Deformation of a Shape Memory Alloy Helical Spring*

(Analysis Based on Stress-Strain-Temperature Relation)

Hisaaki TOBUSHI** and Kikuaki TANAKA***

In the design of a shape memory alloy helical spring, a basic method of designing the spring based on the stress-strain-temperature relation of the material is an important problem to be solved. In the current study, the deformation behavior of the shape memory alloy helical spring was analyzed based on stress-strain-temperature relation of the material. The relationship between load and deflection of the spring as well as the behavior of transformed region is discussed. The main results are summarized as follows. (1) The relation between the load and deflection of the spring is represented by using a simple model of the stress-strain-temperature relation. (2) For a certain maximum deflection, recoverable force and recoverable strain energy increase with temperature. Dissipated strain energy takes a maximum value at a certain temperature. (3) The transformed region in the cross section of the wire expands into the center from the surface under the loading process, but contracts toward the surface under the unloading process. For a certain maximum deflection, the transformed region is small but the load increases as the temperature increases.

Key Words: Inelasticity, Nonferrous Metal, Shape Memory Alloy, Stress-Induced Martensitic Transformation, Helical Spring, Deflection, Pseudoelasticity

1. Introduction

In the application of the shape memory alloy (SMA) elements, SMA helical springs have been used in many cases because a large working stroke can be obtained easily. In the design of a SMA helical spring, it is necessary to obtain knowledge of the deformation behavior of SMA. Nevertheless, because the influence of the progress of stress-induced martensitic (SIM) transformation and reverse transformation on the deformation behavior of SMA is significant if the temperature varies and stress is applied, the stress-strain-temperature relation of the material is very complex^{(1),(2)}. For this reason, the main methods which have been used for convenience in the design of a SMA helical spring are as follows^{(1)~(4)}: (1) the apparent shear modulus of elasticity is used, and (2) the load-deflection relation of the spring obtained

experimentally is used. In the design of a SMA element, a deformation analysis and a basic method of designing the SMA helical spring based on the stress-strain-temperature relation of the material are important problems to be solved^{(3),(4)}. Sato et al.⁽⁵⁾, Tanaka⁽⁶⁾ and Tanaka et al.^{(7)~(9)} have proposed the stress-strain-temperature relation of SMA and have ascertained the propriety of the relation. If the relation is applied for the problem mentioned above, a basic method of designing the SMA helical spring based on the thermomechanical properties of the material can be developed.

In this paper, by using the stress-strain-temperature relation which has been proposed, the deformation property of the SMA helical spring will be analyzed. Based on this analysis, the load-deflection relation, the dependence of the relation on temperature, the recovery force under constraint deformation, and the recoverable strain energy and the dissipated strain energy occurring in the loading-unloading process will be evaluated.

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** Aichi Institute of Technology, 1247 Yachigusa, Yagusa-cho, Toyota 470-03, Japan

*** Tokyo Metropolitan Institute of Technology, 6-6 Asahigaoka, Hino, Tokyo 191, Japan

2. Basic Equations

2.1 Conditions of equilibrium

Concerning the conditions of static equilibrium of the helical spring, we now consider only the equilibrium of the torsional moment. Thus, there is a pure torsional shear stress τ by the torque M_t in the cross sections of the wire of the spring. The equation of equilibrium is

$$M_t = PD/2$$

$$= \int_A \tau r dA = 2\pi \int_0^a \tau r^2 dr, \quad (1)$$

where P , D , a , A and τ represent the load, the mean spring diameter, the wire radius, the area of cross section and the torsional stress at the radius r in the cross section of the wire, respectively.

2.2 Stress-strain-temperature relation

Let us consider the relation between the torsional stress τ and the torsional strain γ based on the assumption in section 2.1. For the stress-strain-temperature relation, a simplified model of the equations in the previous papers⁽⁵⁾⁻⁽⁹⁾ will be employed. The equations in the previous papers are

$$\dot{\tau} = G\dot{\gamma} + \theta\dot{T} + \Omega\dot{\xi}, \quad (2)$$

$$\xi = \Xi(\tau, T), \quad (3)$$

where G , θ and $|\Omega/G|$ represent the shear modulus of elasticity, the thermoelastic constant and the transformation strain, respectively. The variable ξ stands for the volume fraction of induced phase and is the function of stress τ and temperature T as found in Eq. (3). When the values of τ and T vary with time, the progress of the transformation is governed by Eq. (3).

In this study, we make the following assumptions for simplicity. (i) We assume that the influence of thermoelasticity is small, that is, $\theta=0$. As shown by the experimental data, this assumption holds good with sufficient accuracy in the range of temperature considered here. Because the maximum value of γ is smaller than $|\Omega/G|$ in this study, the value of Ω is unnecessary. (ii) We assume that the transformation starts and is completed under the constant critical stress. Because the difference between the starting stress and the completing stress of the transformation is slight especially for TiNi alloy, this assumption holds good with sufficient accuracy from the industrial viewpoint.

