Prediction of Propagation and Nonpropagation of Short Fatigue Cracks at Notches Under Mean Stresses*

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A resistance-curve method was proposed for predicting the growth threshold of short fatigue cracks emanating from the notch root under an arbitrary stress ratio. The resistance-curve was determined from the experimental result obtained from the fatigue tests of single-edge-notched plates of low-carbon steel under several stress ratios. The relation between the crack opening stress intensity factor at threshold, $K_{op\ th}$, and the nonpropagating crack length, c_{np} , was approximated by

$$K_{op\ th} = K_{op\ th\infty} \left[\frac{c_{np} - c_1}{c_2 - c_1} \right]^{1/2} \quad c_1 \leq c_{np} \leq c_2$$

$$K_{op\ th} = K_{op\ th\infty} \quad c_2 \leq c_{np},$$

where $K_{op\ th\infty}$ was the value for long cracks, c_2 was a linear function of $K_{op\ th\infty}$, and c_1 was constant. Under compressive mean stresses, the effect of notch plasticity on crack closure was superposed on the above-mentioned closure. Good agreement between prediction and experiment was obtained for the fatigue limit for fracture and the nonpropagating crack length. The fatigue limit for crack initiation of a Stage II crack was almost constant and equal to the value predicted for the case of $R = -1$.

Key Words: Fatigue, Crack Closure, Short Crack, Resistance Curve, Notch Effect, Mean Stress Effect, Fatigue Limit

1. Introduction

The effect of stress ratio on crack propagation behavior is significant, especially near the threshold. While several empirical equations correlating the stress intensity factor (SIF) range at the threshold and the stress ratio have been proposed for long fatigue cracks^{(1)~(3)}, few reports have dealt with the effect of the stress ratio on the near-threshold propagation of short cracks emanating from a notch⁽⁴⁾⁽⁵⁾.

An increasing number of studies show that the development of crack closure with crack growth plays a key role in the propagation of short fatigue cracks at notches⁽⁴⁾⁽⁶⁾⁽⁷⁾. The mechanism of crack closure of short fatigue cracks at notches can be divided into two aspects: the crack closure due to notch plasticity

which is peculiar to the notched component and the crack closure due to crack itself. In the first stage of our study⁽⁸⁾, a semi-empirical equation for the crack closure due to the crack itself as a function of the crack length was combined with the micromechanical criterion of the nonpropagation of a crack; then a predictive model of the fatigue threshold was derived. However, the concept of an intrinsic microcrack used in the model did not have any clear physical meaning; furthermore, the predicted fatigue limit for fracture was lower than the experimental data. Thus in our previous reports, a resistance-curve method was proposed to clarify the precise relationship between crack closure and crack length⁽⁹⁾⁽¹⁰⁾. The resistance-curve method has given a very accurate prediction of the propagation threshold of a notch-tip crack under the stress ratio $R = -1$. Under a high compressive mean stress, the effect of notch plasticity becomes significant. To account for this effect, we proposed a notch-plasticity model in a separate paper; good agreement was obtained between experiment and prediction⁽¹¹⁾⁽¹²⁾.

In the present study, single-edge-notched plates were fatigued under in-plane cyclic bending at several

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stress ratios. The crack propagation behavior was observed and the crack closure was measured. By combining the two above-mentioned models, the method is extended to predict the fatigue threshold under an arbitrary stress ratio. The predictions are compared with the experimental results.

