Analysis of the HRR Stress Field at the Three-Dimensional Crack TIP* (Analysis of CT and CCT Specimens)

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CT specimens with different thicknesses and a CCT specimen are analyzed precisely using the three-dimensional finite element method in the elastic-plastic stress states. The stress and displacement fields at the crack tip are compared with the HRR singular stress field and discussed. It is found that in the CT specimens, the thickness of them are greater than those recommended by the standard of fracture toughness testing, the stress and displacement fields agree very well with those of HRR fields. For the CT specimen, the thickness of which is a little less than the recommended value, the HRR field exists only at the center of the specimen. It is shown that there exist no HRR fields in the CCT specimen though the thickness is not small.

Key Words: Fracture, CT Specimen, CCT Specimen, HRR Field, Three-Dimensional Analysis

1. Introduction

Due to the increase in the importance of elastic-plastic fracture mechanics, several fracture parameters have been proposed, i.e., CTOD, CTOA, J integral and dJ/da . Among them, the J integral concept is considered to be one of the most important parameters, because it is a unique parameter which can describe the displacement-strain-stress fields at the crack tip. It was proposed theoretically by Hutchinson⁽¹⁾ and Rice and Rosengren⁽²⁾ at the same time and is called the HRR singular field. Many numerical analyses have verified the existence of the HRR fields at the crack tip. The HRR field concept is also important to predict stable crack growth. One parameter called the Tearing Modulus is proposed by Paris et al.⁽³⁾; which is used as a criterion of stable crack growth. The Tearing Modulus criterion assumes J dominant crack growth, which means that the crack growth

occurs in the region where the HRR fields exist. Crack growth simulations have been carried out using this parameter by the finite element analysis⁽⁴⁾.

As mentioned above, though the existence of the HRR field and its availability are well known now, almost all of the research is restricted to two-dimensional problems. Real cracks have three dimensional configuration, and it is important to ascertain the existence of HRR fields in the three dimensional crack tip region. Some experimental results by authors⁽⁵⁾ suggest that the crack growth of surface cracks occurs without the existence of HRR fields, and numerical analysis by Parks and Wang⁽⁶⁾ shows similar results.

To ascertain the existence of HRR fields at the crack tip, finite element analysis using a fine mesh pattern is needed. Three dimensional problems require much CPU time and costly to solve; this has been one important problem. Recently supercomputers have come to be used widely, and CPU time has decreased significantly.

In this paper, several kinds of CT specimens and a CCT specimen are analyzed precisely, and the stress and displacement fields are compared with HRR solu-

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tions. Brocks and Olszewski⁽⁷⁾ have already analyzed a similar problem, but they used a very thick CT specimen, the thickness of which was 200 mm. In this study, the thickness of CT specimens is decreased, and the effect of specimen thickness on singular fields is discussed.

2. Models for Numerical Analysis

Figure 1 shows the dimensions and the mesh pattern of a CT specimen used in the analyses. Only one-quarter of the specimen is shown. The total number of elements is 396, and number of joints is 2 155. In the x - y plane, the number of elements is 66 and the number of joints is 235. By the two-dimensional elastic-plastic analysis using the same mesh pattern, it is shown that the stress field at the crack tip coincides well with the HRR solution in both plane strain and plane stress states. Along the thickness direction, the specimen is divided into 6 layers.

The size of the CT specimen is the same as that of the 1 inch CT specimen, except for the thickness. The thickness varies among 1 inch, 1/2 inch, 1/4 inch, and 1/8 inch. By the standards of the fracture toughness test proposed by ASTM⁽⁸⁾ and JSME⁽⁹⁾, the thickness of the specimen should satisfy the following condition.

$$B \geq 25 \frac{J_{IC}}{\sigma_{fs}} \quad (1)$$

The material is taken to be A 533 B steel with σ_{fs} of 550 MPa, and J_{IC} of 115.0 kN/m. Then the minimum thickness of the specimen is 5.2 mm. The thickness of

Fig. 1 Mesh pattern of CT specimen

the 1/8-inch specimen is 3.1 mm and does not satisfy the thickness condition.

As it is well known that cracks grow faster in the center of the specimen than near the free surface by fatigue precracking, the shapes of four types of specimens are measured. Figure 2 shows the result. The curvature of the crack front becomes steep as the thickness decreases. Their shapes are used in the finite element modelling.

