Dynamic Line-Spring Model for Impact Response of a 3-D Surface Crack*

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A dynamic line-spring model is proposed for the simplified analysis of impact response of a part-through surface crack in an elastic plate. Bending of the plate is considered based on Mindlin's theory. Stiffness of the dynamic line-spring is simply evaluated following the static model of Rice and Levy. The procedure for evaluating the dynamic stress-intensity factor K_{I_d} is developed. The response of a semielliptical surface crack in a plate subjected to impact loading at its ends is analysed by using the present model. It is found that K_{I_d} takes its maximum value at the deepest penetration point of the surface crack and the maximum value is much larger than that in the static case.

Key Words: Fracture Mechanics, Computational Mechanics, Stress Intensity Factor, 3-D Surface Crack, Dynamic Line-Spring Model, Finite-Element Method, Impact Load, Mindlin's Theory for Plate Bending

1. Introduction

It is well-known from the solutions for 2-D crack problems⁽¹⁾ that the stress intensity factor due to dynamic loading takes a higher value than that in a static case. The stress intensity factor for dynamic problems, the dynamic stress-intensity factor, is denoted by K_{I_d} . Many cracks in structures have 3-D configurations as part-through surface cracks. Hence the integrity assessment, which considers dynamic loading caused by a water hammer or earthquake, for example, requires the analysis of K_{I_d} for those 3-D cracks.

Chen and Wilkins⁽²⁾ analysed K_{I_d} of a 3-D surface crack. They used the finite difference method. However the direct application of 3-D finite difference or 3-D finite-element methods is not necessarily suitable

for practical analyses of dynamic 3-D crack problems, since they need a large amount of numerical computation.

Saka et al.⁽³⁾ proposed a dynamic line-spring model as a simplified method for analysis of 3-D dynamic crack problems. A 3-D dynamic crack problem has been modeled by a reduced 2-D problem. The model is an extension from the static model by Rice and Levy⁽⁴⁾ to cover dynamic problems. The problem treated, however, was a fundamental one in which an elastic plate containing two coplanar 3-D surface cracks is subjected to vibrating load at its ends. The dynamic behavior of a 3-D surface crack under vibrating loads was recently analysed by Hua and Abé⁽⁵⁾, with the consideration of plate bending.

Impact load is another fundamental type of dynamic loading. The present paper is concerned with the dynamic line-spring model for impact response of a part-through surface crack in an elastic plate. Bending of the plate is considered based on Mindlin's theory⁽⁶⁾. The dynamic stress-intensity factor K_{I_d} is evaluated for a semielliptical surface crack in a plate subjected to impact loading at its ends.

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2. Dynamic Line-Spring Model

2.1 Modeling

Figure 1 shows a plate containing a 3-D surface crack. The plate thickness is d , the width w and the length l . The cartesian coordinate system (x, y, z) is chosen such that the origin is located at the center of the plate and the surface crack is on the plane $y=0$. The surface length of the crack is $2c$ and the crack depth is denoted by $b(x)$ for $-c \leq x \leq c$. The maximum crack depth is b_0 . The plate is subjected to a dynamic tensile load, $N(t)$, and a bending moment, $M(t)$, per unit length in the x direction at $y = \pm l/2$, where t denotes time.

By introducing the dynamic line-spring model, the problem shown in Fig. 1 is transformed into a 2-D problem of simultaneous plane stress and plate bending for a thin plate in the x - y plane; see Fig. 2. The plate contains a crack of length $2c$, whose surfaces are connected by dynamic line-springs having both stretching and bending resistances. All of the springs are considered to have no length in the undeformed configuration and no mass. Figure 3 shows the deformed line-spring, where δ_s is the displacement

(a) geometry of the plate

(b) schematic illustration of the surface crack

Fig. 1 An elastic plate containing a 3-D surface crack

and θ_s the rotation of one end of the spring relative to the other. The tensile load and moment acting on the spring per unit length in the x direction are denoted by N_s and M_s , respectively, and they are taken to be linear functions of δ_s and θ_s as follows:

$$\begin{cases} N_s(x, t) = k_{11}\delta_s(x, t) + k_{12}\theta_s(x, t) \\ M_s(x, t) = k_{21}\delta_s(x, t) + k_{22}\theta_s(x, t) \end{cases} \quad (1)$$

where k_{ij} represents the stiffness of the spring. The stiffness k_{ij} , which depends on the crack depth $b(x)$, is assumed to be simply evaluated following the result for static load conditions given by Rice and Levy⁽⁴⁾. It is noted that the model by Rice and Levy is already confirmed⁽⁴⁾⁽⁷⁾⁻⁽¹⁰⁾ to have a good accuracy in comparison with direct 3-D computation or experiment for static problems.

2.2 Evaluation of dynamic stress-intensity factor

Consider an edge-cracked strip under the condition of plane strain, having width d and crack depth $b(x)$. Let the stress intensity factor of the edge crack

Fig. 2 Dynamic line-spring model

Fig. 3 Deformed line-spring

for static tension of N per unit thickness be denoted by $K_{I,ten}$, and the factor for static bending of M per unit thickness be $K_{I,ben}$ as shown in Fig. 4. The dynamic stress-intensity factor $K_{I,d}(x, t)$ of the 3-D surface crack at time t and position x can be evaluated by

$$K_{I,d}(x, t) = \frac{K_{I,ten}}{N} N_s(x, t) + \frac{K_{I,ben}}{M} M_s(x, t). \quad (2)$$

In the case of the static problem, Eq. (2) reduces to the model of Rice and Levy.

