^* , which are obtained by reading the maximum values in the distribution diagrams of the maximum tensile stresses occurred during the period up to $\tau=10$ for respective calculation-conditions and by rearranging them. Figure 10 shows also the results of the case of a circular cylinder ($H=1.0$), for refer-

Fig. 7 Distribution of the maximum tensile stress

Fig. 8 Distribution of the maximum tensile stress

ence. From Fig. 10, it can be seen that the shorter the rise time τ_r , the higher the maximum tensile stress σ^* . When H becomes greater than about 0.3, the position corresponding to σ^* is located on the inner surface, and the value of σ^* increases suddenly, as shown in Fig. 10. From Fig. 10, it is also seen that the value for the circular cylinder ($H=1.0$) corresponds approximately to that for the circular tube of $H=0.3$.

5. Conclusions

The authors investigated the two-dimensional wave motion in various types of elastic, circular tubes (or hollow columns), which occurs when the impulsive velocity having a ramp-shaped rising pattern is applied to the end surface of the tube. The findings are summarized as follows.

(1) The shorter the rise time τ_r of the load function acting on the impact end, the greater the stress variation in the circular tube. Unlike the case of the circular cylinder, the tensile stress tends to occur on the inner surface of the circular tube. In the case of the circular tube having a large wall thickness ($H=$

Fig. 9 Distribution of the maximum tensile stress

Fig. 10 Variation in the maximum tensile stresses generated in the circular tube

0.91, or a hollow column), the distribution of the circumferential stress component ($p-q$) varies greatly, resulting in large tensile stress.

(2) The maximum tensile stress $\sigma_{\max}$ occurring at respective points in the tube becomes greater as τ_r becomes shorter. When the wall thickness is large (or when the inner radius is small), the circumferential stress component ($p-q$) on the inner surface becomes larger, and consequently $\sigma_{\max}$ also becomes large. The pattern of the $\sigma_{\max}$ distribution for the circular tube differs from that for the circular cylinder.

(3) The value of the maximum tensile stress σ^* occurring in the circular tube becomes larger as τ_r becomes shorter. When H becomes larger than about 0.3, σ^* increases suddenly, and the position corresponding to the σ^* is located on the inner surface of the tube. The values of σ^* in the case of the circular cylinder correspond approximately to those in the case of the circular tube (or a hollow column) of $H \approx 0.3$. From the above, it follows that when circular tubes (or hollow columns) made of brittle material receive a longitudinal impact, failure could occur on the inner surface of the circular tube if H is greater than about 0.3.

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