The Theoretical Prediction of Forming Limit Strains of Sheet Metals in Press-Forming Processes*

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The theoretical prediction of the forming limit (strain) diagram (FLD), or the forming limit strains of sheet metals by the author's method is reviewed, giving some new and useful information. J₂-Gotoh's corner theory; (or MG-) (J2G) constitutive equation previously proposed by the author is used, together with the localized necking condition as the failure condition. Theoretical FLD nets of sheet metals subjected to various proportional loadings are illustrated with the n -value or ρ -value as the parameter for the convenience of determining the most appropriate value of ρ (the material constant involved in the J2G). Furthermore, discussions on ρ - and n -values and on the material properties of the prestrained sheet are given. The secondary-FLD and the sheet thickness effects on the forming limit strains are also discussed.

Key Words: Plasticity, Forming Limit Strain, FLD, Sheet Metal, Vertex-Hardening Theory, J2G, Localized Necking, Press-Forming

1. Introduction

In the field of sheet metal forming processes, fracture (or breakage) phenomena of the material have long been investigated, both experimentally and theoretically, by a great number of researchers (see e. g. Ref. (1)), because these phenomena constitute one of the most primary factors which governs the formability of sheet metal. The forming limit strain of the sheet depends strongly on the material properties and the loading conditions. Therefore, we cannot easily obtain by experiment alone data which are useful for any circumstance, and thus the importance of the theoretical prediction of it has been recently comprehended. The demand for a theory or an approach which can easily be introduced into the numerical simulation of the deformation process (which has been developing very fast these days) and possibly enable it to predict the punch stroke and the place at which the sheet might break, has become

greater as the so-called computer-aided engineering (CAE) prevails.

The author has long been working on the theoretical prediction of the forming limit strains of sheet metals. He believes that his approach is very appropriate for the recent aforementioned demand. He proposed J₂-Gotoh's corner theory of plasticity (J2G), which is a form of the vertex-hardening theory⁽²⁾, that enables us to sharply simulate the evolution of the localized necking, which will induce almost immediate fracture of the sheet (see e. g. Refs. (3), (4)). He recommends introducing the J2G into the numerical simulation of the deformation processes⁽⁵⁾⁻⁽⁷⁾. In that way, we can recognize the forming limit from the evolution of the localized necking ($\approx$ the onset of fracture), which can be found from the appearance of the deformed pattern with severe strain concentration at some portion of the sheet, or from the sharp drop of the punch load⁽⁶⁾. Or, in a common press-forming operation in which the strain distributes more or less uniformly to a certain extent, we can use the localized necking condition to judge the onset of fracture, which was derived analytically from the so-called localized-type bifurcation condition under the assumption of the

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uniform distribution of strains. This is done by introducing the condition into the numerical simulation of the deformation process as a subroutine, in order to check at every point on the deforming sheet whether or not the condition is satisfied and thus whether or not the localized necking occurs^{(8)~(14)}.

The effectiveness of the author's approach mentioned above, of course, must be examined by experiment at least under the basic conditions. The experiments performed so far under various loading conditions give fairly satisfactory results^{(9)~(14)}. However, some problems remain with regard to the approach; e. g., some assumptions used in it offer no clear explanation about their theoretical background.

In this paper, after a brief review on the author's approach, much new data useful for the determination of the necessary material constants are presented and the aforementioned problems and concerns are answered.

2. Fundamental Theory

The J2G constitutive equation is briefly given below (for detail see Refs. (4), (15)):

$$d\mathbf{e} = d\mathbf{e}^e + d\mathbf{e}^p \\ = (1/2G^*) \dot{\mathbf{d}} \mathbf{T} + \langle P(\Theta) \rangle b/2 \bar{\sigma} h_0 \mathbf{T} \bar{d}\bar{\sigma} \quad (1)$$

$$\text{tr} d\mathbf{e} = (1/3K) \text{tr} d\bar{\sigma} \quad (2)$$

where the bold-face letters are the tensors of second rank, tr means the trace operator, $d\mathbf{e}$ is the deviator of the strain increment $d\mathbf{e}$ and the superscripts e and p denote elasticity and plasticity. The symbol $\mathbf{T}$ is the deviator of the Cauchy stress $\boldsymbol{\sigma}$, $\dot{\mathbf{d}} \mathbf{T} = d\mathbf{T} - d\boldsymbol{\omega} \cdot \mathbf{T} + \mathbf{T} \cdot d\boldsymbol{\omega}$, and $d\boldsymbol{\omega}$ is the increment of rigid-body rotation. The equation $1/G^* = 1/G + \langle P \rangle / H_0$, G is the elastic shear modulus, $P(\Theta) = a + b \cdot \cos \Theta$, $\langle P \rangle = P$ for $P > 0$, $\langle P \rangle = 0$ for $P \leq 0$, $b = 1 - a$, and $a = h_0/H_0 = \cos \Theta_0 / (1 + \cos \Theta_0)$. Also, $\cos \Theta = \text{tr}(\mathbf{T} \dot{\mathbf{d}} \mathbf{T}) / [(\text{tr} \mathbf{T}^2)(\text{tr} \dot{\mathbf{d}} \mathbf{T}^2)]^{1/2}$,

