Prediction of Strength of Stepped-Lap Bonded Joint with Adhesive Resin Under Tensile Shear Load*

Kiyomi MORI** and Toshio SUGIBAYASHI**

The strength of a stepped-lap bonded joint under tensile shear loading is investigated both analytically and experimentally. The joint used in this study consists of adherends of carbon steel and an adhesive of epoxy resin. The strain and stress distributions were analyzed by applying the finite element method on the assumption of elastic deformation. The stress distributions in butt and lap sections varied remarkably on both edges of the steps. The joint strength was calculated by applying strength laws of three kinds of equation to the stress distributions of corresponding parts in the joint. It was predicted that the strength was smaller at the edge of the butt part than at the lap part in every step corner. The calculated strain distribution and strength values of the stepped-lap joint coincided approximately with the experimental ones. On experimental stress-strain curves of the joint, however, it was observed that the joint was completely broken with the increase of load after the initial cracks occurred at the edges of the butt part.

Key Words: Adhesive Joints, Stepped-Lap Joint, Strength Prediction, Tensile Shear Load, Stress Distribution, Finite Element Method

1. Introduction

Adhesively bonded joints have the following advantages for joining electrical parts or machine elements: (1) light weight and (2) no component strength reduction due to fastener holes. After bonding, the joints with adhesive resins are more accurately formed than compared with welded joints, because the adhesive bonding does not require high-temperature or high-pressure processes. When adhesive joints are used in electrical applications, conductivity or insulation functions may be selected, illustratively the varied application of adhesively bonded joints. The present authors^{(1)~(5)} have already reported investigations into the effect of the following factors on the deformation and strength both analytically and experimentally: (1) adherend material, (2) shape and surface roughness of the adhesion interface, (3) adhesive thickness, (4) bonded length and (5) curing condition. The present paper treats a stepped-lap

joint by which the adhesive bonding can be used for multi-functional applications. The mechanical behavior of the stepped-lap joint is investigated both analytically and experimentally based on the results of the previous studies^{(1)~(5)}. Concerning the stepped-lap joint, several papers^{(6)~(8)} on the stress analysis or the strength design have been presented, but there were few papers on the results of strength prediction carried out systematically both experimentally and analytically.

In the following, the strain and stress distributions of a stepped-lap joint under a tensile shear load are investigated in detail. This joint has a complex shape combined with the lap joints and butt joints, so the finite element configuration is determined by comparing the results of analytical strain values with experimental ones. The stress in the joint is analyzed by applying these configuration and boundary conditions to an FEM model. Then the strength of the joint is discussed to establish a suitable method of strength prediction.

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** Takushoku University, Faculty of Engineering, 815-1 Tate-machi, Hachioji, 193, Japan

2. Strain and Stress Distributions

2.1 Analytical method

The dimensions and coordinates for the joint are shown in Fig. 1. The stepped-lap joint has butt parts and lap parts, so the number of butt parts can be represented by notation N_s as the step number of the joint. The step number N_s of the joint shown in Fig. 1 is 5. The joint length, the length of left or right adherend, the total lap length, and the lap length of each step (same on all steps) are represented by notations l , l_1 , l_2 , and l_3 , respectively. The thickness of the adherends and the thickness of the adhesive in both the butt and the lap parts are represented by notations t_1 and t_2 , respectively. Each step is represented by the notation S_i ($i=1, 2, \dots, 5$ in the downstairs direction) and each butt part by S_{ib} ($i=1, \dots, 5$) and lap part by S_{il} ($i=1, \dots, 4$). The longitudinal and thickness directions correspond respectively to the x and z directions. The position in the x direction and z direction are represented by the non-dimensional notations $\bar{X}$ ($\bar{X}=x/l_3$) and $\bar{Z}$ ($\bar{Z}=z/t_3$), and the total overlap length is represented by the notation $\bar{L}_0$ ($\bar{L}_0=l_2/t_1$).

