A Study of Residual Stress for Opening and Closing Behavior of Fatigue Cracks*

Yoshitaka NATUME** and Susumu MIYAKAWA**

The distribution of residual stress in the vicinity of fatigue crack tips and behind them for S15C and SK-5 were measured by X-ray microbeam equipment. Fatigue cracks were loaded and unloaded and then the behaviors of opening and closing were examined analytically. The results obtained are summarized as follows. (1) A monotonic plastic zone ahead of the crack tip and a cyclic plastic zone was found in the crack tip vicinity. (2) The ratios of fatigue crack opening and closing found by X-ray measurement were in good agreement with the value found by the microdisplacement method.

**Key Words:** Fracture Mechanics, Fatigue, Residual Stress, Crack Closing, Crack Opening

1. Introduction

Recent years have seen discussions on the strength of materials and structures with defects or sharp notches from the point of view of fracture mechanics. They have pointed out the importance of stress and strain behavior in the vicinity of the fatigue crack tip generated by the defect or sharp notch(1).

That the stress intensity factor and fatigue crack propagation behavior, which govern the location of local stress in the vicinity of the fatigue crack tip, are intimately related has been reported on numerous occasions(2), and the effects of the area of residual stress on the phenomenon of the crack opening and closing during fatigue crack propagation have also been discussed. Elber(3), for instance, showed experimentally that the fatigue crack tip sometimes closed during propagation even under tensile stress and suggested the application of an effective stress intensity factor of $\Delta K_{ef}$ to the fatigue crack propagation rate analysis by pointing out the importance of the residual stress formed during fatigue crack propagation. Budiansky and Hutchinson(4) analyzed fatigue crack opening and closing behavior under plane stress using the Dugdale model and observed the difference between the ideal crack and the fatigue crack. Nakamura et al.(5) analyzed the opening and closing behavior of the crack via the finite-element method and made additional observations regarding the difference between the ideal crack and the fatigue crack by comparing their results with those of the Budiansky and Hutchinson study. Honda et al.(6) reported that it was possible to calculate the maximum stress intensity factor $K_{max}$ and the stress intensity factor $K_{ef}$ during the opening of the fatigue crack by nondestructively measuring residual stress distribution using X-ray equipment, and, thus, it is possible to estimate the fatigue crack propagation rate using the effective stress intensity factor $\Delta K_{ef}$.

As has been discussed above, major factors in the observation of fatigue crack propagation behavior are the changes in residual stress distribution in the vicinity of the fatigue crack tip accompanying fatigue crack propagation, the relationships between residual stress distribution and the plastic zone, and between residual stress distribution and crack opening-closing behavior.

While the plastic deformation zone of the fatigue crack tip has been examined using the recrystallization and etch pit methods(7), much is expected of the
use of X-ray microbeams, a nondestructive method of measuring stress, for examining the relationship between the applied stress and the stress in the vicinity of the crack tip, and in the observation of fatigue fracture mechanisms.

The authors have developed a device for the measurement of stress using microrange microbeam equipment which makes possible the measurement of stress in a microrange of an X-ray irradiation area of a diameter of 80 \( \mu \text{m} \), proving its effectiveness\(^{(8)} \) and applying it in the analysis of microrange residual stress on the fatigue surface accompanying the generation of a fisheye. It has been shown that this method is effective in the investigation of stress at the tip of a fatigue crack and in determining the plastic zone\(^{(6)} \).

This paper provides additional discussion on the subject of fatigue crack opening and closing behavior through microdisplacement. Measurements were made of X-ray measurement stress distribution in the vicinity of the fatigue crack tip in the loading and unloading processes and of residual stress distribution under no load using an X-ray stress measurement device, and examinations were made of relationships between them and the plastic range formed at the tip of the fatigue crack and fatigue crack opening and closing behavior.

2. Materials and Methods

2.1 Sample materials and test pieces

The test materials consisted of S15C carbon steel used in mechanical structures and SK-5 carbon tool steel, the chemical composition and postheattreatment mechanical properties of which are given in Tables 1 and 2.