2. Model for Fatigue Threshold

2.1 Resistance-curve method

A resistance-curve method was proposed in our previous papers⁽⁹⁾⁽¹⁰⁾. The model is briefly described below. In Fig. 1, the resistance curve is indicated by the solid line. The maximum SIF, $K_{\max}$, is adopted to indicate the resistance of crack propagation, and is given by

$$K_{\max \text{ th}} = K_{\text{op th}} + \Delta K_{\text{eff th}}, \quad (1)$$

where $K_{\text{op th}}$ is the crack opening SIF at the threshold and $\Delta K_{\text{eff th}}$ is the effective component of the SIF range. $\Delta K_{\text{eff th}}$ is constant and equal to the value for long cracks, $\Delta K_{\text{eff th} \infty}$ ⁽⁷⁾. The crack closure is developed from c_1 which corresponds to the length of a Stage I crack, and approaches the value for long cracks, $K_{\text{op th} \infty}$, at c_2 . In the figure, the broken lines indicate the change of $K_{\max}$ calculated for several applied stresses. A crack can be formed when the stress is larger than σ_{w1} . It continues to propagate when the stress is larger than σ_{w2} . A nonpropagating crack can be formed for stresses between σ_{w1} and σ_{w2} . σ_{w1} and σ_{w2} are the fatigue limits of crack initiation and of fracture.

The resistance-curve method can be extended to predict the fatigue threshold at an arbitrary stress ratio once the relationship between the opening stress intensity factor and the crack length at the threshold is established. Our model for the crack closure due to the crack itself is described first.

2.2 Crack closure due to the crack itself

In our previous studies⁽⁹⁾⁽¹⁰⁾, the relation of $K_{\text{op th}}$ to the nonpropagating crack length c_{np} was given by

the following equation for the case of $R = -1$.

$$K_{\text{op th}} = K_{\text{op th} \infty} \left[\frac{c_{np} - c_1}{c_2 - c_1} \right]^{1/2}, \quad c_1 \leq c_{np} \leq c_2 \quad (2 \cdot a)$$

$$K_{\text{op th}} = K_{\text{op th} \infty}, \quad c_2 \leq c_{np}, \quad (2 \cdot b)$$

where $K_{\text{op th} \infty}$ is the crack opening SIF at the threshold for long cracks, and c_1 and c_2 are determined experimentally. The maximum SIF at the threshold is calculated by Eq. (1). The corresponding threshold stress amplitude is given by

$$\sigma_{\text{th}} = (1 - R) K_{\max \text{ th}} / [2\sqrt{\pi c_{np}} F(c_{np})], \quad (3)$$

where $F(c_{np})$ is the geometrical correction factor. In this method, the material properties required are $K_{\max \text{ th} \infty}$, $\Delta K_{\text{eff th} \infty}$, c_1 and c_2 . Those properties are determined experimentally.

2.3 Crack closure due to notch plasticity

Crack closure due to notch plasticity may be predicted as follows⁽¹¹⁾⁽¹²⁾. Fig. 2(a) shows the stress distribution ahead of a notch before crack initiation. The plastic zone, ω , spreads out from a notch root under the maximum compressive stress, and the stress in the plastic zone equals the negative of the yield strength, $-\sigma_y$. When the stress is reversed, reversed yielding takes place and grows to ω^* under the maximum stress. The stress in this zone is σ_y . Figure 1(b) shows the situation after crack formation. It is assumed that the crack closes completely under the minimum applied stress, $\sigma_{\min}$, and that the stress distribution at $\sigma_{\min}$ is the same as that before crack initiation. By denoting the stress ahead of a notch at $\sigma_{\min}$ by $\sigma_{\min}^*$, the SIF is given by

$$K_{\min}^* = \int_t^{t+c} \sigma_{\min}^* M dx, \quad (4)$$

where t is notch depth, c is crack length, and M is a weight function. When the applied stress rises from $\sigma_{\min}$, the SIF can be obtained by adding the increment due to the applied stress to $K_{\min}^*$ as follows:

$$K^* = (K - K_{\min}) + K_{\min}^*. \quad (5)$$

The crack opening SIF is obtained by substituting K^*

Fig. 1 Resistance-curve method

Fig. 2 Notch plasticity model

=0 in Eq. (5).

$$K_{op} = K_{min} - K_{min}^* \quad (6)$$

The corresponding crack opening stress, σ_{op} , is easily determined from K_{op} . The stress σ_{min}^* ahead of the notch at the minimum applied stress is calculated by using the elastic-plastic finite-element method. When the crack propagates beyond the boundary of the plastic zone, the effect of the notch plasticity vanishes completely; the crack closure due to the crack itself operates in isolation.