The stress analysis is conducted with the FEM on the assumption of a plane strain condition. Figure 2 shows the finite element configuration and boundary conditions. The FEM mesh divides the joint triangularly into 4 layers of elements along the interfaces in the adhesive layer. It also divides the joint into 5 layers in the x direction on each step over the whole overlap length l_2 and 30 layers in the x direction over

Fig. 1 Coordinates and dimensions of the joint

Fig. 2 FEM model

the other parts of adherends, and into 10 layers in the z direction over the joint thickness t_1 . The material properties of adherend (carbon steel S45C) and adhesive (epoxy resin) used in this research are given in Table 1.

2.2 Strain distribution

A joint consisting of adherends 25 mm wide, 10 mm thick, an adhesive 0.05 mm thick and overlap length of 50 mm ($\bar{L}_0=5$) is analyzed. Figure 3 represents the strain distributions of ϵ_x , ϵ_z and γ_{xz} along the x axis in the lower adherend at the lap part when an average stress σ_a (σ_a =applied load / cross-sectional area of adherend) of 19.6 MPa occurs in the joint. In Fig. 3, for comparison with experimental results, the calculated strain distributions near the adhesive layer are shown at the positions of $\bar{Z}=4.74, 3.74, 2.74, 1.74, 0.74$ on each step. The calculated strain distributions are not continuous at the points of $\bar{X}=0.0, 1.0, 2.0, 3.0, 4.0$.

The all-strain component is maximum at the left edge of each step, as shown in Fig. 3, and the normal

Table 1 Material properties

	Adherend (Carbon steel)	Adhesive (Epoxy resin)
Young's modulus GPa	206	3.33
Poisson's ratio	0.33	0.34

Fig. 3 Strain distributions in the adherend of the joint (x direction)

strains ε_x and ε_z become smaller with increasing $\bar{X}$. The shear strain γ_{xz} is a negative value at the minimum of $\bar{X}$ (the left edge) on each step, changing to positive with increasing $\bar{X}$, becoming slightly smaller in the center of the step, and varying with $\bar{X}$ from the center to the right edge. The strain distributions show the same tendency at every step.

The strains in the joint, having the same shape as the analytical model under tensile shear load, are measured by the use of a static strain meter, putting strain gauges on the adherend near the adhesive layer. The experimental results under $\sigma_a = 19.6$ MPa are shown in Fig. 3. The analytical results of strain distribution show good agreement with the experimental results. Thus, it is indicated that the boundary conditions used in this analysis are sufficient for the joint in this research.

2.3 Stress distribution

The stress distribution in the joint under average stress σ_a of 19.6 MPa is analyzed by use of the FEM configuration and boundary conditions. The calculated results of σ_x , σ_z and τ_{xz} are shown in Figs. 4 and 5.

Figure 4(a) represents the stress distribution in the x direction through the point of $\bar{Z} = 2.74$ over the whole range of $\bar{X}$, and Fig. 4(b), that in the z direc-

tion through the point of $\bar{X} = 1.1$ over the whole range of $\bar{Z}$. In the x direction, as shown in Fig. 4(a), the stress σ_x in the adhesive layer is smaller than that in the adherends, and the stress σ_z in the adhesive layer is greater than that in the adherends. In the adherends, the compressive stress σ_z near the left adhesion interface ($\bar{X} \approx 2.0$) is greater than that near the right interface, and the tensile stress σ_x near the left interface is greater than that near the right interface. The position of the adhesive layer in Fig. 4(a) ($\bar{X} \approx 2.0$) corresponds to a butt part near the corner. In the z direction, as shown in Fig. 4(b), the normal stresses σ_x and σ_z in the adhesive layer are smaller than those in the adherends and nearly equal to zero. The shear stress τ_{xz} in the adhesive layer is greater than that in the adherends. The position of the adhesive layer in Fig. 4(b) ($\bar{Z} \approx 3.0$) corresponds to a lap part near the corner. Then, in the adhesive layer, it is found that the tensile stresses σ_x and σ_z have high values in the butt part and the shear stress τ_{xz} has a high value in the lap part. The stresses are almost constant over the whole thickness of the adhesive layer in the direction vertical to the adhesive layer. In the adherends near the interface, the changes of stresses are high at the interior corner both in the butt part and in the lap part. This fact suggests that the normal stresses affect the strength of the adhesive layer at the butt parts and the shear stress affects it at the lap parts.