Normalized plates of 10 mm in thickness were obtained. The S15C plates were not heat treated but normalized, and the SK-5 plates were quenched at 820 \( ^\circ \text{C} \), then allowed to stand for 1 hour at 250\( ^\circ \text{C} \) and tempered by being allowed to cool, after which they were machined to produce the test pieces shown in Figs. 1 (a) and (b). After finishing with emery paper, the plates were polished electrolytically using supersaturated phosphate chronic acid solution to remove 50\( \mu \text{m} \) from the surface. Figure 1 (a) shows a test piece used for the evaluation of mechanical properties, while Fig. 1 (b) is a 1/2 CT test piece which a load could be applied on the fatigue crack tip using M4 screws. The direction in which the test material is taken is L-T. The microstructure of these test pieces is indicated in Figs. 2 (a) and (b). Figure 2 (a) is an S15C plate after normalization, and gives the 10-30 \( \mu \text{m} \) ferrite grain diameter structure, while Fig. 2 (b) shows a quenched and tempered SK-5 sorbite structure.

2.2 Fatigue crack propagation testing and measuring residual stress

Fatigue testing was accomplished utilizing an electrohydraulic servo-type fatigue testing machine (Shimazu Manufacturing Lab-5 U) under a pulsating load with a stress ratio of \( R = 0.05 \). The repeated load wave was sinusoidal with a repetition frequency of 10 Hz. In order to ensure that fatigue crack propagation testing conformed to ASTM E647-81\(^{(10)} \), the fatigue

<table>
<thead>
<tr>
<th>Materials</th>
<th>C</th>
<th>Si</th>
<th>Mn</th>
<th>P</th>
<th>S</th>
</tr>
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<tbody>
<tr>
<td>S15C</td>
<td>0.17</td>
<td>0.25</td>
<td>0.46</td>
<td>0.020</td>
<td>0.012</td>
</tr>
<tr>
<td>SK-5</td>
<td>0.88</td>
<td>0.27</td>
<td>0.49</td>
<td>0.014</td>
<td>0.004</td>
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<table>
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<tr>
<th>Materials</th>
<th>Yield strength (MPa)</th>
<th>Tensile strength (MPa)</th>
<th>Elongation (%)</th>
<th>Reduction of area (%)</th>
<th>Microvickers hardness (HV)</th>
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<tbody>
<tr>
<td>S15C</td>
<td>380</td>
<td>540</td>
<td>24.5</td>
<td>65</td>
<td>140</td>
</tr>
<tr>
<td>SK-5</td>
<td>1020</td>
<td>1250</td>
<td>11.8</td>
<td>17</td>
<td>590</td>
</tr>
</tbody>
</table>

Fig. 1 Dimensions of specimen
crack length was measured using an optical microscope at ×100 magnification, the fatigue crack propagation rate was calculated using the secant method, and the stress intensity factor range \( \Delta K \) was calculated using the formula derived by Srawley.\(^{11}\)

Residual stress in the direction of the load in the vicinity of the fatigue crack tip was measured using microrange microbeam equipment, with measurements made under the conditions given in Table 3. The diffraction plane was CoKα (200), and the diameter of the surface irradiated with X-ray beams was 80 \( \mu m \) in the case of SK-5 and 160 \( \mu m \) in the case of S15C. The center of the area irradiated by X-rays coincided with the fatigue crack tip as the original point. In order to assess the residual stress surface distribution in the vicinity of the fatigue crack tip, measurements were made while moving the center of the area irradiated by X-rays away from the original point in the direction of fatigue crack propagation (X axis) and perpendicularly to that (Y axis) at unspecified intervals, as shown in Fig. 3. Minimum intervals for SK-5 were 0.05 mm, and for S15C, 0.1 mm. Residual stress after electrolytic polishing prior to the fatigue testing was, for SK-5, −3.9 MPa, and −0.2 MPa for S15C.

