Prediction of the Tension-Softening Relation for Short-Fiber-Reinforced SMC Composites by a Probabilistic Model*

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A new probabilistic model has been proposed to predict the tension-softening relation for short-fiber-reinforced SMC composites by using the measured debond and pull-out lengths of fiber bundles. The relationship between cohesive stress and crack opening displacement, namely the tension-softening relation, is determined statistically by the probabilistic expectation of loads acting on the bridging fiber bundles at a fictitious crack. It is found that the area below the tension-softening curve is nearly equal to the apparent fracture energy of the composite, while the maximum value of the tension-softening curve is somewhat higher than the unnotched tensile strength of the composite. Comparison of the tension-softening relation with that estimated by a technique based on the J -integral is also made. Furthermore, the influences of fiber bundle-matrix interfacial conditions on the tension-softening relation are clarified.

Key Words: Composite Material, Probabilistic Model, Fracture Toughness, Tension-Softening Relation, Debond Length, Pull-Out Length

1. Introduction

With the increasing use of short-fiber-reinforced SMC composites as structural components, the study of the fracture behavior of the SMC composites has received wide attention. Aronsson and Bäcklund⁽¹⁾ have conducted the failure analysis of the composites using a damage zone model. In their model, the damage zone ahead of a crack tip is characterized by the tension-softening relation on the assumption of a localized planar process zone in which tensile stress decays from tensile strength to zero as the fictitious crack opening displacement increases. They have claimed that the method can be utilized to predict failure of the SMC composites with cracks. In their model, however the critical separation distance must be determined to fit the experimental results in the application.

In the fracture of SMC composites by tensile

stress, fiber bundles debond at the interfaces between the fiber bundles and matrix and break just like fibers. It is known that the fracture energy of the SMC composites increases with the increasing debond and pull-out lengths of fiber bundles. That is to say, there is a correlation between the fracture energy of the SMC composites and the debond and pull-out lengths. By taking account of this fact, the tension-softening relation for the SMC composites may be predicted from the debond and pull-out lengths of fiber bundles.

Several probabilistic fracture models^{(2)~(4)} for short-fiber-reinforced composites have been proposed to predict the strength and fracture behavior. However, the tension-softening relation for short-fiber-reinforced SMC composites has not been considered by the probabilistic fracture model which is based on the fractographic information on fiber-matrix debonding and fiber pull-out.

Recently, the authors⁽⁵⁾ have proposed a probabilistic model to estimate the debond lengths and pull-out lengths of fibers in random short-fiber-reinforced SMC composites subjected to monotonously increasing load point displacement. In this paper, an extension of the probabilistic model is made to predict

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the tension-softening relation for short-fiber-reinforced SMC composites. The relationship between cohesive stress and crack opening displacement is determined statistically by the probabilistic expectation of loads acting on the fiber bundles at a fictitious crack. Comparison of the tension-softening relation with that estimated by a technique based on the J -integral which has been developed by Li et al.⁽⁶⁾ is also made. Furthermore, the influences of the fiber bundle-matrix interfacial conditions on the tension-softening relation are clarified.

2. Description of the Failure Process of SMC Composites

The damage zone at a notch tip in the SMC composites is modeled as a fictitious crack ahead of the notch tip, in which the fiber bundles bridge. With applied load increasing, fiber bundle-matrix interfacial debondings initiate at the crack surfaces and tend to grow on both sides of the crack surfaces. With the load further increasing, several bridging fiber bundles break. Then, although the load acting on surviving fiber bundles increases, the cohesive stress acting on the crack surfaces decreases. When almost all bridging fiber bundles have broken, the pull-out process begins and the cohesive stress is mainly caused by the frictional forces on the fiber bundle-matrix interfaces.