2.4 Superposed model

The crack opening SIF at the threshold due to the crack itself is denoted by $K_{op\ th1}$ and that due to notch plasticity by $K_{op\ th2}$. It is assumed that the value of $K_{op\ th}$ is given by

$$K_{op\ th} = K_{op\ th1} + K_{op\ th2} \quad (7)$$

when those two types of crack closure exist simultaneously.

3. Experimental Procedure

3.1 Material and specimen

The experimental material is a low-carbon steel (JIS SM 41 B) with the following chemical composition in weight percent: 0.17 C, 0.19 Si, 0.79 Mn, 0.016 P, 0.020 S. The yield stress of the material is 194 MPa, the tensile strength is 423 MPa, and the fatigue limit of the smooth specimen is 163 MPa. The specimen is a single-edge-notched plate whose width and thickness are 20 mm and 6 mm, respectively. The notch has a length of 3 mm and a root-radius of 0.16 mm.

After machining, specimens were annealed at 1200°C for 5 h in vacuum and cooled in the furnace. After having been notched by an electro-discharge machine, the specimens were heat treated again at 650°C for 90 min to relieve the residual stress. The material microstructure after the heat treatment was ferritic and pearlitic with an average ferrite grain size of 64 μm . The surface of all specimens was polished by emery paper to remove the decarburized layer, and was finally finished by electropolishing. The elastic stress concentration factor of the specimen calculated by the finite-element method is 6.887 under in-plane bending.

3.2 Fatigue tests

Single-edge-notched specimens were subjected to cyclic in-plane bending in a computer-controlled servo-hydraulic testing machine. The stress ratios selected were $R = -2$, -0.5 , and 0.1 . The load frequency was 30 Hz.

3.3 Measurement of crack length and crack closure

The fatigue test was interrupted at a certain number of stress cycles and the crack length was measured on the specimen surface with a traveling

microscope at $\times 200$ magnification. The crack length was taken as the average of the crack lengths on both surfaces. All the specimens were broken at liquid nitrogen temperature after fatigue tests. The length of a nonpropagating crack was measured with a scanning electron microscope. The length of a through-thickness crack was taken as the average of nine measurements over the specimen thickness.

The crack opening point of a short fatigue crack at the notch was measured by the compliance method⁽¹³⁾ under $R = -2$ and -0.5 . The displacement parameter adopted was the output of the strain gage glued on the front or back face of the notch.

3.4 Calculation of stress intensity factor and stress distribution

The geometrical correction factor of the SIF used was the same as used in our previous study⁽¹²⁾. The weight function M in Eq. (5) for a single-edge-cracked plate⁽¹⁴⁾ was used by regarding the notch as a crack and ignoring the effect of the notch root radius. The stress distribution near the notch was calculated by using the elastic-plastic finite-element method. The program used was the 138 Y 6/EPFEMV file of the Kyoto University Program Library, and the computer was a FACOM VP 200 in the Data Processing Center, Kyoto University.

4. Experimental Results

4.1 Crack propagation behavior

The crack propagation curve is presented in Fig. 3, where (a) and (b) indicate the results under $R = -2$ and -0.5 , respectively. For the case of $R = -2$, the specimen broke under $\sigma_a = 90$ MPa and a nonpropagating part-through crack of Stage I was formed under $\sigma_a = 40$ MPa. For the case of $R = -0.5$, the specimen broke under $\sigma_a = 60$ MPa and no nonpropagating crack was observed under $\sigma_a = 30$ MPa. The length of nonpropagating cracks increases with increasing stress amplitude under each stress ratio. When compared at the same stress amplitude, the length of nonpropagating cracks increases with increasing stress ratio. For the case of $R = 0.1$, the crack propagation behavior was not examined. The specimen broke under $\sigma_a = 50$, 65, and 80 MPa, and a nonpropagating part-through crack of Stage I was formed under $\sigma_a = 35$ and 38 MPa. No crack was formed under $\sigma_a = 30$ MPa.