### Table 3 Conditions of the X-ray diffraction technique

<table>
<thead>
<tr>
<th>Conditions</th>
<th>SK-5</th>
<th>S15C</th>
</tr>
</thead>
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<tr>
<td>Target</td>
<td>Co</td>
<td></td>
</tr>
<tr>
<td>Tube voltage</td>
<td>40kV</td>
<td></td>
</tr>
<tr>
<td>Tube current</td>
<td>200mA</td>
<td></td>
</tr>
<tr>
<td>Projection area</td>
<td>80 ( \mu m )</td>
<td>160 ( \mu m )</td>
</tr>
<tr>
<td>Diffraction plane</td>
<td>(200)</td>
<td></td>
</tr>
</tbody>
</table>

2.3 Fatigue crack opening and closing measurement

The measurement of the opening and closing of the fatigue crack was performed under the load and strain conditions indicated in Fig. 4, changing the load at equal intervals with M4 screws. After photographing the amount of opening and closing of the fatigue crack tip during the loading and unloading processes with an optical microscope at ×200 magnification, a magnifying lens was used on these photographs to further magnify them ten times, measuring according to the microdisplacement method.

The relationship between load and the amount of opening and closing is approximated by two straight lines, and the load at the fatigue crack opening and closing is defined by the intersection of these lines. Because it is impossible to apply a test load on the loading point due to the fact that a 2 mm-thick test piece, in which the plane stress condition is dominant, is used in this test, M4 screws were used to apply the load.

3. Results and Observations

3.1 Fatigue crack propagation behavior

The relationship between fatigue crack propagation speed \( dL/dN \) and the stress intensity factor range \( \Delta K \) of S15C and SK-5 test pieces, the hardnesses of which differ greatly, when the same repeated load is applied is shown in Fig. 5.

As is shown in Fig. 5, the relationship between
fatigue crack propagation rate \( \frac{dl}{dN} \) and the stress intensity factor range \( \Delta K \) follows the Paris rule as shown in Eq. (1).

\[
\frac{dl}{dN} = C(\Delta K)^n
\]  

(1)

When a comparison is made on the fatigue crack propagation rates of the S15C and SK-5 test pieces, SK-5 has a faster fatigue crack propagation rate than S15C\textsuperscript{12} if the stress intensity factor range is the same. The influence of metallurgical structures, strength and thickness have been statistically analyzed by Masuda et al. based on the literature; they reported that low-temperature tempered steel has a faster fatigue crack propagation rate than ferrite pearlite. The result of the analysis agrees with the result obtained in this study.

In the following study, the fatigue crack test piece as shown by the arrows in Fig. 5 is examined. Both the S15C and SK-5 fatigue crack tips may have been created at the point at which the steady propagation zone was entered; for the S15C, the length of the crack was \( l=2.5 \) mm and the stress intensity factor range \( \Delta K=24 \text{ MPa} \sqrt{m} \), while for the SK-5, the length of the crack was \( l=2.6 \) mm, and the stress intensity factor range was \( \Delta K=24 \text{ MPa} \sqrt{m} \).

3.2 Residual stress distribution in the vicinity of the fatigue crack tip

Figure 6 shows the residual stress line distribution in the vicinity around the fatigue crack tip propagated in the steady propagation zone according to the distance from the crack tip. With both test pieces, the maximum value of compressive residual stress is found at the fatigue crack tip, and the maximum value of tensile residual stress occurs ahead of the crack tip. Residual stress changed rapidly between compressive residual stress at the crack tip and tensile residual stress ahead of the crack tip. Surface distribution of the residual stress, summarized by contour line processing based on the relationship between the measured coordinate position and the residual stress value, is given in Figs. 7 (a) and (b). The residual stress surface distribution for S15C is given in Fig. 7 (a), and that for the SK-5 in Fig. 7 (b). With both test pieces, the compressive residual stress that could have acted to produce the fatigue crack opening and closing behavior was distributed parabolically at the
front and rear, centered on the fatigue crack tip.