Let us consider the results obtained from the assumption (ii). Authors have found that the following equations are adequate for transformation kinetics as⁽⁷⁾⁻⁽⁹⁾

$$\xi = 1 - \exp\{b_A c_M (M_s - T) + b_M \tau\}, \quad (4)$$

$$\xi = \exp\{b_A c_A (A_s - T) + b_A \tau\}, \quad (5)$$

where Eq. (4) is employed for SIM transformation

and Eq. (5) for reverse transformation. In both equations, the variable ξ means the volume fraction of martensite. The coefficients b_M , b_A , c_M and c_A are the material constants. The transformation lines which describe the starting and completing conditions read as, for example,

$$\tau = c_M (T - M_s), \quad (6)$$

$$\tau = -2 \ln 10 / b_M + c_M (T - M_s), \quad (7)$$

in the case of SIM transformation, respectively. Similar equations are obtained in the case of reverse transformation. The progress of the transformation is observed in the region between lines (6) and (7) in the plane (τ, T) , and the width of the transformation strip along the ordinate is $-2 \ln 10 / b_M$. The assumption (ii) means that the value of b_M is very large and the transformation lines given by Eqs. (6) and (7) coincide. Thus the starting and completing lines of SIM transformation and reverse transformation lead to

$$\tau = c_M (T - M_s), \quad (6)$$

$$\tau = c_A (T - A_s), \quad (8)$$

respectively. In this case, there is the relation: $M_s = M_f < A_s = A_f$ among the transformation temperatures under no stress.

The stress-strain relation under the constant temperature is obtained as follows. From Eq. (2), the deformation before the start of SIM transformation and after the completion of reverse transformation is governed by

$$\dot{\tau} = G\dot{\gamma}. \quad (9)$$

Though G is the function of temperature and induced phase, we assume that G takes the same value in the loading and unloading processes. The transformation starts when the stress reaches a certain critical value k as follows:

$$\tau = k = \begin{cases} \tau_M : \text{SIM transformation} \\ \tau_A : \text{reverse transformation.} \end{cases} \quad (10)$$

The values of τ_M and τ_A are determined by Eqs. (6) and (8). The schematic $\tau \sim \gamma$ relation under the constant temperature is shown in Fig. 1. Figure 1(a) shows the shape memory effect in the low-temperature range below A_s and Fig. 1(b) shows the pseudoelasticity in the high-temperature range above A_s . It should be noted that the $\tau \sim \gamma$ relation in the loading process is the same as the stress-strain relation for a perfectly elastic-plastic body.

2.3 Deflection-strain relation

Suppose that only the pure torsional stress, which is balanced with the torque caused by the load, acts in the cross section of the wire, as assumed in section 2.1. Thus the deformation in the cross section is only the torsional strain γ . It is assumed that the uniform deformation by torsion appears in the cross section and γ is proportional to the radius. Thus the SIM

transformation and the reverse transformation start at the surface where γ takes a maximum value.

In the untransformed region (elastic region), the torsional stress τ is proportional to the radius r . At the surface ($r=a$), where τ reaches the critical stress k , the transformation starts. The torsional strain at the surface $\gamma_{\max}$ is

$$\gamma_{\max} = a\theta, \quad (11)$$

where θ denotes the angle of twist per unit length for the wire. When the transformed region progresses to the radius r_b , as the torsional stress τ is equal to the critical stress k , the torsional strain γ_b at the boundary of the transformed region shown in Fig. 2 is

$$\gamma_b = r_b\theta = k/G. \quad (12)$$

If the number of active coils is denoted by N , the deflection δ is obtained as ⁽¹⁾⁻⁽³⁾

$$\delta = \int_0^{\pi DN} (D/2)\theta ds = \pi D^2 N \theta / 2$$

Fig. 1 Schematic stress-strain curves under constant temperature

Fig. 2 Torsional strain and torsional stress distribution when the transformed region reaches the boundary radius r_b

$$= \pi D^2 N k / 2 G r_b, \quad (13)$$

where $\int \dots ds$ denotes the integration along the wire. Thus the relation between the deflection δ and the boundary radius of transformed region r_b is given as

$$r_b = \pi D^2 N k / 2 G \delta. \quad (14)$$

2.4 Load-deflection relation

The relation between the load P and the deflection δ is determined by using the equations shown in sections 2.1 through 2.3. The relation between P and δ at the constant temperature is given as follows. In the elastic region where the transformation does not progress, the relation is

$$P = d^4 G \delta / 8 D^3 N, \quad (15)$$

which is the same as a normal spring. When the transformed regions of SIM transformation and reverse transformation progress to the boundary radius r_b , the relation is

$$P = \frac{\pi k d^3}{6 D} \left\{ 1 - \frac{1}{4} \left(\frac{\pi D^2 N k}{d \delta G} \right)^3 \right\}. \quad (16)$$