Fig. 4 Stress distributions (parallel to axes)

Figure 5 represents the stress distributions near the interface between the left adherend and the adhesive layer along the stepped adhesive layer. In the adhesive layer, the stress distributions of σ_x and σ_z show the same tendency; i.e., the stress values are high at the butt parts and nearly equal to zero at the lap parts. The shear stress τ_{xz} is small at the butt parts, and its variations are high on both edges of the lap parts. On the other hand, in the adherend, the stress σ_x is great near the interior corners (the left edges) at the lap parts, and it becomes greater as $\bar{X}$ approaches the right edge of each step, reaching a minimum at the butt parts. The stresses σ_x and τ_{xz} are almost zero at the center of the lap parts, and change to negative values at the butt parts. Figure 5 shows the same tendency in the stress distributions on each step, but the tensile stresses become slightly greater at $\bar{Z}=0$ and 5 corresponding to both the edges of the adhesion interface. In the case of a stepped-lap joint, the results of stress analysis as shown in Figs. 4 and 5 indicate that the stress in the adhesive layer at butt parts or in the adherends near the interface at lap

parts affects the joint strength.

3. Strength Prediction of Joints and Experimental Results

3.1 Strength law

The strengths of adhesive bonded joints are predicted by applying the strength laws of the adherend, the adhesive layer and the adhesion interface to the stress distributions of corresponding parts^{(1)~(5)}. Von Mises criterion is applied to both the adherend and the adhesive layer. The adhesion strength criterion is obtained by the results of the combined tensile and torsional load testing on the butt joints of thin-walled tubes.

Von Mises criterion is represented as follows:

$$F_1 = (\sigma_z^2 - \sigma_x \sigma_z + \sigma_z^2 + 3\tau_{xz}^2)^{1/2} / \sigma_{01} = 1 \quad (1)$$

for the adherends, and

$$F_2 = (\sigma_z^2 - \sigma_x \sigma_z + \sigma_z^2 + 3\tau_{xz}^2)^{1/2} / \sigma_{02} = 1 \quad (2)$$

for the adhesive layer, where σ_{01} and σ_{02} denote the yield stresses of the adherend and adhesive material, respectively.

The adhesion strength criterion is represented by

Fig. 5 Stress distributions (along steps)

$$F_3 = |\sigma_z/\sigma_{04}|^m + |\tau_{xz}/\tau_{01}|^m = 1 \quad (3)$$

where σ_z and τ_{xz} respectively denote the tensile and shear stresses on the adhesion interface. For the material in this research, $\sigma_{01}=343$ MPa, $\sigma_{02}=64$ MPa, $\sigma_{04}=31.5$ MPa, $\tau_{01}=30.8$ MPa and $m=8.66$.

3.2 Strength prediction

The critical loads of adherends, adhesive layer and adhesion interface are calculated by applying our strength laws, Eqs. (1), (2) and (3), to the corresponding parts in the joint. The minimum value among the obtained critical loads is determined as the joint strength. Table 2 shows evaluated stresses σ_c in each part of the joint corresponding to the positions illustrated in Fig. 6. The stress σ_c is equal to the critical load divided by the cross-sectional area of the joint. The evaluated strength values of the corresponding parts shown in Fig. 6 are smaller than those of the other parts in the joint. These values, in the parts I_i and A_i ($i=1, \dots, 10$), are shown in Table 2 because a crack must initiate in one of these parts. The strength σ_c at the position of I_2 at the interface is the lowest of that at any part in the joint. The strength σ_c at I_2 near the corner is lower than that at

I_1 corresponding to an edge of the adhesion interface because of stress concentration near the interior corner. The same tendency is observed in the strength of the interface in every step. In the adhesive layer, the strength σ_c at A_1 and A_{10} corresponding to both edges of the adhesive layer is lower than that in the other parts, and becomes higher as the position approaches the center in the direction of the joint thickness. These results of strength prediction imply that an initial crack occurs at the position of the interface I_2 at which the evaluated strength value is lowest. Then the crack may grow into the lap part, and finally, the joint will fracture.