$\bar{\sigma} = \sqrt{3/2}(\text{tr} \mathbf{T}^2)^{1/2}$, $\bar{d}\bar{\sigma} = \sqrt{3/2}(\text{tr} \dot{\mathbf{d}} \mathbf{T}^2)^{1/2} \neq d\bar{\sigma}$, $h_0 = (1/3)d\bar{\sigma}/d\epsilon$ for the uniaxial tensile test (for example), and K is the elastic bulk modulus. Incompressibility in plastic deformation is assumed.

The J2G allows the evolution of a vertex at the current stress state on the subsequent loading surface. The symbol Θ_0 is half of the vertex angle, where it is expressed as follows:

$$\Theta_0 = (\pi/2) - \rho \epsilon^2; \epsilon = \int d\epsilon^p. \quad (3)$$

In general, $0 \leq \rho \leq 1$. If $\rho = 0$, then $\Theta_0 = \pi/2$, and the J2G reduces to the J2F (J₂-flow theory) which allows no vertex. In the common deformation processes, the vertex evolves rather slightly within the range of $70^\circ < \Theta_0 < 90^\circ$. However, this slight evolution of the vertex

contributes strongly to the occurrence of the plastic strain localization phenomenon, such as localized necking, (see e. g. Ref. (10)).

On the other hand, the localized necking which induces instantaneous fracture is considered to be a narrow band along which the increments of stresses (or strains) become discontinuous, and thus its occurrence is a type of the localized bifurcation phenomenon. Considering a small element of the sheet with almost uniform strain, on the basis of the J2G, we can lead the fourth-order algebraic equation with respect to the components (g_1, g_2) of the unit normal vector $\mathbf{g}$ to the possible localized necking band by using a procedure similar to that used by Stören and Rice⁽¹⁶⁾ for the derivation of the necking condition, (see e. g. Refs. (9)~(14)). The coefficients involved in the equation are the functions of the values of stress components, the relations between the components of stress increments and so forth, and are thus dependent on the magnitude of deformation and the strain trajectory. The algebraic equation possesses no real roots, therefore such a necking band cannot exist until deformation exceeds a certain level. At the instant the equation acquires any real roots, the localized necking band can exist along the direction whose normal is parallel to the vector $\mathbf{g}$ with the real components (g_1, g_2). Then we can recognize that the sheet will break along the band.

When we can introduce the J2G into the existing computer program for the numerical simulation of sheet metal forming, we can detect the onset of fracture by introducing the algebraic equation into the program as a subroutine by which to check the reality of the roots, or by directly observing the deformation pattern to find the occurrence of localized necking.

3. FLD Under Proportional Loading; Determination of Appropriate Values of n and ρ

When the material is assumed to be isotropic, the main material properties governing the forming limit strain are the ρ -value in the J2G through Eq. (3), and the n -value in the power law ($\sigma = c\epsilon^n$) expressing the strain-hardening property of the sheet. The n -value is usually determined by the uniaxial tensile test. Of course, the power law is just a simple approximation of the actual strain-hardening property. In the common press-forming of sheet metals, the ratio of the in-plane principal strains $\alpha (= \epsilon_2/\epsilon_1)$ is primarily positive (especially at the critical area). Therefore, the strain level at which the n -value is determined is rather low when compared with that at the critical area of the sheet in the actual press-forming, and, moreover, the strain ratio α is different for the two cases. For these reasons, the n -value thus determined is not necessar-

ily relevant to the estimation of the limiting strain in usual press-forming. For example, the limiting strain for $\alpha=0$ (plane strain state) $[(\epsilon_1)_{cr}]_{\alpha=0}$ (denoted by n'), evaluated by the approach described in the previous section, is equal to n , which is the same as in Stören-Rice's theory⁽¹⁶⁾. However, as seen in Table 1, the experimental results sometimes deviate from the theory to some extent. Therefore, it is recommended to determine the n -value at the strain level as high as possible by using an equi-biaxial tensile test such as hydraulic bulging, or to use n' instead of n , because the experimental determination of n' is comparatively easy by performing the tension test of a short and wide test specimen. [Note : It is not the n -value (which expresses the mean strain-hardening property over a certain strain range), but the instantaneous strain-hardening rate $h_0=(1/3)d\sigma/d\epsilon^p$ at the critical position and time that directly governs the occurrence of localized necking (see e. g. Refs. (9), (10) etc.). Therefore, within the scope of the author's approach, it is even more reasonable to use n' instead of n , because n' reflects the actual h_0 .] Of course, as seen in Table 1, there exist many materials with n' close to n . For such materials, the above argument makes no sense, and no such problems remain.