3. Numerical Results and Discussion

3.1 Deformation properties of spring

Let us analyze the deformation behavior of the SMA helical spring by using the relations obtained in section 2. At first, the load-deflection relation which describes the basic properties of the helical spring will be discussed. Concerning the material constants and the object of analysis, the experimental data in Ref. (3) are references. The values of the slopes of transformation lines C_M and C_A are estimated by the relation between the torsional stress and the torsional strain at the surface of the wire in torsion. The material is TiNi alloy and the values of the material constants used in the analysis are shown in Table 1. The transformation temperatures of the material are $M_s = 288$ K and $A_s = 318$ K.

The dimensions of the helical spring are the wire diameter $d = 1$ mm, the mean spring diameter $D = 9$ mm and the number of active coils $N = 8$. The maximum deflection given to the spring is $\delta_{\max} = 40$ mm.

The relations between the load P and the deflection δ in the loading-unloading process at the constant temperatures are shown in Fig. 3. As found from Fig. 3, the simulated results shown by the dashed curve well explain the overall experimental data shown by the solid curve. In the temperature range $T < A_s = 318$ K, the SIM transformation in the cross

Table 1 Material constants

G (GPa)		C_M MPa/K	C_A MPa/K
$T < A_s$	$T > A_s$		
10	24	4.9	6.3

section of the wire progresses in the loading process, but no reverse transformation starts. Consequently, residual deflection after the perfect unloading appears. In the temperature range $T > A_s$, the reverse transformation progresses in the unloading process, the original configuration recovers and thus the deflection becomes $\delta = 0$. It should be noted that, in every temperature range, the suddenly bent phenomenon (corner) at the transformation starting point which is observed in the $\tau \sim \gamma$ relation as shown in Fig. 1 disappears, and the $P \sim \delta$ relation shows the gently bent curve.

If the spring is heated under the constant maximum deflection $\delta_{\max}$ after the loading process at the constant temperature ($T < A_s$) or heated after unloading, the $P \sim \delta$ relations become as shown in Fig. 4. It is ascertained from the calculation that after a successive loading-unloading process at $T_i = 303 \text{ K} < A_s$, there appears the residual deflection OR . However, the deflection δ diminishes by heating at another temperature above A_s under no load, and thus the spring recovers the original configuration. This behavior is called the shape memory effect.

If the spring is heated at $T_h = 333 \text{ K}$ under the constant $\delta_{\max}$ which is obtained in the loading point L at 303 K , the axial force P_h at point H appears. The difference of forces between P_h and P_i at point L , $P_r = P_h - P_i$, is called the recoverable force which is used as the driving force in the SMA elements. The analysis explains that, if the constraint is removed, the deflection decreases along the load-deflection curve at this temperature and finally the transformed deflection recovers perfectly.

The calculated results between the recoverable

Fig. 3 Load-deflection relation under constant temperature

force P_r from a base load P_i at 303 K and the heating temperature are shown in Fig. 5. It is found that, by heating, the recoverable force (driving force), which is several times larger than the maximum load $P_i = 4 \text{ N}$ in the initial loading process, appears.

In the temperature range $T > A_s$, after the successive loading-unloading process, δ becomes zero and the original configuration recovers, which was ascertained in Figs. 3 and 4. This behavior is called the pseudoelasticity. The schematic $P \sim \delta$ curve in this case is shown in Fig. 6. The area under the unloading curve denotes the recoverable strain energy E_r . Evaluating E_r at $T = 353 \text{ K}$ in Fig. 3, we obtain $E_r = 0.36 \text{ J}$. The comparison of this value with E_r of steel is as follows. Concerning a steel helical spring of the same configuration and size, taking the shear modulus of elasticity $G = 80 \text{ GPa}$ and the load $P = 17.5 \text{ N}$ in Eq. (15), we obtain $\delta = 10.2 \text{ mm}$ and thus $E_r = P \cdot \delta / 2 = 0.089 \text{ J}$. Therefore the recoverable strain energy E_r of a TiNi alloy helical spring is about four times larger than E_r of a steel helical spring. In the case of a SMA helical spring, as δ increases nonlinearly with an increase in P , the value of E_r becomes very large. Therefore the ratio of E_r to that of steel becomes

Fig. 4 Load-deflection relation under the general thermoloading path

Fig. 5 Dependence of recoverable force on heating temperature

larger with an increase in δ .

This reason is further clarified by considering the relation on uniaxial tension as follows. Concerning axial strain under uniaxial tension, the elastic strain of steel is about 0.1%, but the recoverable strain of TiNi alloy is about 6-10%. Thus, because the level of yield stress of steel is almost the same as that of critical stress of TiNi alloy, E_r of SMA under uniaxial tension may be about 120-200 times larger than that of steel. The study on energy storage using this property of SMA has been carried out^{(10),(11)}.

