Moire Analysis of Strain by Fourier Transform*

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A "complex moire pattern" is obtained by shifting the discrete Fourier spectrum of the image of a deformed grating. The strain distribution is given as the derivatives of the phases of the complex moire fringes. The analysis is completely automated by digital image processing. All of the laborious and subjective procedures required in conventional analysis, such as fringe sign determination, fringe ordering, and fringe interpolation, are completely eliminated, and objective, fast and accurate analysis is performed.

Key Words: Experimental Stress Analysis, Strain Measurement, Moire Method, Image Processing, Fourier Transform

1. Introduction

The moire method has been used as a technique for measuring the full-field deformation of a specimen surface⁽¹⁾. A moire fringe pattern is generated by the superposition of two gratings: a model grating and a master grating. The fringes show contour lines of equal displacement. Conventional analysis of fringes to determine strain distribution can be characterized by the next three steps:

- (1) Pinpointing of center lines of fringes,
- (2) Allocation of fringe orders which signify equal displacement on each line,
- (3) Differentiation of displacement, i. e., determination of strain.

However, the following demerits are involved in the process mentioned above. In step (1), inconsis-

encies of the position occur due to the human factor in extracting center lines of fringes. In step (2), it is difficult to determine fringe orders when they increase and decrease by turns in a general complicated situation of deformation. In step (3), displacement data given only sporadically on the specimen require some kind of interpolation before numerical differentiation. Choice of interpolating function more or less affects the resulting strain.

Recently, Takeda and Mutoh⁽²⁾ developed, in the field of moire topography, "Fourier transform profilometry (FTP)", in which a profile of a 3-D object with a projected grating is automatically evaluated by calculating the discrete Fourier transform of the grating image.

The procedures are the discrete Fourier transform of the deformed grating image followed by extraction of the first harmonic and the inverse discrete Fourier transform of the extracted first harmonic. The output shows the twin grating patterns whose phase difference is $\pi/2$. The brightness intensity at any point of the grating patterns, which has almost never been utilized, in addition to fringe centers, provides information to form a continuous

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profile. The extraction process of the first harmonic contributes so that all the intensities are so naturally interpolated as to carry profile information. As a result, the efficiency and accuracy of quantitative analysis of moire topography are remarkably promoted.

The authors have independently developed the "scanning moire method"⁽³⁾ in which the grating image is thinned out by digital image processing in order to induce aliasing which results in the generation of a moire pattern available to strain analysis. This method is based on the principle that shifting of the spectrum of the deformed grating yields a moire pattern.

In this paper, the authors propose the strain analysis method consolidating and developing the FTP and the scanning moire method, and open the way for automatic and highly accurate moire analysis of strain without the demerits of the conventional method mentioned above.

2. Theory of Strain Analysis

Let us develop a theory of generation of moire pattern and a strain analysis method, considering an initial phase of grating lines in addition to the theory of FTP proposed by Takeda and Mutoh.

Figure 1 shows a rectangular intensity distribution of a model grating in the direction normal to the grating lines before deformation. This grating consists of alternate dark and bright stripes whose brightnesses are zero and E , respectively.

The expansion of the grating intensity function, $f(x)$, in Fourier series is

$$f(x) = \sum_{n=-\infty}^{+\infty} C_n \exp\left\{j \frac{2\pi n}{p_0} (x - x_0)\right\}, \quad (1)$$

where

$$C_0 = Eb_0/p_0,$$

$$C_n = C_0 \sin\left(\frac{n\pi b_0}{p_0}\right) / \left(\frac{n\pi b_0}{p_0}\right),$$

and j is the imaginary unit, n is an integer, p_0 is the

Fig. 1 Brightness intensity distribution of model grating along x axis

grating pitch, and b_0 is the width of the bright part of the grating. x_0 is the initial position of the grating line, expressed by $2\pi n x_0/p_0$. The ratio α of the bright line width b_0 to the pitch p_0

$$\alpha = b_0/p_0, \quad (2)$$

is called the opening ratio of the grating and is a specific property of the grating together with the grating pitch p_0 . The initial spatial frequency of the grating is defined as

$$\omega_0 = 1/p_0, \quad (3)$$

With this, Eq. (1) can be rewritten as

$$f(x) = \sum_{n=-\infty}^{+\infty} C_n \exp\{j2\pi n \omega_0 (x - x_0)\}. \quad (4)$$