A comparison between SK-5, which has a high yield point, and S 15 C, which has a low yield point, shows the compressive residual stress zone in the vicinity of the fatigue crack tip and tensile residual stress zone ahead of the fatigue crack tip of SK-5 to be narrow and both compressive and tensile residual stress values to be high.

By applying the Dugdale-Barenblatt model for fatigue where plastic stretch is formed at the fatigue crack tip and where plastic stretch remains on the fractured surface of the propagated crack, under the hypothesis of plane stress and small-scale yield, the monotonic plastic zone dimension \( W_m \) at the crack tip when the maximum stress intensity factor \( K_{max} \) is applied is obtained using Eq. (2) assuming that \( \sigma_y \) is yield stress for perfectly plastic-elastic material\(^{13}\).

\[
W_m = \pi/8(K_{max}/\sigma_y)^2 \quad (2)
\]

If \( K = K_{max} \) is unloaded to a desired \( K \) value, reverse yield is generated within the monotonic plastic zone, as in a cyclic plastic zone. The cyclic plastic zone dimension \( W_c \) is defined by Eq. (3), considering the fact that the fatigue crack closes at \( K = K_{ci} \), which is obtained before \( K = 0 \text{MPa} \sqrt{m} \). The fatigue crack closing ratio \( K_{ci}/K_{max} \) is expressed by Eq. (4), and if the ratio between the monotonic plastic zone dimension \( W_m \) and cyclic plastic zone \( W_c \) can be determined from the residual stress distribution, the fatigue crack closing ratio \( K_{ci}/K_{max} \) may be obtained.

\[
W_c = \pi/8((K_{max} - K_{ci})/2\sigma_y)^2 \quad (3)
\]

\[
K_{ci}/K_{max} = 1 - 2W_c/W_m \quad (4)
\]

Honda et al.\(^{16}\) confirmed that the applied stress distribution when static tension is applied in the vicinity of a slit-shaped notch tip corresponds exactly to the position where the maximum tensile residual stress value is generated in the unloaded condition, and evaluated the monotonic plastic zone dimension \( W_m \) from the residual stress distribution by applying this fact to the fatigue crack.

In this study, the distance from the fatigue crack tip to the point of the maximum tensile residual stress shown in Fig. 6 is taken as the monotonic plastic zone dimension \( W_m \), and the distance from the fatigue crack tip to the point the maximum compressive residual stress is taken as the cyclic plastic zone dimension \( W_c \). Determination of each plastic zone dimension by approximating residual stress distribution by a parabolic curve allows the fatigue crack closing ratio to be obtained using Eq. (4). Taking SK-5 as an example, the monotonic plastic zone dimension \( W_m \) and the cyclic plastic zone dimension \( W_c \) become 0.3 mm and 0.026 mm, respectively, and the fatigue crack closing ratio is obtained as \( K_{ci}/K_{max} = 0.40 \).

### 3.3 Stress behavior in the vicinity of the fatigue crack tip during the loading and unloading processes

Based on the relationship between load and stress shown in Fig. 4, as with the microdisplacement method, stress in the vicinity of the crack tip was measured, using an M 4 screw, during the loading and unloading processes through the same process cycle as that of fatigue testing. Figure 8 shows the X-ray measurement stress variation in the SK-5 and S15C at the fatigue crack tip and at 0.2 mm behind the crack tip. Examination of the loading process proved that the X-ray measurement stress at the fatigue crack tip changed from compression to tension, regardless of the type of material, and the highest value was observed at the point where the maximum load was applied. It is approximately 900 MPa for SK-5 and 280 MPa for S15C, these values being equivalent to yield stress. In the unloading process, reverse behavior to that in the loading process is observed and, at the fatigue crack tip, the reversed stress condition is observed. On the other hand, the compression stress of X-ray measurement stress at a point 0.2 mm behind

\[\text{Fig. 7}\]
the fatigue crack tip fell rapidly to nearly 0 MPa with the increase in tensile load, remaining at this level thereafter with the increase of tensile load to the maximum. In the unloading process, behavior opposite to that observed during the loading process was observed, as was true with X-ray measurement stress behavior at the fatigue crack tip.