The change in the crack propagation rate, dc/dN , with the crack length, c , is shown in Fig. 4. When compared at the same crack length, dc/dN increases with increasing σ_a or R . For the case of $R = -2$, the solid lines in Fig. 4(a) indicate the relationship predicted by the notch-plasticity model (see section 2.3). Most of the data lie below the solid lines; the

prediction gives an upper bound of the crack propagation rate. This is because the crack opening stress prediction is based only on the effect of notch plasticity. For more precise prediction, the effect of crack closure due to the crack itself must be taken into account simultaneously.

4.2 Crack closure behavior

The crack opening stress σ_{op} is plotted against the crack length in Fig. 5 for the case of $R=-2$. For simplicity, only the results of the cases of $\sigma_a=60$ and 80 MPa are plotted in the figure. When compared at the same crack length, σ_{op} decreases with increasing σ_a , namely with decreasing σ_{min} . The solid lines in the figure indicate the values predicted from the notch plasticity model. The predicted values are lower than the experimental results as described above. The relationship between dc/dN and the effective stress intensity range, ΔK_{eff} , is shown in Fig. 6. The solid line indicates the relationship obtained for long cracks in center-cracked plates under $R=-1$. A unique rela-

Fig. 3 Crack propagation curve

Fig. 4 Change of crack propagation rate with crack length

Fig. 5 Change of crack opening stress with crack length

tionship can be seen independent of the stress ratio or stress amplitude, and it agrees well with that for long cracks.

5. Discussion

5.1 Effect of stress ratio on crack closure due to the crack itself

On the basis of the experimental results, the resistance curve due to the crack itself can be constructed at an arbitrary stress ratio. The threshold SIF range for long cracks, $\Delta K_{th\infty}$ (MPa), is shown as a function of the stress ratio in Fig. 7, where a straight-line relationship is obtained⁽³⁾. The relation

Fig. 6 Relationship between crack propagation rate and effective stress intensity range

Fig. 7 Relationship between threshold stress intensity range and stress ratio for long cracks

between $\Delta K_{th\infty}$ and R can be approximated by

$$\Delta K_{th\infty} = -4.35R + 8.16. \quad (8)$$

Since $\Delta K_{eff th}$ is a constant independent of stress ratio, $K_{op th\infty}$ is obtained under a given value of the stress ratios by

$$K_{op th\infty} = \Delta K_{th\infty} / (1 - R) - \Delta K_{eff th\infty}. \quad (9)$$

The relationship between the crack opening SIF at the threshold and the nonpropagating crack length can be approximated by Eq. (2) and is shown in Fig. 8. The solid line in the figure indicates the relationship obtained for the case of $R = -1$. The length of c_2 is determined through a least-squares regression method by assuming a constant c_1 irrespective of the stress ratio. The c_1 value adopted is 0.057 mm obtained for $R = -1$. For the case of $R = -2$, data at the crack length larger than 0.21 mm are acceptable for calculation because the effect of notch plasticity is operating when $c < 0.21$ mm, as will be described later. For the case of $R = 0.1$, the crack opening stress was not measured. $K_{op th}$ is obtained by subtracting $\Delta K_{eff th\infty}$ from $K_{max th}$ which is calculated from the applied stress and the nonpropagating crack length. Although the data are scarce, it can be seen that c_2 (mm) decreases with increasing $K_{op th\infty}$ (MPa√m). The following linear relationship can be obtained:

$$c_2 = -0.304 K_{op th\infty} + 2.281. \quad (10)$$

For an arbitrary stress ratio, $K_{op th\infty}$ and c_2 can be calculated from Eqs. (9) and (10), so the $K_{op th}$ -vs- c_{np} relationship can be determined.