3. Tensile Stress Acting on a Fiber Bundle in Debonding Process and Pull-Out Process

In debonding and pull-out processes, frictional forces act on the debonding interfaces of the fiber bundle. The frictional forces are due to normal compressive stress acting on the fiber bundle, which arises from resin shrinkage during fabrication of the SMC composite. When we estimate the compressive stress acting on the fiber bundle, the fiber bundle is assumed to be an ellipsoidal cylinder embedded in an infinite isotropic matrix under uniform shrinkage. In general, there is a difference between the compressive stress acting on the cylinder with a perfect bonding interface and that acting on the cylinder with a sliding interface. However, we neglect the difference to facilitate the analysis because the difference is small. When the fiber bundle is subjected to tensile stress in the longitudinal direction, the fiber bundle contracts in the transverse direction. By taking account of the contraction of the fiber bundle, the compressive stress acting on the fiber bundle in the infinite matrix under uniform shrinkage is given in closed form⁽⁷⁾.

In the debonding process, the equilibrium of frictional forces and tensile stresses acting on an element of the fiber bundle at a distance ξ from the tip of the

debonding region gives (Fig. 1(a))

$$\frac{d\sigma_s(\xi)}{d\xi} = \frac{4S_d}{\pi ab} \mu \bar{\sigma}_n, \quad (1)$$

where $\sigma_s(\xi)$ is the tensile stress acting on the fiber bundle at a distance ξ from the tip of the debonding region, $\bar{\sigma}_n$ the mean value of the compressive stress acting on the fiber bundle, S_d the peripheral length of the elliptical cross section of the fiber bundle, a and b the major and minor axes of the elliptical cross section and, μ the coefficient of friction. It is well known that the coefficient of friction increases with the relative displacement on contact surfaces for the relative displacement to be small and it becomes constant for large relative displacement⁽⁸⁾. On the debonding interfaces, the relative displacement between the fiber bundle and the matrix is so small that the coefficient of friction μ is assumed to increase proportionally with the relative displacement. On the other hand, in the pull-out process, the coefficient of friction μ is assumed to be a constant. As the first approximation, in the debonding process, the relative displacement between the fiber bundle and the matrix is replaced by the elongation of the fiber bundle measured from the tip of the debonding region. Therefore, the coefficient of friction μ is expressed by

$$\mu = \alpha \int_0^\xi \frac{\sigma_s(\xi')}{E_L} d\xi', \quad (2)$$

where E_L is Young's modulus of the fiber bundle in the fiber bundle direction and α is a constant. Substituting Eq.(2) into Eq.(1) and solving the equation under the condition that the debonding occurs when the tensile stress at the position of the tip of the debonding region attains the debond stress σ_d , we can obtain the tensile stress acting on the fiber bundle $\sigma_s(\xi)$.

In the pull-out process, $\sigma_s(\xi)$ and ξ in Eq.(1) are replaced by the tensile stress $\sigma_f(\zeta)$ and the distance ζ

(a) Debonding fiber bundle (b) Pull-out fiber bundle

Fig. 1 Schematic view of a debonding fiber bundle and a pull-out fiber bundle

from the broken location (Fig. 1(b)). By solving the equation under the condition that no tensile stress acts on the fiber bundle at the broken location, i.e., $\sigma_f(0) = 0$, the tensile stress acting on the fiber bundle $\sigma_f(\xi)$ is given.

4. Probabilistic Expectation of Tensile Stress Acting on a Bridging Fiber Bundle

Consider a fiber bundle of length l_f , which is embedded in an infinite isotropic matrix. The strength distribution of the fiber bundle is assumed to be uniform in the longitudinal direction. When the uniform tensile stress is applied to the matrix at infinity parallel to the fiber bundle, a matrix crack extends perpendicular to the load direction across the fiber bundles and fiber bundle bridging occurs, as shown in Fig. 2(a). As the applied stress increases, the interfacial debondings grow on both sides of the crack and the fiber bundle breaks, at last, in the debonding region.