Now, let us consider deformation of this grating printed on an object before deformation. When the displacement at any point X is $u(X)$, the relationship between point X after deformation and the initial position x of the same point before deformation is expressed as

$$X = x + u(X). \quad (5)$$

The grating intensity function $g(X)$ at a point X after deformation corresponds to the grating intensity $f(x)$ in the initial position x . Thus,

$$\begin{aligned} g(X) &= f(x) \\ &= f(X - u(X)) \\ &= \sum_{n=-\infty}^{+\infty} C_n \exp[j2\pi n \omega_0 \\ &\quad \times \{X - u(X) - x_0\}] \\ &= \sum_{n=-\infty}^{+\infty} C_n \exp[-j2\pi n \omega_0 \{u(X) + x_0\}] \\ &\quad \times \exp(j2\pi n \omega_0 X) \\ &= \sum_{n=-\infty}^{+\infty} i_n(X) \exp(j2\pi n \omega_0 X), \end{aligned} \quad (6)$$

where $i_n(X)$ is

$$i_n(X) = C_n \exp[-j2\pi n \omega_0 \{u(X) + x_0\}]. \quad (7)$$

The computation of the Fourier transform of the grating pattern expressed in Eq. (6) gives

$$\begin{aligned} G(\Omega) &= \int_{-\infty}^{+\infty} g(X) \exp(-j2\pi \Omega X) dX \\ &= \sum_{n=-\infty}^{+\infty} I_n(\Omega - n\omega_0), \end{aligned} \quad (8)$$

where $I_n(\Omega)$ is defined as the Fourier transform of the function $i_n(X)$. Figure 2(a) shows spectra obtained by the transform. Each spectrum corresponds to the n th harmonic. Although the spectra of $I_n(\Omega)$ have real and imaginary parts, this figure shows the power spectra schematically.

Now, from Fig. 2(a), the first harmonic is extracted from these spectra using a band-pass filter, as indicated by oblique lines. If both the real and imaginary parts of the first harmonic are shifted by ω_0 toward the origin of the frequency axis Ω , $I_1(\Omega)$ is obtained, as shown in Fig. 2(b). Computing the inverse Fourier transform of $I_1(\Omega)$, we obtain

$$i_1(X) = \int_{-\infty}^{+\infty} I_1(\Omega) \exp(j2\pi\Omega X) d\Omega \\ = C_1 \exp[-j2\pi\omega_0\{u(X) + x_0\}]. \quad (9)$$

When Eq. (3) is substituted into Eq. (9), the real part of Eq. (9) becomes

$$\operatorname{Re}\{i_1(X)\} = C_1 \cos 2\pi \frac{u(X) + x_0}{p_0}. \quad (10)$$

This function forms moiré fringes that have bright intensity peaks where displacements $u(X)$ are equal to an integer multiplied by the master grating pitch p_0 ; that is, $u(X) + x_0 = mp_0$ (m is an integer). This phenomenon corresponds to the well-known law of moiré fringe formation in the conventional moiré method: Moiré fringes generated by the superposition of master and model gratings show the contours of equal displacements. Furthermore, because the moiré pattern expressed by Eq. (10) is purely reproduced by the inversion of the first harmonic if the first harmonic does not overlap the other harmonics in Fig. 2(a), it never involves heterogeneous moiré fringes of the other harmonics, which often complicate the conventional moiré analysis.

The imaginary part of Eq. (9) shows that the phase of the moiré fringes is delayed by $\pi/2$ compared with that of the real part. It is noted that Eq. (9) consists of the real and imaginary parts of moiré fringes whose phase difference is $\pi/2$. The authors call this moiré pattern expressed by Eq. (9) the "complex moiré pattern".