5.2 Crack closure due to notch plasticity

When the compressive mean stress increases, compressive plastic deformation takes place at the notch root and influences the crack closure. The $K_{op th}$ -vs- c_{np} relationship due to notch plasticity is discussed on the basis of our notch plasticity model.

Fig. 8 Change of crack opening stress intensity factor with crack length at threshold

The plastic zone size, ω (mm), calculated by using the elastic-plastic finite-element method, is plotted against the square of the minimum applied stress, $\sigma_{\min}$, (MPa) in Fig. 9(a). The following relation can be obtained.

$$\omega = 3.66 \times 10^{-5} \sigma_{\min}^2 - 0.030 \quad (11)$$

Figure 9(b) indicates the relationship between the crack opening stress σ_{op} and the crack length where the ordinate is $\sigma_{op}/|\sigma_{\min}|$ and the abscissa is $(c/\omega)^{1/2}$. When compared at the same crack length, $\sigma_{op}/|\sigma_{\min}|$ increases with the increasing absolute value of the minimum stress. This tendency is the same as that described in our previous study⁽¹¹⁾. This relationship is approximated by the following simple equation.

$$\sigma_{op}/|\sigma_{\min}| = 0.752 \sqrt{c/\omega} - 0.840 \quad (12)$$

Nishikawa et al.⁽⁴⁾ also obtained a similar relation by using the finite-element simulation.

The change of K_{op} with crack length for the case

(a) Relationship between plastic zone size with minimum applied stress

(b) Change of crack opening stress with crack length

Fig. 9 Finite-element analysis

of $R = -2$ is shown in Fig. 10, together with the changes of $K_{\max}$ and ΔK_{eff} . The SIF values are calculated for crack lengths shorter than the plastic zone size. The maximum SIF, $K_{\max}$, increases with increasing crack length. K_{op} decreases with increasing crack length at short crack lengths, and then K_{op} increases with increasing crack length after taking the minimum value. ΔK_{eff} is calculated from $(K_{\max} - K_{op})$. For the case of $\sigma_{\min} = -46.7$ MPa, ΔK_{eff} is lower than the threshold value for long cracks, $\Delta K_{\text{eff th}}$, at any crack length, indicating the nonpropagation of a fatigue crack. For the case of $\sigma_{\min} = -60$ MPa, ΔK_{eff} equals $\Delta K_{\text{eff th}}$ at $c = 0.105$ mm, and a nonpropagating crack of this length is formed. K_{op} at $c = 0.105$ mm means $K_{op th}$. For the case of $\sigma_{\min} = -80$ MPa, ΔK_{eff} is larger than $\Delta K_{\text{eff th}}$ even at a crack length equal to the plastic zone size. The crack propagates causing eventual fracture.

Similar calculations are conducted under various conditions of stress amplitude and stress ratio. The relationship between $K_{op th}$ and c_{np} obtained is shown in Fig. 11, where the stress ratios are $R = -2, -3.1, -5.5, -22$, and 11 (compressive loading). The relationship is a monotonically increasing function for the case of positive $\sigma_{\max}$ ($R < 0$), and a decreasing function for the case of negative $\sigma_{\max}$ ($R > 0$).

5.3 Resistance curve at arbitrary stress ratio

By adding $K_{op th1}$ to $K_{op th2}$, Fig. 12 is obtained. Experimental data fall on the predicted lines.

Once the resistance curve is determined by the method described above, the relationship between the stress amplitude at the threshold and the nonpropagating crack length can be obtained under a given stress

Fig. 10 Change of crack driving force with crack length

ratio. Figures 13(a), (b), and (c) are the results obtained for $R = -2$, -0.5 , and 0.1 respectively. In these figures, the open circles represent the through cracks. The solid circles and barred marks indicate nonpropagating part-through cracks of Stage I and those of Stage II, respectively. The predicted value of nonpropagating crack lengths and fatigue limits for fracture agree well with the experimental data. While the fatigue limit for crack initiation predicted increases with increasing stress ratio, the experimental data is about 40 MPa irrespective of the stress ratio.