Until the debonding region attains a certain length l the bridging fiber bundle may either survive or be broken in the debonding region. Then, the probabilistic expectation of tensile stress acting on the bridging fiber bundle at the position of the matrix crack σ_h is the sum of the probabilistic expectation of the tensile stress in a surviving fiber bundle σ_{s0} and that in a broken fiber bundle σ_{f0} . Therefore, the probabilistic expectation of the tensile load acting on the fiber bundle at the position of the matrix crack P_h is expressed as

$$P_h = \frac{\pi ab}{4} \sigma_h = \frac{\pi ab}{4} (\sigma_{s0} + \sigma_{f0}). \quad (3)$$

Fig. 2 Schematic view of a bridging fiber bundle

4.1 Probabilistic expectation of tensile stress in a surviving fiber bundle σ_{s0}

When the debonding region extends to a certain length l , the probabilistic expectation of tensile stress in a surviving fiber bundle at the position of the matrix crack σ_{s0} is the product of $\sigma_s(l)$ obtained from Eq. (1) and the survival probability of the fiber bundle $S(l)$, i.e.,

$$\sigma_{s0}(l) = \sigma_s(l) S(l). \quad (4)$$

Let us consider the survival probability of the fiber bundle $S(l)$. When the distance from the position of the matrix crack to the closer tip of the fiber bundle, which is named 'embedded length', is l_e , the survival probability of the fiber bundle $S'(l, l_e)$ is expressed, on the basis of the Weibull weakest link theory, as⁽⁹⁾

$$S'(l, l_e) = \begin{cases} \exp\left[-2 \int_0^l \left\{ \frac{\sigma_s(\xi)}{\sigma_0} \right\}^m d\xi\right] & (l < l_e) \\ 0 & (l \geq l_e), \end{cases} \quad (5)$$

where σ_0 and m are material constants. When the probability of the position of the matrix crack is uniform along the fiber bundle, the survival probability of the fiber bundle $S(l)$ is obtained in the form

$$S(l) = \int_0^{l_f/2} S'(l, l_e) \frac{2}{l_f} dl_e. \quad (6)$$

4.2 Probabilistic expectation of tensile stress in a broken fiber bundle σ_{f0}

In the pull-out process, the tensile stress acting on the fiber bundle σ_f is determined from Eq. (1) as a function of the distance from the broken location. If we define the pull-out displacement by the contraction of the debonding region of the broken fiber bundle, the pull-out displacement u_0 is approximately estimated by using the tensile stress $\sigma_s(\xi)$ as follows:

$$u_0(l) = 2 \int_0^l \frac{\sigma_s(\xi)}{E_L} d\xi. \quad (7)$$

The difference between the pull-out length l_p and the pull-out displacement u_0 of the broken fiber bundle is denoted by l_b , i.e., $l_b = l_p - u_0$ (Fig. 1(b)); then the probabilistic expectation of the tensile stress in a broken fiber bundle at the position of the matrix crack σ_{f0} is given by $\sigma_f(l_b)$ and the probability density of the length l_b . When the probability density of the pull-out length l_p at the debonding length of the surviving fiber bundles is denoted by $f_{pd}(l_p, l)$, the probability density of the length l_b can be expressed by $f_{pd}(l_b + u_0, l)$. By using $f_{pd}(l_b + u_0, l)$, the probabilistic expectation of the tensile stress in a broken fiber bundle at the position of the matrix crack is given by

$$\sigma_{f0}(l) = \int_0^{l-u_0} \sigma_f(l_b) f_{pd}(l_b + u_0, l) dl_b. \quad (8)$$