Next, we are concerned with the experimental methods for determination of the appropriate value of ρ . One method is to use the relation $\rho=0.0417n/[x_1^3-0.6667nx_1^2]$, which is derived from the theory described in the previous section, where $x_1=[(\epsilon_1)_{cr}]_{\alpha=1}$

=the limiting strain for $\alpha=1$ (equi-biaxial tension), and $n>0$. From this relation, using the values of n and x_1 obtained experimentally, we can evaluate the value of ρ . In Table 1, the ρ -values thus determined are listed. Figure 1 illustrates, for convenience' sake, the relation between x_1 and ρ with respect to the various values of n . Another method is to use the FLD (forming limit (strain) diagram) nets under the proportional loadings illustrated in Fig. 2 (with the constant values of ρ) and Fig. 3 (with the constant values of n). If we know from experiment the limiting strain for an arbitrary strain ratio α , then by plotting it on these diagrams, we can easily estimate the ρ -value.

From Figs. 1, 2 and 3, it follows that the influence of the variation in the ρ -value on the limiting strain decreases as ρ tends to 1 or as α approaches 0, though the limiting strain increases very sharply as ρ tends to 0 except for $\alpha=0$. The value of ρ for a common dead-soft material is usually greater than 0.3 or more, which means that we do not need to pursue the accuracy of the ρ -value so rigorously, and Table 1 (for example) can be referred to effectively for such materials.

For reference, the equi-equivalent strain diagrams in strain space are illustrated in Fig. 4. This is useful in determining the strain level at an arbitrary point on strain space. Furthermore, this diagram shows that the classical simple criterion of the constant equivalent strain for fracture is very unreliable when compared with Figs. 2 and 3.

Table 1 Typical values of ρ

material	n or n'	ρ
killed-steel	$n = 0.26$	0.5
	$n' = 0.33$	0.85
aluminium - O	$n = 0.22$	0.5
	~ 0.27	~ 0.6
	$n' \sim n$	--
aluminium - 1/2H	$n = 0.05$	0.25
	$n' \sim n$	--
copper - O	$n = 0.45$	0.7
	$n' \sim n$	--
copper - 1/4H	$n = 0.3$	0.4
	$n' \sim n$	--
brass - O	$n \geq 0.45$	—
	$n' = 0.3$	0.3
brass - 1/4H	$n = 0.32$	—
	$n' = 0.28$	0.5
	~ 0.30	—

$$n' = [(\epsilon_1)_{cr}]_{\alpha=0}$$

Fig. 1 Relationship between the forming limit strain for equi-biaxial tension and the ρ -value

4. The Forming Limit Strain Under Nonproportional Loadings

The forming limit strain for arbitrary nonproportional loading, without unloading during deformation, can also be predicted by the theory described in the second section which employs the n -(or n' -) value

and the ρ -value as determined by the method in the above section. A few examples are demonstrated in Figs. 5, (a)~(c)⁽¹⁴⁾. The experimental plots in the figures are obtained by the so-called out-of-plane operation such as pure stretching or deep drawing, with the aid of rigid punches of various shapes. Therefore, the effect of bending is also more or less

Fig. 2 FLD nets under proportional loadings (ρ : constant in each figure ;
 $n=0.50, 0.35, 0.30, \dots, 0.05, 0.01$ for top curve to bottom curve in each figure)

involved. This figure shows that the theoretical data (Δ) coincide well with experimental data ($\bullet$). Note that the curves in Fig. 5 are experimentally determined strain paths which were used for the theoretical prediction of the limiting strains. Therefore, the effect of the possible inaccuracy involved in the smoothed

strain paths is also counted in the theoretical plots. (The notation 'prop. FLD' in the figures means the FLD for proportional loadings.)

5. Forming Limit Strains of Prestrained Sheets

We concern ourselves with the a situation where

Fig. 3 FLD nets under proportional loadings (n : constant in each figure ; $\rho=0.05, 0.1, \dots, 1.0$ for top curve to bottom curve, where ρ is limited ≥ 0.20 in (i))

the sheet is strained under proportional loading, and unloaded once, and then re-strained under another type of loading up to fracture. In such a case, how to estimate the properties of the prestrained sheet becomes questionable. In previous papers by the author^{(11),(13),(17)} and the author et al.⁽¹²⁾, a simple, practical method for such an estimation was applied without introduction of any additional material properties. However, sufficient theoretical background for that method has not been presented so far. It will be provided below.