The stress-strain relationship of A 533 B steel is expressed as follows:

$$\begin{aligned} \sigma &\leq \sigma_0; \epsilon = \sigma/E \\ \sigma &\geq \sigma_0; \epsilon = \frac{\sigma}{E} + \left\{ \left(\frac{\sigma}{E'} \right)^n + \left(\frac{\sigma_0}{E'} \right)^n \right\}^{1/n} \end{aligned} \quad (2)$$

$n=3 \quad E=206 \text{ GPa} \quad \nu=0.3$
 $\sigma_0=0.549 \text{ GPa} \quad E'=1.609 \text{ GPa}$

Equation (2) is approximated by the Ramberg-Osgood type equation as follows:

$$\frac{\epsilon}{\epsilon_0} = \frac{\sigma}{\sigma_0} + \alpha \left(\frac{\sigma}{\sigma_0} \right)^n \quad (3)$$

where n is equal to 4.0, and α is 6.73. Equations (2) and (3) are compared to each other in Fig. 3. It is shown that they agree well for a large strain range. The material and shape of the CCT specimen are the same as those of the one tested by Machida et al., and are shown in ref. 10. The thickness of the CCT speci-

Fig. 2 Shapes of fatigue precracks

Fig. 3 Ramberg-Osgood type approximation of stress-strain relationship

men is 18 mm, which is larger than the 1/2-inch thick CT specimen. n and α values of the CCT specimen are 4.0 and 9.5, respectively.

The elastic-plastic FEM analyses are carried out until the J value exceeds the J_{IC} value.

3. Results and Discussions

3.1 Distributions of J values along the crack front

Figures 4(a)~(e) show the changes of distributions of J values along the crack front for every specimen, where the ordinate is normalized by the J value at the center of the specimen. The deformation is denoted by the normalized value, $L/(J/\sigma_0)$, where L is the ligament length, and J is the value at the center of the specimen. In every specimen, the J value at the free surface is the largest at the beginning of deformation, but as the deformation proceeds, it relatively decreases to the J value at the center of the specimen. In the 1-inch thick specimen, the area where J is nearly constant exists at the inner part of the specimen. As the thickness decreases, the width of this area

also decreases and vanishes when the thickness is 1/4 inch, which relates with the decrease of the region of the plane strain stress state. The distribution pattern of the J value of the CCT specimen is similar to that of the 1/2-inch thick specimen except very near the free surface.

3.2 Comparison with HRR solution

The stress and displacement fields at the crack tip are expressed as follows by the HRR solution:

$$\sigma_{ij} = \sigma_0 \left(\frac{J}{\alpha \sigma_0 \epsilon_0 I_n r} \right)^{1/(n+1)} \bar{\sigma}_{ij}(\theta, n) \quad (4)$$

$$u_i = \alpha \epsilon_0 r \left(\frac{J}{\alpha \sigma_0 \epsilon_0 I_n r} \right)^{n/(n+1)} \bar{u}_i(\theta, n) \quad (5)$$

where n and α are constants determined by Eq. (3), I_n is a constant determined by the n value, and $\bar{u}_i(n, \theta)$ and $\bar{\sigma}_{ij}(n, \theta)$ are nondimensional parameters determined by n and angle θ , which is defined by the polar coordinate at the crack tip. In a similar manner, the relationship between CTOD and J is determined by the following equation based on the HRR solution.

$$\text{CTOD} = d_n(\alpha \epsilon_0, n) \frac{J}{\sigma_0} \quad (6)$$

Fig. 4 J distributions along the thickness direction

where

$$d_n(\alpha\epsilon_0, n) = 2(\alpha\epsilon_0)^{1/n} \{ \tilde{u}_x(\pi) + \tilde{u}_y(\pi) \}^{1/n} \frac{\tilde{u}_y(\pi)}{I_n} \quad (7)$$

Figures 5(a)~(c) show the stress fields for three types of specimens with HRR solutions. They are the results at the centers of the specimens. In the 1-inch-thick CT specimen, the stress field coincides well with the plane strain HRR solution from the beginning of the deformation until the J value becomes the J_{IC} value. In the 1/8-inch-thick specimen, though the stress field at the beginning of the deformation agrees well with the plane strain HRR solution, it deviates from it due to the increase of deformation

(a) 1 CT

(b) 1/8 CT

(c) CCT

Fig. 5 Stress distributions at the crack tip ($2Z/B=0.15$)

and comes nearer to the plane stress HRR solution. In the CCT specimen, the stress field does not agree with the HRR solution except at the beginning of the deformation. McMeeking and Tvergaard⁽¹¹⁾ concluded by their two-dimensional analysis that $L/(J/\sigma_0)$ should be larger than 200 for the existence of a HRR field in the CCT specimen, and Aoki et al.⁽¹²⁾ say that it should be larger than 300~500. By the three dimensional analysis, it seems that this value should be larger than those values determined by the two dimensional analyses.