Let us consider the probability density $f_{pd}(l_p, l)$. When the embedded length is l_e , the probability den-

sity of the pull-out length l_p at the debonding length l of the surviving fiber bundles, $f'_{pd}(l_p, l, l_e)$, is given by⁽⁹⁾

$$f'_{pd}(l_p, l, l_e) = p \begin{cases} 2 \int_{\sigma_a}^{\sigma_s(l-l_p)} \exp\left[-2 \int_0^{l^*} \left\{ \frac{\sigma_s(\xi)}{\sigma_0} \right\}^m d\xi\right] \frac{m}{\sigma_0} \left(\frac{\sigma}{\sigma_0} \right)^{m-1} d\sigma & (l_p \leq l < l_e) \\ 2 \int_{\sigma_a}^{\sigma_s(l_e-l_p)} \exp\left[-2 \int_0^{l^*} \left\{ \frac{\sigma_s(\xi)}{\sigma_0} \right\}^m d\xi\right] \frac{m}{\sigma_0} \left(\frac{\sigma}{\sigma_0} \right)^{m-1} d\sigma \\ + \exp\left[-2 \int_0^{l_e} \left\{ \frac{\sigma_s(\xi)}{\sigma_0} \right\}^m d\xi\right] \delta(l_p - l_e) & (l_p \leq l_e \leq l) \\ 0 & (l_p > l_e \text{ or } l_p > l), \end{cases} \quad (9)$$

where δ is the Dirac delta function and l^* is the debonding length satisfying the following equation :

$$\sigma_s(l^* - l_p) = \sigma. \quad (10)$$

By taking account of the embedded length l_e being less than $l_f/2$, the probability density of the pull-out length l_p at the debonding length l of the surviving fiber bundles is given, similarly to Eq.(6), by

$$f_{pd}(l_p, l) = \int_0^{l_f/2} f'_{pd}(l_p, l, l_e) \frac{2}{l_f} dl_e. \quad (11)$$

5. Relationship between Cohesive Stress and Crack Opening Displacement

Let us consider the opening of a matrix crack extending perpendicular to load direction across the fiber bundles. When the crack opening displacement is u_c , the pull-out displacement of the fiber bundle which makes an angle θ with the load direction, as shown in Fig. 2(b), is approximately given by

$$u_0 = \frac{u_c}{\cos \theta}. \quad (12)$$

If u_c is prescribed the debonding length l , the survival probability $S(l)$ and the probabilistic expectations of the tensile stress in the surviving fiber bundle and in the broken fiber bundle can be evaluated for the fiber bundle which makes an angle θ with the load direction. Since the probabilistic expectation of the tensile load acting on the bridging fiber bundles which make an angle θ with the load direction is expressed in the form of Eq.(3), the probabilistic expectation of the tensile load component in the load direction is given by

$$P_c = \frac{\pi ab}{4} (\sigma_{s0} + \sigma_{f0}) \cos \theta. \quad (13)$$

Therefore, the cohesive stress acting on the crack surfaces σ_c is expressed as

$$\sigma_c = \frac{\pi nab}{2} \int_0^{\pi/2} f_h(\sigma_{s0} + \sigma_{f0}) \cos \theta d\theta, \quad (14)$$

where n is the average number of the bridging fiber bundles per unit area of crack surface and f_h is the probability density of the number of bridging fiber bundles. When the fiber bundles in the SMC composites are in two-dimensional random distribution in the plane, f_h can be expressed in $-\pi/2 \leq \theta \leq \pi/2$ by the

following equation :

$$f_h = \frac{1}{2} \cos \theta. \quad (15)$$

Note that Eq.(14) provides the relationship between cohesive stress and crack opening displacement, namely tension-softening relation.

6. Experimental Procedure and Experimental Results

The material used in the experiments is a random short-fiber-reinforced SMC composite which contains E-glass fibers of 25% by volume in an unsaturated polyester resin matrix. The chopped fiber strands consist of 150 E-glass fibers of 13 μ m in diameter and 50 mm in length. The compact tension specimens were machined from the SMC composite, as shown in Fig. 3. Initial notch length a_0 is 25~30 mm. The notch root radius is less than 0.25 mm.