On the diagram demonstrating the relationship between residual stress in the vicinity of the fatigue crack tip and the distance from the crack tip given in Fig. 6, the X-ray measurement stress at maximum load and the stress distribution in the vicinity around the crack tip calculated through the combination of residual stress and X-ray measurement stress for residual stress and at maximum load are drawn in broken and phantom lines (see Figs. 9 and 10).

X-ray measurement stress at the maximum load of the fatigue crack tip is equivalent to the yield stress value for both S15C and SK-5, indicating the yield of the crack tip. Stress distribution in the vicinity of the crack tip under the condition that there is no residual stress at the crack tip, i.e., when local yielding is not generated, is represented by the phantom line which is obtained by overlapping residual stress when there is no load with X-ray measurement stress at maximum load. These indicate a good correspondence with elasticity analysis accomplished separately using the finite-element method.

Thus, due to stress redistribution occurring as a result of locally yielding, under the maximum load condition, the plastic zone expands as much as two times the distance X indicated by the intersection of the yield stress and the elastic stress distribution, represented by the phantom line. On the other hand, an examination of stress distribution during unloading shows that permanent strain remains within the plastic zone during loading, and that even when, during unloading, permanent strain prevents the elastic zone from returning to its original position and compressive stress develops in the immediate surroundings of the fatigue crack tip, this stress results in reyielding.

The above examination proved that the fatigue crack tip is the area specific to fatigue where plastic

![Fig. 8 X-ray measurement stress under loading](image)

![Fig. 9 Distribution of residual stress at the vicinity of the fatigue crack tip (S15C)](image)

![Fig. 10 Distribution of residual stress at the vicinity of the fatigue crack tip (SK-5)](image)
flow occurs repeatedly in the reverse direction due to the loading and unloading processes and that a reverse stress condition is thus created.

Because, in the vicinity of fatigue crack tip, it is considered that X-ray measurement stress behavior corresponds to fatigue crack opening/closing behavior, and under the hypothesis that fatigue crack opening/closing load of $P_{o}$ and $P_{c}$ is given at the load which causes X-ray measurement stress and closing ratio to be obtained as $K_{op}/K_{max} = P_{o}/P_{max}$ and $K_{cl}/K_{max} = P_{c}/P_{max}$, where $P_{max}$ reprints the maximum load. Taking SK-5 as an example, the fatigue crack opening and closing ratio is calculated as $K_{op}/K_{max} = 0.49$ and $K_{cl}/K_{max} = 0.47$.

It has been discussed above that the fatigue crack opening and closing ratio may be obtained from the results of fatigue crack tip X-ray stress measurements. The following discusses the correspondence between the results obtained through the use of X-rays and the previously suggested microdisplacement method. The relationship between displacement and load measured at three positions on an SK-5 test piece, 0.25 mm, 0.15 mm and 0.05 mm from the rear of the fatigue crack tip, is shown in Fig. 11 (a). The distance $X$ from the rear of the fatigue crack tip is the measurement result in Fig. 11 (a) approximated to a straight line, and the relationship between the fatigue crack opening ratio $K_{op}/K_{max}$ and fatigue crack closing ratio $K_{cl}/K_{max}$ obtained by this approximation is given in Fig. 11 (b). $K_{op}/K_{max}$ and $K_{cl}/K_{max}$ both show a tendency to decrease from the front of the fatigue crack tip toward the rear.