The impact loads $N(t)$ and $M(t)$ are assumed to be

$$\left. \begin{aligned} N(t) &= N_0 \{1 - \exp(-t/T_c)\} \\ M(t) &= M_0 \{1 - \exp(-t/T_c)\} \end{aligned} \right\} \quad (3)$$

where N_0 and M_0 are the limiting values of $N(t)$ and $M(t)$ at $t \rightarrow \infty$. The intensity of the impact is characterized by a parameter T_c . Figure 5 shows the $N(t)$

Fig. 4 An edge-cracked strip

Fig. 5 Impact tensile load $N(t)$ and bending moment $M(t)$

$-t$ and $M(t)-t$ relations, where T is the time-of-flight of the elastic wave to propagate the distance $l/2$. The time T is given by using Young's modulus E , Poisson's ratio ν and density ρ as

$$T = \frac{l}{2} \sqrt{\frac{E}{1-\nu^2} \frac{1}{\rho}}. \quad (4)$$

3. Verification of the Dynamic Line-Spring Model

Hua and Abé⁽⁵⁾ have shown the validity of using the stiffness of the static model for the plate subjected to vibrating loads. The validity of the dynamic line-spring model for impact load, however, remains to be proven.

Let us consider the limiting case of $c, w \rightarrow \infty$ for the 3-D cracked plate shown in Fig. 1. Figure 6 shows the edge-cracked strip under plane strain obtained by the limiting procedure and its dynamic line-spring model.

In order to verify the dynamic line-spring model, $K_{I,d}$ obtained from the analysis of the model (Fig. 6 (b)) is compared with that calculated directly based on 2-D finite-element analysis of Fig. 6(a). The values of E , ν , and ρ are taken to be 210 GPa, 0.3, and 7.85×10^3 kg/m³ respectively. Figure 7 shows the comparison calculated for the case of $T_c/T = 0.5$, $l/d = 8$, and $b_0/d = 0.3$, where $K_{I,d}$ is normalized by the stress intensity factor K_I for static loads of N_0 (Fig. 7(a)) and M_0 (Fig. 7(b)). The result of the dynamic line-spring model (LSM) coincides with that of 2-D finite-element analysis (FEM). It is also found for several combinations of T_c/T ($=0.2, 0.5, 1.0$), l/d ($=8, 40$), and b_0/d ($=0.1, 0.3$) that the results of LSM and FEM coincide with each other for the cases of $T_c/T \geq 0.5$,

Fig. 6 An edge-cracked strip under plane strain

independently of the crack depth and aspect ratio of the cracked plate. In the case of $l/d=40$, the dynamic line-spring model has yielded good results even for the strong impact load of $T_c/T=0.2$. Thus the dynamic line-spring model can evaluate K_{Id} accurately for edge-cracked strips under plane strain.

4. Application Example

The impact behavior of a 3-D surface cracked plate is analysed for tension load, $N(t)$, by applying the present model. The dimensions of the plate are taken to be $l/d=40$, $w/d=40$, $b_0/d=0.3$, and $b_0/c=0.2$. Figures 8 and 9 are the results obtained for the cases of $T_c/T=1.0$ and 0.5 respectively. The variation of K_{Id} along the crack front is shown in Figs. 8(a) and 9(a) with the normalization of K_{Id} by $\sigma_0\sqrt{\pi b_0}$, where σ_0 is N_0/d . It is found that K_{Id} takes its maximum value at $x/c=0$, the deepest penetration point of the surface crack. The transient response of $K_{Id}(x=0)$ is shown in Figs. 8(b) and 9(b). The maximum value of K_{Id} just obtained is compared with that of the stress intensity factor $K_I^{(11)}$ obtained for static tensile stress of σ_0

acting at the ends of the 3-D cracked plate. The comparison is shown in Table 1. The maximum value of K_{Id} is found to increase up to about twice that of K_I .

5. Conclusions

The dynamic line-spring model is proposed for the simplified analysis of the impact response of a 3-D surface crack, where the plate bending is considered. Since the present model simply evaluates the stiffness of the dynamic line-spring following the static model by Rice and Levy, it can easily be used for the analysis of dynamic problems. By considering limiting cases of the 3-D dynamic crack problem, it is

Table 1 Comparison of the maximum values of K_{Id} and K_I for 3-D surface crack

Static	Dynamic		Maximum of K_{Id} Maximum of K_I
Maximum of $K_I/\sigma_0\sqrt{\pi b_0}$	T_c/T	Maximum of $K_{Id}/\sigma_0\sqrt{\pi b_0}$	
1.179 ⁽¹¹⁾	1.0	2.023	1.72
	0.5	2.399	2.03

(a) impact tensile problem

(b) impact bending problem

Fig. 7 Comparison of normalized K_{Id} calculated based on the dynamic line-spring model (LSM) with that obtained from 2-D finite-element analysis (FEM) of an edge-cracked strip under plane strain

(a) variation of K_{Id} along the crack front

(b) the transient response of $K_{Id}(x=0)$

Fig. 8 Dynamic behavior of 3-D surface crack for impact tensile load of $T_c/T=1.0$

(a) variation of K_{Ia} along the crack front (b) the transient response of $K_{Ia}(x=0)$ Fig. 9 Dynamic behavior of 3-D surface crack for impact tensile load of $T_c/T=0.5$

found that the present model is valid as a simplified analysis method. An application example is shown by analysing the response of a semielliptical surface crack in a plate subjected to impact tensile load at its ends. It is found that K_{Ia} takes its maximum value at the deepest penetration point of the surface crack and the maximum value is much larger than that in the static case.

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