The area surrounded by the hysteresis loop in Fig. 6 denotes the dissipated strain energy E_d which is wasted mainly as the heat during the loading-unloading process. Because the recoverable deformation of the normal metal is the elastic deformation, E_d is nearly equal to zero. This means that the dissipated strain energy of SMA is much larger than that of the normal metal within the elastic range. Thus the properties of SMA as the high damping material are excellent and the application of the property has attracted attention^{(10),(12)}.

The recoverable strain energy E_r and the dissipated strain energy E_d of the material evaluated at each temperature are shown in Fig. 7. E_r increases with temperature T , while E_d takes a maximum value at about 323 K. This explains the experimental fact that, if SMA is used for a vibration-proof material, there is a temperature range where the damping capacity takes a maximum value⁽¹⁰⁾.

3.2 Behavior of transformed region

If the spring is successively loaded and unloaded at the constant temperature above A_s where the pseudoelasticity appears, the torsional stress τ induced in the cross section of the wire varies, as shown schematically in Fig. 8. In the loading process, elastic

Fig. 6 Schematic load-deflection curve showing recoverable strain energy and dissipated strain energy

deformation occurs at first. In the elastic region, the torsional stress τ is proportional to the radius r . If τ reaches the critical stress of SIM transformation $k(=\tau_M)$ at the surface ($r=a$), SIM transformation starts (Fig. 8(a)). The transformation region expands with the load from the surface into the center and reaches the boundary radius of the transformed region r_b at a point of maximum load (Fig. 8(b)). In this state, the structure is the mother phase in $0 \leq r < r_b$ and the mixed structure with martensite phase and mother phase in $r_b \leq r \leq a$. It should be noted that the mixed ratio (fraction) could not be determined. This reason for this is that the fraction is determined not by the strain but by the stress in the transformation kinetics used in this analysis. In the unloading process, at first,

Fig. 7 Dependence of recoverable strain energy and dissipated strain energy on temperature

Fig. 8 Variation of stress distribution ($T > A_s$)

Fig. 9 Relation between load and boundary radius of transformed region

the torsional stress τ decreases in proportion to the radius by the elastic deformation. If τ reaches the critical stress of reverse transformation $k(=\tau_A)$ at the surface, the reverse transformation starts and the ratio of martensite phase in the mixed structure begins to decrease (Fig. 8(c)). The starting position of reverse transformation proceeds into the center with a decrease in load. If the starting position of reverse transformation reaches the boundary radius r_b , the boundary between mother phase and mixed phase (boundary of transformed region) begins to move toward the surface (Fig. 8(d)). Because the reverse transformation is completed at the surface, the transformed region diminishes in this state (Fig. 8(e)). Finally, the original configuration is recovered by the elastic deformation.

Let us show the calculated results. The relation between the load P and the boundary radius r_b which corresponds to the $P \sim \delta$ relation in Fig. 3 is shown in Fig. 9. As mentioned above, in the loading process, the SIM transformation starts at a point S_M on the surface, expands into the center with an increase in P and reaches a point B_M at the maximum load. In the unloading process, the reverse transformation starts at the surface, the starting position of reverse transformation proceeds into the center with a decrease in P , and reaches a point B_A . After that, the transformed region contracts toward the surface and diminishes perfectly at a point F_A on the surface. If the temperature is high, the load which is necessary to move the transformed region increases and the amount of load variation is large. If the temperature is high, the variation of the boundary radius of the transformed region r_b is small, and the transformation occurs only in the layer near the surface; thus the transformed region is narrow. In the temperature range $T < A_s$, no

reverse transformation starts and the martensite phase remains after the perfect unloading.

4. Conclusions

The deformation behavior of the shape memory alloy helical spring was analyzed based on the stress-strain-temperature relation of the material. The relationship between the load and deflection of the spring as well as the behavior of the transformed region was discussed. The main results are summarized as follows.

(1) The relation between the load and deflection of the spring is represented by using a simple model of the stress-strain-temperature relation.

(2) For a certain maximum deflection, recoverable force and recoverable strain energy increase with temperature. Dissipated strain energy takes a maximum value at a certain temperature.

(3) The transformed region in the cross section of the wire expands into the center from the surface under the loading process, but contracts toward the surface under the unloading process. For a certain maximum deflection, the transformed region is small but the load increases as the temperature increases.

Further studies should be pursued for the subjects to establish the equations applicable for the tensile test which is easy to perform as the material test on the wire, to establish the methods which are possible to apply directly for the design of the helical spring and to construct equations considering the cyclic deformation properties of the helical spring⁽¹³⁾.

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