The fatigue limit for crack initiation, σ_{w1} , is given by the condition of the propagation of a crack of length c_1 . The effective component of the crack driving force is constant irrespective of the stress ratio. The crack opening stress is zero when the effect of notch plasticity is absent. The predicted value of σ_{w1} thus decreases with increasing stress ratio. In reality, however, a certain stress amplitude is necessary to

Fig. 11 Change of crack opening stress intensity factor due to notch plasticity with crack length at threshold

Fig. 12 Change of crack opening stress intensity factor with crack length after superposition

(a) $R = -2$

(b) $R = -0.5$

(c) $R = 0.1$

Fig. 13 Limiting curves for nonpropagation of fatigue crack

Table 1 Effect of stress ratio on fatigue threshold

Load ratio R	Fatigue limit for crack initiation		Fatigue limit for fracture	
	Experiment	Prediction	Experiment	Prediction
	σ_{w1} (MPa)		σ_{w2} (MPa)	
0.1	38 - 40	41.3	46 - 50	48.1
-0.5	30 - 50	41.3	55 - 60	60.0
-2	40 - 50	41.3	85 - 90	94.0

initiate a Stage I crack. Frost and Dugdale⁽¹⁵⁾ and Nisitani and Okasaka⁽¹⁶⁾ indicated that σ_{w1} was constant irrespective of mean stress; this means that the fatigue limit for the initiation of a Stage I crack is independent of the stress ratio and is determined by the stress amplitude. In our previous studies⁽⁹⁾⁽¹⁰⁾, center-notched plates and single-edge-notched plates having various notch geometries were fatigued under axial tension-compression and in-plane bending, respectively, with $R = -1$. We found that the predicted value of σ_{w1} agreed very well with the experimental data. In the present experimental conditions, the value of σ_{w1} is predicted as 41.3 MPa under $R = -1$ and this value agrees with the present experimental data, so this predicted value is now adopted for all stress ratios. The fatigue limits predicted from the model and the experimental data are summarized in Table 1.

Figure 14 indicates the relationship between the stress concentration factor and the fatigue limits predicted from the model for a center-notched plate under the mean stresses of $\sigma_m = 0, 17$ and 50 MPa. The fatigue limit for fracture, σ_{w2} , decreases with increasing mean stress; the notch-tip radius at the branch point decreases with increasing σ_m . This tendency agrees with the experimental result reported by Frost and Dugdale⁽¹⁵⁾ and Nisitani and Okasaka⁽¹⁶⁾.

6. Conclusion

(1) The relationship between the propagation rate of short fatigue cracks emanating from a notch and the effective stress intensity range was unique, irrespective of stress ratio, and agreed very well with that for long cracks at $R = -1$.

(2) A resistance-curve is constructed under an arbitrary stress ratio. The crack length of c_1 , from which the crack closure started developing, was constant irrespective of the stress ratio. The crack length of c_2 , at which the crack opening stress intensity factor at the threshold $K_{op th}$ approached the threshold value for long cracks, decreased with increasing stress ratio. The relationship between the crack opening stress intensity factor for long cracks and c_2 was approximated by a linear relationship. The ratio of

Fig. 14 Relationship between fatigue limits and stress concentration factor for center-notched plate

the crack opening stress due to notch plasticity to the minimum applied stress increased linearly with the square root of the ratio of crack length to the plastic zone size. The relationship was unique and independent of the minimum applied stress.

(3) The fatigue limit for the initiation of a Stage II crack was almost constantly independent of the stress ratio and agreed well with the predicted value under $R = -1$. Good agreement between prediction and experiment was obtained for the fatigue limit for fracture and the nonpropagating crack length.

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