The fracture toughness tests were carried out with an Instron testing machine at a crosshead speed of 0.2 mm/min. Applied load P and load line displacement u_L were measured. Furthermore, crack opening displacements were measured at a distance of 3 mm from the initial notch tip by LVDT (linear variable displacement transducer) in order to estimate the crack tip displacements u_t . When the fiber bundle-matrix interfaces are debonded, the whitening regions appear in the specimen. After failure of the specimen, the area of the whitening regions was measured on the surface of the specimen, and the mean value of the debonding distance⁽⁵⁾ $\bar{l}_{dt}$ was obtained by dividing the area by the ligament length of the specimen. Also, the area of the silhouette of the pull-out fiber bundles above the initial notch extension line was measured. The mean value of the pull-out distance $\bar{l}_{pt}$ was obtained by dividing the area by the ligament length. The values were $\bar{l}_{dt} = 1.21$ mm and $\bar{l}_{pt} = 0.678$ mm.

Fig. 3 Geometry of compact tension specimen

7. Determination of the Tension-Softening Relation

Many material constants are required to determine the tension-softening relation by means of the method mentioned above. Among them, the debond stress σ_d and the constant α in Eq. (2) are difficult to obtain experimentally. However, σ_d and α can be evaluated easily from $\bar{l}_{dt}$ and $\bar{l}_{pt}$, by means of the method developed by the authors⁽⁵⁾, and we have obtained $\sigma_d=1.14$ GPa and $\alpha=3.59 \times 10^4 \text{ m}^{-1}$. In evaluating σ_d and α the average strength of a single E-glass fiber, of which the length is 250 mm, has been taken as 1.3 GPa*¹. The major and minor axes of the elliptical cross section of the fiber bundles are $a=0.568$ mm and $b=0.0568$ mm, respectively. The values of the other material constants are the same as in Ref (5).

The relationship between cohesive stress and crack opening displacement, namely tension-softening relation, has been determined by using Eq. (14). In numerical calculation, the coefficient of friction in the pull-out process and Young's modulus of the composite have been taken as $\mu=0.25$ and $E_c=9.29$ GPa. The result is shown by the solid line in Fig. 4. The cohesive stress decreases abruptly as the crack opening displacement increases. When the crack opening displacement becomes 0.1 mm, almost all of the bridging fiber bundles break, only friction in the pull-out process causes the cohesive stress.

Fig. 4 Tension-softening relation

*1 The average strength of commercial glass fibers is 1.0~1.5 GPa

8. Comparison of the Tension-Softening Relation Estimated by the Probabilistic Model with that by a Technique Based on the J -Integral

The tension-softening relation estimated by a technique based on the J -integral⁽⁶⁾ has been obtained by using the load P -load line displacement u_L curves and the crack tip displacement u_t for the specimens with different crack lengths. The result is shown by the broken line in Fig. 4. In this figure, the tension-softening relation estimated by the linear model that the cohesive stress decreases linearly with the crack opening displacement is also shown. In the linear model, the maximum value of the cohesive stress is taken so as to equal the unnotched tensile strength of the SMC composite, and the area below the linear line is equal to the area below the tension-softening curve estimated by the technique based on the J -integral. The area below the tension-softening curve coincides with the apparent fracture energy of the composite. It is seen from the figure that the area below the tension-softening curve for the probabilistic model is nearly equal to the apparent fracture energy obtained experimentally. This means that the fracture energy of the SMC composite can be estimated by the probabilistic model. With respect to the value of the tension-softening curve at $u_c=0$, that for the probabilistic model is somewhat higher than the unnotched tensile strength of the composite. This can be understood from the fact that in the probabilistic model, the position of fracture is restricted to the crack tip. In addition, note that the value of the tension-softening curve at $u_c=0$ obtained by the technique based on the J -integral is generally lower than the unnotched tensile strength of the composite⁽⁶⁾.