(a) Fourier spectra of deformed model grating

(b) Shifting of first harmonic

Fig. 2 Frequency spectra of model grating and shifting of first harmonic

The argument of Eq. (9) or the phase of the fringe is expressed as

$$\theta(X) = -2\pi\omega_0\{u(X) + x_0\} \quad (11)$$

and contains the displacements $u(X)$.

Calculating the arctangent of the ratio of the imaginary part to the real part of the complex moiré pattern, we compute the argument as follows:

$$\theta(X) = \tan^{-1}[\operatorname{Im}\{i_1(X)\}/\operatorname{Re}\{i_1(X)\}]. \quad (12)$$

The Eulerian strain $\epsilon(X)$ at a point X is given as a derivative of the displacement $u(X)$ obtained from the argument:

$$\epsilon(X) = \frac{du(X)}{dX} \\ = -\frac{1}{2\pi\omega_0} - \frac{d\theta(X)}{dX}. \quad (13)$$

3. Application

3.1 Procedures and tips for analysis

In analysis, image processing is introduced. The procedures required in strain analysis are as follows:

- (1) Sampling of a deformed grating by a TV camera. Here, care must be taken not to induce aliasing⁽⁴⁾,
- (2) Computation of the discrete Fourier transform (DFT) of the image to obtain the spectra,
- (3) Extracting and shifting of the real and imaginary parts of the first harmonic of the spectra,
- (4) Computation of the inverse discrete Fourier transform (IDFT) of the spectrum and production of a "complex moiré pattern" composed of real and imaginary parts,
- (5) Determination of displacement contained compactly from the arguments of the complex moiré pattern,
- (6) Differentiation of the continuous displacement and determination of strain.

Needless to say, displacement is regarded as "continuous" as viewed from the pixels-constituents of image.

In computation of DFT and IDFT, the MRFFT algorithm⁽⁵⁾ is employed to emphasize process speed at the cost of freedom of the number of data.

In step (3), the principle of image processing is required to be shifted with the pixel as the unit. Moreover, the coordinate, on the frequency axis, of the initial spatial frequency of the master grating (Eq. (3)) is as large as the number of master grating lines within the region studied. Therefore, magnification of a sampling camera should be adjusted so that the number of grating lines before deformation is just an integer.

3.2 Application to a rubber plate with a hole

Let us analyze a rubber plate with a hole in the center. In computation, a minicomputer (Toshiba DS-

600) and FORTRAN language are used. The grating lines, whose pitch is 1 mm, are drawn on the specimen before deformation⁽⁶⁾. The magnification of a TV

camera is set so that the grating pitch coincides with a 4-pixel length. Figure 3(a) shows the image of the tensile loaded specimen 240×240 pixels in size near the hole. In this area, there are 60 grating lines before deformation.

For the deformed grating, one-dimensional analysis is performed along each Y directional single line, line by line, by changing its X coordinate. Figure 3(b) shows the real part of the spectra obtained by the Y directional DFT of the grating. In this figure, the intensity of brightness corresponds to the intensity of the frequency. The vertical direction shows the frequency. The origin of the frequency is centered; the upper domain is the positive higher frequency in proportion to the distance from the origin, and the lower domain is the negative higher frequency. Both the real and imaginary parts of the first harmonic, from within a range of 30 to 60 pixels from the frequency origin, are extracted and shifted by as much as the initial frequency (60 pixels) toward the origin.

(a) Original deformed grating

(b) Fourier spectra (Real part)

(d) Complex moire pattern (Real part)

(c) Shifted spectrum (Real part)

(e) Complex moire pattern (Imaginary part)

Fig. 3 Application for rubber plate with hole

The real part of the shifted spectrum is shown in Fig. 3(c).

Figures 3(d) and (e) are the real and imaginary parts of the complex moire pattern obtained by the IDFT of the shifted spectrum, respectively.