Figure 6(a) illustrates the σ - ϵ curve for loading $\rightarrow$ unloading $\rightarrow$ (another type of) loading. As shown by the shaded area in this figure, there exists a "transient region" which corresponds to the vertexed portion on the subsequent loading surface illustrated in Fig. 6(b). When the second loading path is different from that of the first, the n -value for the second one ($n^{(2)}$) is considered to vary from the original value of n . The magnitude of that variation is considered to be dependent on the variation θ^* of the direction of T , which is expressed by the use of the third invariant of T , and on the magnitude of the preloading, which is expres-

sed by the equivalent plastic strain ϵ_a at the end of the preloading. Furthermore, it is of no use to consider the variation of n for the case of $\epsilon_a = \epsilon_{cr}^{(1)}$ (=the limiting strain for the first proportional loading) because the sheet breaks due only to the first loading. For these reasons, $n^{(2)}$ is expressed as follows:

$$n^{(2)} = n \times \{1 + \theta^* \epsilon_a (\epsilon_{cr}^{(1)} - \epsilon_a)\}, \quad (4)$$

where

$$\theta^* = \theta_2 - \theta_1,$$

$$\theta_i = \cos^{-1}\{\text{III}_{(i)}^*\}, \quad i=1, 2$$

$$\text{III}^* = \text{III}/(\text{III})_{m=0} = (2-m)(1-2m)(1+m)/2$$

$$m = \sigma_2/\sigma_1 = (1+2\alpha')/(2+\alpha')$$

and α' is the ratio of $d\epsilon_2$ to $d\epsilon_1$.

On the other hand, we assume that the narrow transient part on the subsequent loading surface shaded in Fig. 6(b) will vanish as unloading is applied. Then, as reloading acts, half of the vertex angle θ_0 will change rapidly from $\pi/2$ to θ_0^* during the increase of the equivalent stress from $\sigma^* = \sigma \cdot \sin \theta_0^*$ to σ , as shown in the figure. However, σ^* is close to σ . Therefore, we can finally express the half-vertex angle θ_0 and σ for the second loading as follows:

$$\theta_0^{(2)} = \pi/2 - \rho \cdot (\epsilon + \epsilon_a)^2 \quad (5)$$

$$\sigma^{(2)} = C \cdot (\epsilon + \epsilon_a)^{n^{(2)}}$$

In the previous papers by the author^{(11),(13),(17)} and the

Fig. 4 Equi-equivalent strain diagram

Fig. 6 Schematic illustration of material property for loading-unloading-reloading process

Fig. 5 Examples of the forming limit strain under nonproportional loadings

author et al.⁽¹²⁾, the ideas mentioned above are adopted to give satisfactory results.

6. Dependency of Forming Limit Strain on Sheet Thickness

As shown by the solid curve in Fig. 7, the dependency of the forming limit strain for a constant α on the sheet thickness is assumed to be expressed as follows:

$$[(\epsilon_1)_{cr}]_{\alpha=1} = [(\epsilon_1)_{cr}^*]_{\alpha=1} \times [1 + \xi_1 \{(t/t^*)^2 - 1\} - \xi_2 \{(t/t^*)^4 - 1\}], \quad (6)$$

where α is representatively set equal to 1, t is the thickness, and the asterisk mark * designates the reference values. If we put $\xi_2=0$, Eq. (6) reduces approximately to the relation derived from the condition of the localized-type bifurcation⁽¹⁸⁾ (see also Refs. (12) and (13)). The third term in Eq. (6) is introduced in order to fit the data reported by other investigators⁽¹⁹⁾. The forming limit strain decreases very rapidly as the thickness reduces to beyond t_c (~ 0.1 mm) because of the crystallographic effect⁽²⁰⁾, as shown by the dashed line in the figure.

7. Additional Comments

For the cases such as severe bending and bore-expanding processes where a strong gradient of strains exists in the direction along which the localized necking band will proceed, or for the case where the dependency of the material properties on the strain rate is not negligible⁽²¹⁾, it is recommended to directly find the onset of localized necking from the appearance of the deformed pattern by performing the numerical simulation of deformation on the basis of the J2G (Eq. (1)).

8. Conclusions

Some uncertainties involved in the approach for the theoretical prediction of the forming limit strains of sheet metals, which has been recently proposed by the author, are pointed out and explained clearly. The experimental method to determine the appropriate values of ρ and n , which play important roles in this approach based on the J2G (J_2 -Gotoh's corner theory), and the necessary diagrams (e. g., FLD nets) are presented. The material behavior for a loading-unloading-reloading process relating to the vertex-hardening theory (J2G), the dependency of the limiting strain on the sheet thickness, and the effects of the strain gradient and the strain-rate sensitivity on the limiting strain are also discussed.

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Fig. 7 Schematic illustration of thickness dependence of the forming limit strain

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