9. Influences of the Debond Stress σ_d and the Constant α on the Tension-Softening Relation

Influences of the debond stress σ_d and the constant α on the tension-softening relation are examined. First of all, the tension-softening relations are given for the debond stresses $\sigma_d=0.5, 1.0, 1.14, 1.5$ and 2.0 GPa and the same values of the other material constants as in section 7. The result is shown in Fig. 5. The figure shows that the cohesive stress σ_c at $u_c=0$ increases with increasing σ_d . When $\sigma_d \leq 1.0$ GPa, the cohesive stress σ_c tends to increase with the crack opening displacement u_c at the early stage of crack opening. This is attributed to the fact that few fiber bundles break until the debonding region grows to a certain extent, whereas, for $\sigma_d \geq 1.14$ GPa, the cohesive stress σ_c decreases monotonously with increasing u_c . In this case, several fiber bundles break even at the early stage of crack opening, causing the decrease of

Fig. 5 Influence of debond stress σ_d on tension-softening relation

Fig. 6 Influence of constant α on tension-softening relation

the area below the tension-softening curve, i.e., the apparent fracture energy as the debond stress σ_d increases.

Figure 6 shows the influence of the constant α on the tension-softening relation. The area below the tension-softening curve decreases with increasing α , since the tensile stress acting on the fiber bundle in the debonding process increases with increasing α and the fiber bundle breakage occurs.

10. Conclusions

A new probabilistic model has been proposed to predict the tension-softening relation for short-fiber-reinforced SMC composites by using the debond lengths and the pull-out lengths of fiber bundles. The tension-softening relation of a SMC composite has been determined by the probabilistic model. The main results are summarized as follows :

- (1) The cohesive stress of the SMC composite

decreases as the crack opening displacement increases. Finally, almost all of bridging fiber bundles break, and only friction in the pull-out process causes the cohesive stress.

- (2) The area below the tension-softening curve for the probabilistic model is nearly equal to the apparent fracture energy obtained experimentally. This means that the fracture energy of the SMC composite can be estimated by the probabilistic model.

- (3) The value of the tension-softening curve at $u_c=0$ for the probabilistic model is somewhat higher than the unnotched tensile strength of the composite.

- (4) By examining the influences of the debond stress σ_d and the constant α on the tension-softening relation, it is clarified that the apparent fracture energy decreases with increasing σ_d and α .

References

- (1) Aronsson, C. G. and Bäcklund, J., Tensile Fracture of Laminates with Cracks, *J. Comp. Mater.*, Vol. 20, No. 3 (1986), p. 287.
- (2) Fukuda, H. and Chou, T.W., A Probabilistic Theory of the Strength of Short-Fibre Composites with Variable Fibre Length and Orientation, *J. Mater. Sci.*, Vol. 17, No. 4 (1982), p. 1003.
- (3) Wang, Y., Backer, S. and Li, V. C., A Statistical Tensile Model of Fibre Reinforced Cementitious Composites, *Composites*, Vol. 20, No. 3 (1989), p. 265.
- (4) Wells, J. K. and Beaumont, P. W. R., The Toughness of a Composite Containing Short Brittle Fibres, *J. Mater. Sci.*, Vol. 23, No. 4 (1988), p. 1274.
- (5) Suzuki, H. and Sekine, H., Probabilistic Study of Debond Length and Pull-Out Length of Fibers in a Short Fiber Reinforced SMC Composite, *Proc. 14th Comp. Mater. Sympo.*, (in Japanese), (1989), p. 13.
- (6) Li, V. C., Chan, C. M. and Leung, C. K. Y., Experimental Determination of the Tension-Softening Relations for Cementitious Composites, *Cement and Concrete Research*, Vol. 17, No. 3 (1987), p. 441.
- (7) Mura, T. and Mori, T., *Micromechanics*, (in Japanese), (1976), p. 31, Baifukan.
- (8) Rabinowicz, E., *Friction and Wear of Materials*, (1965), p. 54, John Wiley and Sons, Inc.
- (9) Cassenti, B. N., Probabilistic Static Failure of Composite Material, *AIAA J.*, Vol. 22, No. 1 (1984), p. 103.
- (10) Hayashi, T. ed. *Fukugozairyokogaku*, (in Japanese), (1971), p. 85, Nikkagiren.