Next, arguments of the complex moire fringes are calculated. It should be mentioned that the output of the arctangent function ATAN 2 in FORTRAN language is confined to the range of $-\pi$ to π . For this reason, sudden jumping of the θ_p often occurs, as shown in Fig. 4(a). Therefore the θ_p should be connected into the smooth arguments θ_m . Because the minimum pitch of the reconstructed grating has a length of 2 pixels, the maximum nonabnormal increment of the arguments between two neighboring pixels must range from $-\pi$ to π . If the increment is out of this range, the jump should be corrected. When the jumping is detected by computing the increment between two neighboring pixels, the offset value 2π is added or subtracted automatically for smooth connection, as shown in Fig. 4(b).

Lack of the grating on the specimen surface because of the holes, background, etc., may exist. The increment of the arguments in such regions should be set at zero by automatically detecting these points. Such regions can be easily detected by introducing the threshold process since the power spectrum of the complex moire pattern is considerably weaker within these regions when the average of the brightness, i. e., the component of the frequency, is set at zero. In this analysis, the threshold is set at 2 % of the average of

(a) Band-limited computer output of argument

(b) Connected arguments

Fig. 4 Connection of arguments

the power in the whole region. In the region where the power is less than the threshold, the increment of the argument is set at zero. Since the final purpose is to obtain strain, it is not necessary to calculate the absolute displacement, but it is necessary to obtain the relative displacement.

The strains are calculated by differentiating the continuous arguments, i. e., displacements. Since the arguments fluctuate due to such factors as noise, a smoothing operation⁽⁷⁾ is employed in the differential process. The data of 10 continuous points near a point are used in the smoothing. The result of the strain distribution is shown in Fig. 5.

The calculation time from the sampling of the image to the analysis of the strain distribution is about 30 seconds.

4. Limits of Measurable Strain

4.1 Overlapping of harmonics depending on deformation

As mentioned previously, this method requires the extraction of the first harmonic from all of the harmonics of the grating. The overlappings between the first harmonic and other harmonics depend on the deformation state or the strain distribution. There is no problem in extracting the first harmonic when the strain is small or almost uniform. However, when the deformation is large or complicated, the first harmonic is overlapped by the higher harmonics and it is difficult to extract only the first harmonic. Let us consider the conditions of strain not to cause overlapping phenomenon. The local Euler strain $\epsilon(X)$ is approximated using the initial pitch p_0 and local pitch after deformation $p(X)$ as follows:

Fig. 5 Strain distribution obtained by present method (%)

$$\varepsilon(X) = \frac{du}{dX} = \frac{p(X) - p_0}{p(X)}. \quad (14)$$

The local frequency of the local pitch is defined as

$$\omega(X) = \frac{1}{p(X)}. \quad (15)$$

By substituting Eq. (15) and the initial frequency of the grating $\omega_0 = 1/p_0$ into Eq. (14), we obtain

$$\omega(X) = \omega_0 \{1 - \varepsilon(X)\}. \quad (16)$$

If the maximum strain is denoted as $\varepsilon_{\max}$, the minimum strain as $\varepsilon_{\min}$, the maximum grating frequency as $\omega_{\max}$ and the minimum grating frequency as $\omega_{\min}$, the following relation can be derived among them in consideration of the correspondences between $\varepsilon_{\max}$ and $\omega_{\min}$ and between $\varepsilon_{\min}$ and $\omega_{\max}$:

$$\omega_{\max} = \omega_0 \{1 - \varepsilon_{\min}\} \quad (17)$$

$$\omega_{\min} = \omega_0 \{1 - \varepsilon_{\max}\}. \quad (18)$$

Because the strains on the specimen surface range from $\varepsilon_{\min}$ to $\varepsilon_{\max}$, the corresponding first harmonic of the grating ranges from $\omega_{\max}$ to $\omega_{\min}$ on the frequency axis. In general, the distribution range of the harmonic of the order n is n times as wide as that of the first harmonic. That is, it ranges from $n\omega_{\max}$ to $n\omega_{\min}$, as shown in Fig. 6. It is evident that the overlapping phenomenon is initiated by the encounter between the highest frequency component of the first harmonic and the lowest frequency component of another harmonic of the order n . Therefore, in order to avoid overlapping of the harmonics, the following condition must be satisfied:

$$n\omega_{\min} > \omega_{\max}. \quad (19)$$

There are two cases to be considered in the limit of strain:

(1) Overlapping between the first harmonic and the second harmonic,

(2) Overlapping between the first harmonic and the third harmonic.