3.3 Experimental results

The changes of strains are shown in Fig. 7 when the tensile shear load acts progressively on the same-shaped joint as an analytical model. The strain values, at points from 1 to 7 illustrated in Fig. 6, are measured by placing strain gauges on the joint surface until the joint is completely broken. In Fig. 7, the ordinate and abscissa show the average stress σ_a and the strain ϵ_x , respectively. With increased applied load, the strain values at the positions 1, 5 and 7 start decreasing abruptly as stress σ_a reaches about 30 MPa. With a greater load increase, the relation between σ_a and ϵ_x varies slightly at position 2; then the joint finally fractures when $\sigma_a=86$ MPa. From this fact, it is presumed that cracks initiate simultaneously at positions 1, 5 and 7, and that they propagate with increased applied load, resulting in final fracture.

3.4 Comparison between calculated and experimental results

To compare the experimental results with the calculated ones, the predicted strength values of corresponding parts are also shown in Fig. 7. Near position 1, the evaluated strength of the adhesive layer is particularly low; hence its value is shown in Fig. 7(a) with strain variation measured until the joint fractures. The experimental initial cracking strength at positions 1 and 5 almost equals the predicted values at I_2 and I_6 . At position 7, a crack initiates as the predicted value not at I_{10} but at I_2 . Thus cracks appear to be initiated simultaneously at these 3 points near the interior corner in the butt parts on the interface. This fact coincides with the findings of the stress distributions in Figs. 4 and 5 for the fracture behavior of the joint.

Figure 8(a) shows a photograph of the joint with gauges for the strain measurement in place. Figure 8(b) shows a fracture surface of the joint. From Fig. 8, it is found that the joint fractures in the interface between an adherend and an adhesive layer.

Table 2 Predicted strength values at each position in Fig. 6

i	Interface I_i σ_c MPa	Adhesive A_i σ_c MPa
1	37.6	84.3
2	29.0	104.0
3	45.0	112.0
4	30.4	115.0
5	46.4	114.2
6	30.7	116.3
7	46.0	113.1
8	30.4	114.8
9	37.1	101.6
10	35.2	83.9

Fig. 6 Positions of strain gauges and parts for strength values in Table 2

(a) At position 1 in
Fig. 6 until joint fractures

(b) At each position in Fig. 6

Fig. 7 Strain variation

(b) Fracture surface
Fig. 8 Fractured joint

4. Conclusions

The strain and stress distributions in the stepped-lap joint are analyzed and investigated in detail to find an adequate method for the prediction of the joint strength under tensile shear load. The tensile shear loading test for a joint of carbon steel adherends and epoxy resin adhesive is carried out. The validity of the analytical results is confirmed by the experiments. The strength of the joint is predicted by substituting the calculated stress values in our adhesion strength criterion. The predicted strength values are compared with experimental results. The conclusions are as follows:

(1) The stress distribution in the adhesive layer is almost constant over the whole thickness, but that in the butt parts is very different from that in the lap parts: in the butt parts near the corner, the tensile stress σ_z is greater and σ_x is smaller than that in the

Fig. 9 Calculated stress values on which Fig. 4 is based

adherend, and the shear stress τ_{xz} is almost zero. On the other hand, in the lap part near the corner, stress τ_{xz} is high and stresses σ_x and σ_z are almost zero.

(2) In adherends near the adhesive layer, the tensile stress σ_x is remarkably high near the interior corner in both butt and lap parts because of stress concentration in the corner edge. Near another corner, the compressive stress σ_z is great in the butt part and negative shear stress τ_{xz} is great in the lap part.

(3) From the results of strength prediction, it is found that the strength values in the adhesion interface of the butt part are small especially at the interior corners near the edges of the adhesive layer.

These results show good agreement with the experimental ones.

For reference, the plot of the calculated stress values in each element of an FEM model, on which Fig.4 is based, is shown in Fig. 9.

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