Figures 6(a)~(e) show the stress fields along the crack front for every specimen. They are the results when the J value at the center of the specimen becomes nearly equal to J_{IC} . The results of 1-inch- and 1/2-inch-thick specimens are nearly the same, and show that a plane strain HRR field exists at the center of the specimens. In the 1/4-inch-thick specimen, though the stress field at the center of the specimen is a little smaller than the plane strain HRR solution, the difference between them is small. But in the 1/8-inch-thick specimen, the difference of the stress field from the plane strain HRR solution is large. As only this type of specimen does not satisfy the fracture toughness testing standard, these results show that the standards of ASME and JSME are valid. In the CCT specimen, as shown in Fig. 5(c), the stress field deviates largely from the HRR solution when the J value is nearly J_{IC} . Machida et al.⁽¹⁰⁾ measured the fracture toughness of this specimen and obtained a J_{in} value which was 20% smaller than the J_{IC} value obtained from the standard CT specimen. The decrease of the J_{in} value is due to the deviation of the stress field from the HRR solution shown in this figure.

Figures 7(a) and (b) show the comparison of displacement fields with HRR solutions. The ordinate is denoted by the following nondimensional parameter.

$$\tilde{u}_i(\theta, n) \cdot \left(\frac{\alpha\epsilon_0}{I_n} \right)^{1/(n+1)} = \frac{u_i}{r} \left(\frac{r}{J/\sigma_0} \right)^{n/(n+1)} \quad (8)$$

The solid lines in Fig. 7 mean both plane strain and plane stress HRR solutions. In the center of the specimen (Fig. 7(a)), the results of every specimen agree with each other and coincide with the plane strain HRR solution. Though the effect of the specimen thickness is recognized, they are rather small. At the free surface (Fig. 7(b)), the numerical results differ from each other. As the thickness decreases, the results deviate greatly from the plane strain HRR solution, but do not come nearer to the plane stress HRR solution. For $\theta=120^\circ$, the result of the 1/8-inch-thick specimen is greater than either the plane strain or plane stress solution. This result agrees well with the experimental results of Yagawa et al.⁽¹³⁾.

(a) 1 CT ($L/(J/\sigma_0)=102$) (c) 1/4 CT ($L/(J/\sigma_0)=107$) (b) 1/2 CT ($L/(J/\sigma_0)=107$) (d) 1/8 CT ($L/(J/\sigma_0)=114$) (e) CCT ($L/(J/\sigma_0)=57$)

Fig. 6 Change of stress fields along the thickness direction

(a) At the center of the specimen ($2Z/B=0.15$) (b) Near the free surface ($2Z/B=0.98$)

Fig. 7 Displacement fields at the crack tip

(a) At the center of the specimen ($2Z/B=0.15$) (b) Near the free surface ($2Z/B=0.98$)

(c) CCT specimen

Fig. 8 Relationship between CTOD and J

Figures 8(a)~(c) show the relationship between CTOD and J . CTOD is defined as the opening displacement between two joints nearest the crack tip in the FEM model. Figure 8(a) shows the results at the centers of four types of CT specimens. In every specimen, the numerical results agree well with the plane strain HRR solution. As shown in Fig. 6, the stress field of the 1/8-inch-thick specimen does not agree with the plane strain HRR solution, but the displacement fields, shown in Fig. 7(a) and Fig. 8(a), agree well with it. These results mean that at the center of the specimen, there may exist a HRR field even for the

1/8-inch-thick specimen. Ootsuka⁽¹⁴⁾ measured the fracture toughness value using the CT specimen which does not satisfy the condition of Eq. (1), and obtained a valid J_{IC} value by measuring the crack growth length only at the center of the specimen. Ootsuka's result may be explained by these numerical analyses. Figure 8(b) shows the results of CT specimens at the free surface. For 1/4- and 1/8- inch thick specimens, the results are greater than those of the plane stress HRR solution. Figure 8(c) is the result of the CCT specimen. Both at the center of the specimen and at the free surface, the relationship between CTOD and J is different from that of the plane stress HRR solution. As the deformation proceeds, this difference becomes larger. It may be concluded that for the CCT specimen analyzed in this study, both stress and displacement fields are different from the HRR solution and there is no HRR region in this specimen.

4. Conclusions

In this paper, standard specimens, CT and CCT, are numerically analyzed by three dimensional FEM, and the stress and displacement fields are compared with HRR solutions.

(1) In CT specimens which satisfy the thickness condition, Eq. (1), the plane strain HRR field exists at the center of the specimen from the beginning of the deformation until the J value becomes the J_{IC} value.

(2) In the 1/8-inch-thick CT specimen, which does not satisfy Eq. (1), there may exist a plane strain HRR region at the center of the specimen.

(3) In the CCT specimen analyzed here, there is no stress or displacement field which agrees with the plane strain HRR solution.

These conclusions are not new; for example, (1) and (3) have already been cited by others based on two dimensional analyses. But from the viewpoint of the three dimensional HRR field, many problems remain for future study. One is to study the stress field of a growing crack, and another is the stress field of surface cracks. It was shown through this paper that the HRR field is affected by the specimen shape, dimensions, and loading conditions. Parametric studies may be needed in the future.

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