It depends on the opening ratio α , defined as Eq. (2), which of the two should be taken into account. In

case $\alpha = 1/2$, the coefficient C_2 of the second harmonic is theoretically zero, and all the second harmonic components vanish; then case (2) presents itself. In practical use, case (1) often occurs, because the opening ratio α is uneven throughout the specimen surface, and C_2 cannot be zero. We can neglect other harmonics over the fourth degree, because the overlapping phenomena do not appear until case (1) or case (2) occurs.

Thus, for $n=2$, the next relation is obtained by substituting Eqs. (17) and (18) into Eq. (19):

$$2\omega_0 \{1 - \varepsilon_{\max}\} > \omega_0 \{1 - \varepsilon_{\min}\}; \quad (20)$$

that is,

$$\varepsilon_{\min} > 2\varepsilon_{\max} - 1. \quad (21)$$

For $n=3$,

$$3\omega_0 \{1 - \varepsilon_{\max}\} > \omega_0 \{1 - \varepsilon_{\min}\}, \quad (22)$$

$$\varepsilon_{\min} > 3\varepsilon_{\max} - 2.$$

If the strain conditions Eqs. (21) or (22) are satisfied, the overlapping does not occur and the first harmonic can be extracted from all of the harmonics.

4.2 Sensitivity and limits of strain due to image processor

In the proposed method, the number of pixels used for sampling the image is limited by the capacity of a system. In our system, the available maximum image size is a square of 512×480 pixels. The sensitivity and the measurable range of the strain in this method depend on the resolution of the system, or the maximum amount of grating information that can be sampled by the system.

When h (pixels) is the image size in the X direction, the range of the grating pitch that can be expressed by pixels is clearly

$$2 \leq p_0 \text{ and } p(X) \leq h-1. \quad (23)$$

The maximum tensile strain that can be measured comes into existence if the initial grating pitch of the 2-pixel length is deformed into that of $h-1$ pixel length. Thus,

$$\varepsilon_{\text{ten}} = 1 - \frac{2}{h-1} \approx 1 \quad (\text{when } h \gg 1). \quad (24)$$

Likewise, the maximum compressive strain that can be measured occurs when the initial grating pitch of the $h-1$ pixel length is deformed into that of the 2-pixel length. Then,

$$\varepsilon_{\text{comp}} = 1 - \frac{h-1}{2} \approx -\frac{h}{2}. \quad (25)$$

Practically speaking, the range is possibly narrowed down because the number of grating lines is less than two when the pitch is $h-1$. Figure 7 shows the theoretical measurable strain range which takes Eqs. (21), (22), (24) and (25) into consideration.

Sensitivity is defined as the smallest strain that can be detected. The strain expressed in Eq. (14) can be minimized when $p(X) = h$ and $p_0 = h-1$; then the

Fig. 6 Overlapping of harmonics

Fig. 7 Limits of measurable strain

sensitivity is given as

$$\varepsilon_{fine} = \frac{1}{h}. \quad (26)$$

5. Conclusions

In-plane strain analysis has been fully automated based on our theory and by virtue of digital image processing. Fringe center tracing is no longer necessary, nor is fringe ordering, since displacement information is contained in, and extracted out of, the arguments of the complex moire pattern. Also, exclusive access to the first harmonic, the single source of the moire patterns of displacement contour free from other higher harmonics causing noises and heterogeneous fringes, ensures highly accurate solution of strain distribution.

What is better, since the analysis algorithm is composed of a series of objective steps, consistent strain distribution, independent of human error, can be obtained.

Present situations only allow us a large strain

measurement due to the small number of technically available pixels of a TV and/or image processing system. However, when the number of available pixels increases in the near future, it will be possible to measure a smaller strain.

In conclusion, it might be mentioned that this approach provides a way to rapidly and accurately evaluate strain in a plane.

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