Taking work hardening and softening characteristics of the material into consideration using Eq. (5) [14], fatigue crack opening and closing ratios $K_{op}/K_{max}$ and $K_{cl}/K_{max}$ were obtained using the Budiansky-Hutchinson theorem.

$$\sigma_x = 0.772\sigma_y + 143(\text{MPa})$$

(5)

where $\sigma_y$ is yield stress during tensile testing, and $\sigma_x$ is yield stress in the plastic zone.

The fatigue crack closing ratio calculated from the monotonic plastic zone dimensions and the cyclic plastic zone dimensions, obtained from the residual stress distribution discussed above, results of fatigue crack tip X-ray stress measurements taken over one cycle of the fatigue test, and fatigue crack opening and closing ratios obtained using the microdisplacement method and the Budiansky-Hutchinson theorem are summarized in Table 4.

As may be seen from the table, fatigue crack opening and closing ratios determined using any of the methods resulted in nearly identical values.

<table>
<thead>
<tr>
<th>Table 4</th>
<th>The results of $K_{op}/K_{max}$, $K_{cl}/K_{max}$</th>
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<tbody>
<tr>
<td></td>
<td>$K_{op}/K_{max}$</td>
</tr>
<tr>
<td>Budiansky-Hutchinson model</td>
<td>0.52</td>
</tr>
<tr>
<td>Microdisplacement method</td>
<td>0.49</td>
</tr>
<tr>
<td>Plastic zone size method</td>
<td>0.44</td>
</tr>
<tr>
<td>X-ray measurement method</td>
<td>0.40</td>
</tr>
</tbody>
</table>

This fact indicates the plausibility of the fatigue crack closing ratio calculated from the monotonic plastic zone dimensions and the cyclic plastic zone dimensions, obtained from the residual stress in the vicinity of the fatigue crack, as well as the plausibility of the evaluation of the fatigue crack opening and closing ratio calculated from X-ray stress measurements during the loading and unloading processes.

It is thus possible to directly measure stress behavior and residual stress distribution in the localized zone in the vicinity of the fatigue crack tip in a nondestructive manner, and it may be possible to
explain the stress ratio effect of the fatigue crack propagation rate, the fluctuating load effect, and the such effects as these on the area of residual stress of welding.

It is further possible to directly measure, for example, the contact area and contact stress, at the fatigue crack tip and rear in the direction vertical to the fractured surface in the X-ray fractography techniques, which is used to examine the stress condition to discover the cause of fracture by radiating X-rays to the fractured surface. Thus, it will be possible to closely analyze the mechanisms of the residual stress and plastic zone which are obtained by X-ray fractography technique.

4. Conclusions

Using microzone microbeam stress measurement equipment, X-ray stress measurement was carried out on S 15 C and SK-5, having different hardnesses. For residual stress distribution in the vicinity of and behind fatigue crack tips, and for stress in the vicinity of the fatigue crack during loading and unloading processes, the stress condition of the fatigue crack tip was analyzed. A comparative examination was also made regarding the fatigue crack opening and closing behavior based on the stress condition with the microdisplacement method and the Budiansky-Hutchinson model; the following results were obtained.

1. Residual stress in the vicinity of the fatigue crack in the steady crack propagation zone, regardless of the type of material, displayed compressive stress equivalent to the yield stress at the fatigue crack tip, and tensile residual stress persisted ahead of the fatigue crack.

2. It was proven, from the residual stress distribution at the fatigue crack tip and the results of X-ray stress measurements performed on fatigue crack tips at the same maximum load as in the fatigue testing, that reverse stress existed in the fatigue crack tip.

3. Fatigue crack opening and closing ratios obtained from fatigue crack tip X-ray measurement stress during the loading and unloading processes, the fatigue crack closing ratio calculated from the monotonic plastic zone dimensions and the cyclic plastic zone dimensions obtained from the residual stress distribution in the no-load condition, yielded the same results as the fatigue crack opening and closing ratios obtained from the microdisplacement method. It was clarified that the X-ray measurement and evaluation methods for determining fatigue crack opening and closing behavior are effective.

References

(10) Annual Book of ASTM Standards.